Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.1-c-trig-^m-d-trig-^n

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July 17, 2021 Compiled on July 17, 2021 at 10:03pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [254]. This is test number [135].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (254)	% 0.00 (0)
Mathematica	% 99.21 (252)	% 0.79 (2)
Maple	% 84.65 (215)	% 15.35 (39)
Maxima	% 62.60 (159)	% 37.40 (95)
Fricas	% 82.28 (209)	% 17.72 (45)
Sympy	% 21.26 (54)	% 78.74 (200)
Giac	% 54.72 (139)	% 45.28 (115)
Mupad	% 66.54 (169)	% 33.46 (85)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

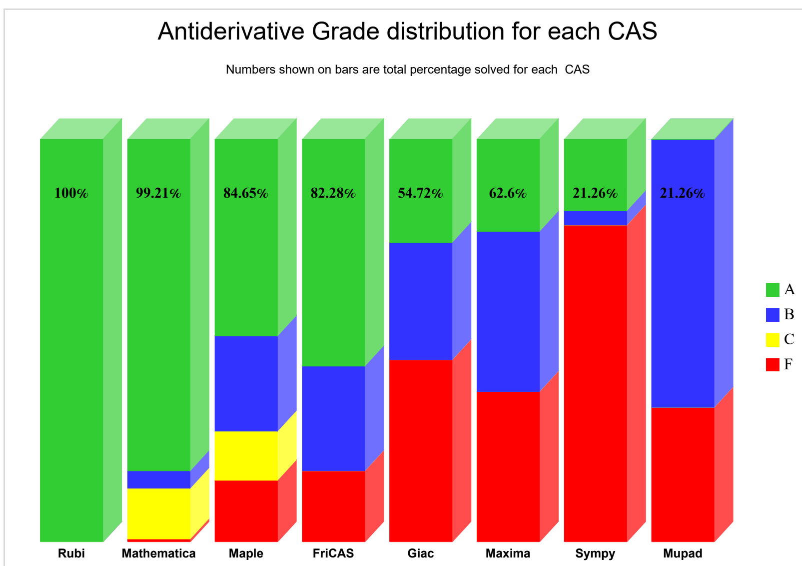
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

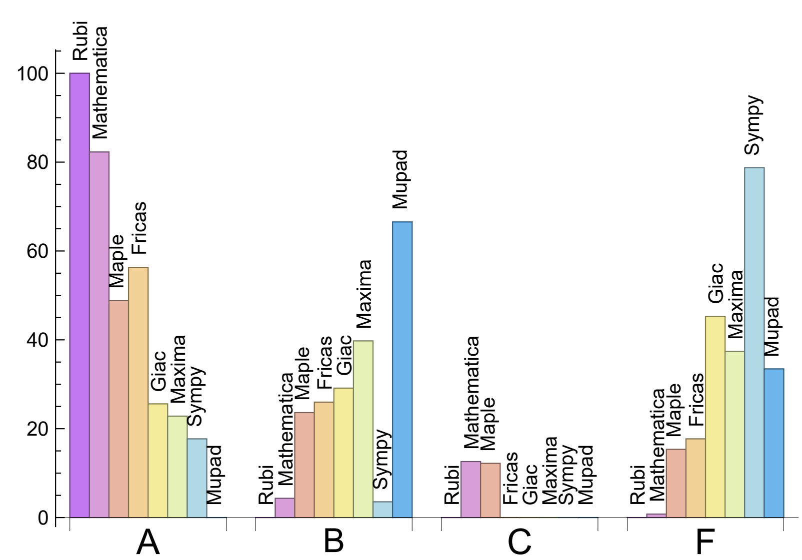
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	82.28	4.33	12.60	0.79
Maple	48.82	23.62	12.20	15.35
Maxima	22.83	39.76	0.00	37.40
Fricas	56.30	25.98	0.00	17.72
Sympy	17.72	3.54	0.00	78.74
Giac	25.59	29.13	0.00	45.28
Mupad	0.00	66.54	0.00	33.46

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	39	48.72 %	46.15 %	5.13 %
Maxima	95	100.00 %	0.00 %	0.00 %
Fricas	45	100.00 %	0.00 %	0.00 %
Sympy	200	16.00 %	83.00 %	1.00 %
Giac	115	54.78 %	32.17 %	13.04 %
Mupad	85	83.53 %	16.47 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

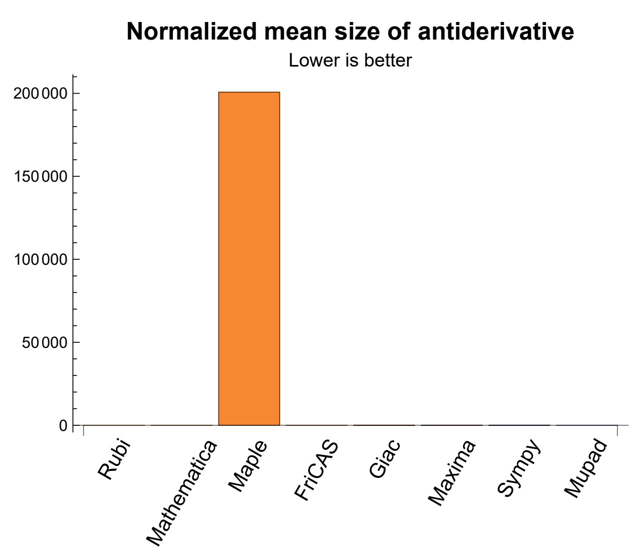
1.3 Performance

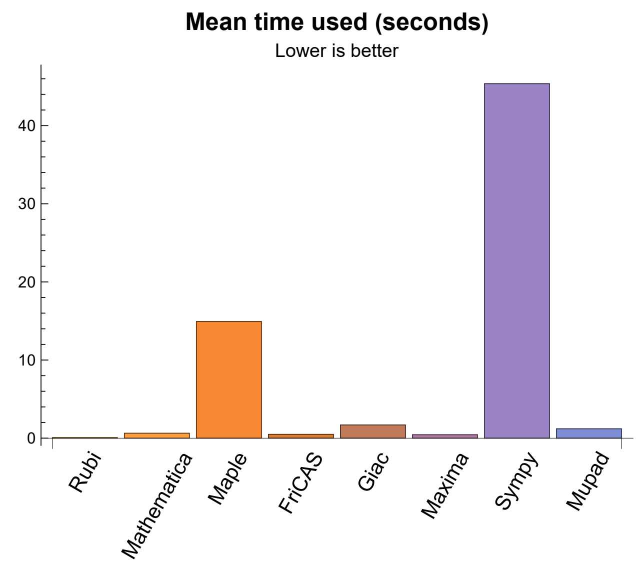
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	72.75	1.00	60.00	1.00
Mathematica	0.63	99.40	1.48	62.00	0.95
Maple	14.93	14465659.84	200708.18	93.00	1.50
Maxima	0.44	702.83	10.38	124.00	6.38
Fricas	0.49	109.40	1.79	71.00	1.30
Sympy	45.37	886.04	22.82	383.50	8.80
Giac	1.69	584.94	10.13	89.00	1.93
Mupad	1.20	123.10	1.96	49.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {123, 124, 125, 126, 127, 128, 187, 188, 189, 191, 209, 219, 223}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

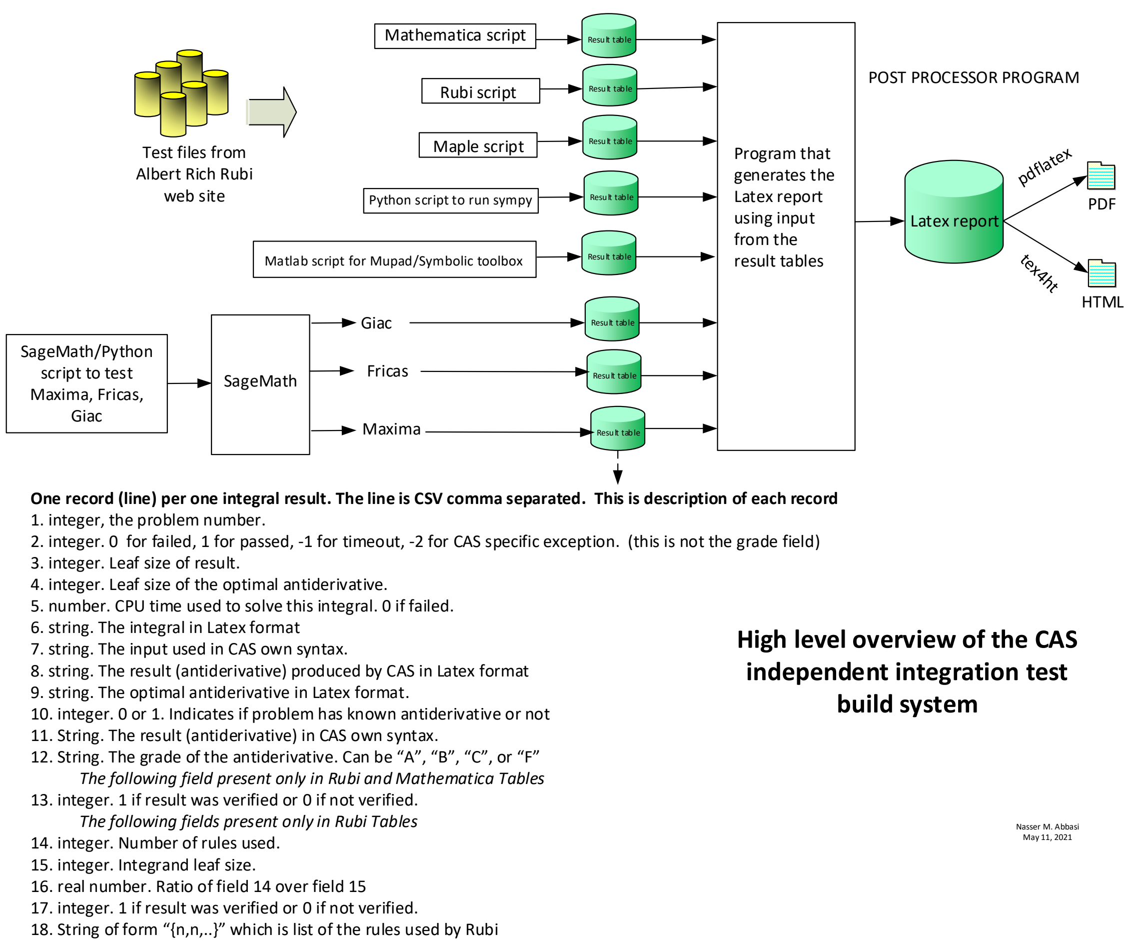
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 252, 253, 254 }

B grade: { 11, 40, 42, 44, 61, 72, 136, 138, 140, 158, 160 }

C grade: { 10, 12, 32, 41, 43, 69, 71, 123, 124, 125, 126, 127, 128, 137, 139, 159, 187, 188, 189, 196, 214, 227, 228, 231, 232, 233, 234, 242, 248, 249, 250, 251 }

F grade: { 201, 205 }

2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 87, 105, 106, 107, 108, 112, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 153, 154, 156, 157, 158, 159, 160, 175, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 229, 238, 239, 240, 243, 244, 245, 246 }

B grade: { 1, 3, 13, 15, 17, 23, 27, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 89, 90, 91, 92, 109, 111, 129, 131, 141, 143, 145, 151, 155, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 178, 179, 180, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 241, 242 }

C grade: { 78, 93, 97, 98, 99, 100, 101, 102, 103, 115, 116, 117, 118, 119, 121, 166, 181, 185, 186, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

F grade: { 79, 80, 81, 88, 94, 95, 96, 104, 110, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 167, 168, 169, 177, 182, 183, 184, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 66, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 190, 194, 212, 240 }

B grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 52, 53, 54, 55, 56, 57, 58, 65, 67, 68, 69, 70, 71, 72, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 210, 211, 213, 214, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252 }

C grade: { }

F grade: { 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 115, 116, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 165, 166, 167, 168, 182, 183, 184, 190, 192, 193, 194, 195, 196, 197, 199, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 229, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

B grade: { 8, 9, 20, 22, 27, 30, 41, 42, 43, 53, 55, 57, 69, 70, 71, 73, 74, 75, 76, 89, 90, 91, 92, 97, 98, 99, 100, 117, 118, 119, 136, 137, 138, 139, 148, 150, 158, 159, 160, 161, 162, 163, 164, 177, 178, 179, 180, 181, 185, 186, 198, 200, 214, 228, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

C grade: { }

F grade: { 81, 82, 83, 84, 85, 86, 87, 88, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 169, 170, 171, 172, 173, 174, 175, 176, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.6 Sympy

A grade: { 3, 4, 5, 6, 7, 14, 15, 16, 17, 25, 26, 27, 131, 132, 133, 134, 135, 142, 143, 144, 145, 153, 154, 155, 192, 193, 194, 202, 203, 204, 206, 207, 210, 211, 212, 220, 221, 222, 224, 225, 238, 239, 240, 244, 245 }

B grade: { 40, 195, 196, 213, 214, 227, 228, 241, 242 }

C grade: { }

F grade: { 1, 2, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 197, 198, 199, 200, 201, 205, 208, 209, 215, 216, 217, 218, 219, 223, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

2.1.7 Giac

A grade: { 2, 4, 5, 6, 7, 14, 16, 24, 25, 26, 34, 36, 38, 40, 45, 47, 49, 51, 60, 62, 64, 66, 68, 130, 132, 133, 134, 135, 136, 137, 139, 142, 144, 147, 149, 152, 153, 154, 157, 159, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 238, 239, 240, 244, 245, 246 }

B grade: { 1, 3, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 46, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 63, 65, 67, 69, 70, 71, 129, 131, 138, 140, 141, 143, 145, 146, 148, 150, 151, 155, 156, 158, 160, 197, 199, 213, 215, 217, 227, 228, 229, 241, 242, 243 }

C grade: { }

F grade: { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 195, 196, 198, 200, 201, 205, 209, 214, 216, 218, 219, 223, 230, 231, 232, 233, 234, 235, 236, 237, 247, 248, 249, 250, 251, 252, 253, 254 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 165, 166, 167, 168, 181, 182, 183, 184, 190, 192, 193, 194, 195, 196, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 214, 220, 221, 222, 224, 225, 226, 227, 228, 231, 232, 233, 234, 238, 239, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251 }

C grade: { }

F grade: { 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 209, 215, 216, 217, 218, 219, 223, 229, 230, 235, 236, 237, 243, 247, 252, 253, 254 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	91	83	0	110	45
normalized size	1	1.00	0.77	1.82	1.49	1.36	0.00	1.80	0.74
time (sec)	N/A	0.057	0.477	1.945	0.367	0.481	0.000	3.139	0.079

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	80	46	0	96	46
normalized size	1	1.00	0.77	1.59	1.31	0.75	0.00	1.57	0.75
time (sec)	N/A	0.058	0.333	0.261	0.349	0.429	0.000	1.906	0.134

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	69	63	197	82	36
normalized size	1	1.00	0.80	1.80	1.50	1.37	4.28	1.78	0.78
time (sec)	N/A	0.055	0.257	1.287	0.339	0.415	124.076	0.709	0.088

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	36	163	68	36
normalized size	1	1.00	0.80	1.50	1.26	0.78	3.54	1.48	0.78
time (sec)	N/A	0.055	0.150	0.243	0.336	0.415	40.277	0.449	0.140

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	41	126	54	26
normalized size	1	1.00	0.87	1.77	1.52	1.32	4.06	1.74	0.84
time (sec)	N/A	0.050	0.094	0.930	0.339	0.473	11.906	0.374	0.051

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	36	26	92	40	26
normalized size	1	1.00	0.87	1.32	1.16	0.84	2.97	1.29	0.84
time (sec)	N/A	0.050	0.067	0.266	0.338	0.508	3.397	0.353	0.106

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	21	51	26	44
normalized size	1	1.00	0.50	0.90	0.87	0.70	1.70	0.87	1.47
time (sec)	N/A	0.011	0.034	0.483	0.321	0.436	0.813	0.321	0.360

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	20	115	28	0	174	12
normalized size	1	1.00	1.00	1.43	8.21	2.00	0.00	12.43	0.86
time (sec)	N/A	0.017	0.005	0.681	0.463	0.485	0.000	0.801	0.149

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	36	236	52	0	412	26
normalized size	1	1.00	1.79	1.29	8.43	1.86	0.00	14.71	0.93
time (sec)	N/A	0.038	0.041	0.975	0.350	0.451	0.000	2.371	0.111

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	55	808	85	0	1421	48
normalized size	1	1.00	0.59	1.12	16.49	1.73	0.00	29.00	0.98
time (sec)	N/A	0.063	0.019	0.985	0.502	0.494	0.000	4.654	0.121

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	205	78	2174	112	0	3028	60
normalized size	1	1.00	3.11	1.18	32.94	1.70	0.00	45.88	0.91
time (sec)	N/A	0.067	0.480	0.866	0.405	0.479	0.000	8.639	0.098

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	31	97	3088	140	0	5647	79
normalized size	1	1.00	0.35	1.09	34.70	1.57	0.00	63.45	0.89
time (sec)	N/A	0.069	0.031	0.979	0.619	0.515	0.000	12.384	0.172

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	72	46	0	85	46
normalized size	1	1.00	1.55	1.95	1.64	1.05	0.00	1.93	1.05
time (sec)	N/A	0.069	0.357	0.393	0.333	0.479	0.000	0.761	0.128

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	65	67	434	74	110
normalized size	1	1.00	0.82	0.99	0.86	0.88	5.71	0.97	1.45
time (sec)	N/A	0.068	0.196	0.959	0.335	0.624	117.945	0.599	1.675

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	48	58	50	36	362	57	33
normalized size	1	1.00	1.66	2.00	1.72	1.24	12.48	1.97	1.14
time (sec)	N/A	0.059	0.117	0.292	0.327	0.455	40.096	0.502	0.109

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	41	46	231	46	43
normalized size	1	1.00	0.82	0.96	0.84	0.94	4.71	0.94	0.88
time (sec)	N/A	0.056	0.075	0.669	0.329	0.493	11.960	0.418	0.302

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	24	133	29	13
normalized size	1	1.00	1.00	2.00	1.73	1.60	8.87	1.93	0.87
time (sec)	N/A	0.033	0.005	0.142	0.330	0.520	3.298	0.459	0.096

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	55	14	0	95	12
normalized size	1	1.00	1.00	0.93	3.93	1.00	0.00	6.79	0.86
time (sec)	N/A	0.026	0.013	0.396	0.332	0.444	0.000	0.770	0.084

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	152	11
normalized size	1	1.00	1.00	0.92	4.08	1.46	0.00	11.69	0.85
time (sec)	N/A	0.035	0.008	1.071	0.330	0.405	0.000	1.522	0.109

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	641	56	0	734	35
normalized size	1	1.00	1.20	0.90	21.37	1.87	0.00	24.47	1.17
time (sec)	N/A	0.048	0.038	0.872	0.351	0.428	0.000	3.551	0.146

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	43	0	1034	33
normalized size	1	1.00	1.14	1.21	7.33	1.02	0.00	24.62	0.79
time (sec)	N/A	0.063	0.056	1.217	0.350	0.393	0.000	2.260	0.135

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	69	3164	112	0	2657	74
normalized size	1	1.00	0.93	1.15	52.73	1.87	0.00	44.28	1.23
time (sec)	N/A	0.071	0.248	0.922	0.441	0.430	0.000	2.354	0.155

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	80	73	0	96	36
normalized size	1	1.00	0.80	2.11	1.74	1.59	0.00	2.09	0.78
time (sec)	N/A	0.063	0.369	1.478	0.352	0.423	0.000	0.618	0.074

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	69	46	0	82	46
normalized size	1	1.00	0.77	1.36	1.13	0.75	0.00	1.34	0.75
time (sec)	N/A	0.069	0.228	0.305	0.344	0.422	0.000	0.798	0.118

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	53	284	54	26
normalized size	1	1.00	0.87	1.77	1.52	1.71	9.16	1.74	0.84
time (sec)	N/A	0.060	0.188	0.882	0.346	0.413	115.462	0.448	0.053

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	47	36	202	54	36
normalized size	1	1.00	0.80	1.20	1.02	0.78	4.39	1.17	0.78
time (sec)	N/A	0.064	0.113	0.302	0.340	0.445	38.634	0.328	0.125

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	31	117	40	13
normalized size	1	1.00	1.00	2.73	2.27	2.07	7.80	2.67	0.87
time (sec)	N/A	0.033	0.008	0.582	0.331	0.458	11.216	0.333	0.099

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	32	124	36	0	618	23
normalized size	1	1.00	0.96	1.14	4.43	1.29	0.00	22.07	0.82
time (sec)	N/A	0.037	0.014	0.523	0.465	0.489	0.000	0.986	0.124

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	83	13	0	319	13
normalized size	1	1.00	1.00	1.08	6.38	1.00	0.00	24.54	1.00
time (sec)	N/A	0.035	0.010	0.685	0.326	0.440	0.000	3.869	0.091

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	38	480	61	0	1111	36
normalized size	1	1.00	1.12	1.12	14.12	1.79	0.00	32.68	1.06
time (sec)	N/A	0.041	0.012	10.773	0.467	0.451	0.000	2.945	0.167

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	49	987	67	0	2114	37
normalized size	1	1.00	1.42	1.14	22.95	1.56	0.00	49.16	0.86
time (sec)	N/A	0.053	0.029	0.917	0.359	0.484	0.000	6.790	0.075

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	29	76	1805	95	0	4032	67
normalized size	1	1.00	0.41	1.09	25.79	1.36	0.00	57.60	0.96
time (sec)	N/A	0.072	0.036	0.916	0.523	0.595	0.000	14.593	0.192

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	91	46	0	270	46
normalized size	1	1.00	1.95	1.16	1.49	0.75	0.00	4.43	0.75
time (sec)	N/A	0.061	0.097	0.834	0.332	0.514	0.000	1.178	0.153

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	97	80	73	0	46	45
normalized size	1	1.00	0.79	1.59	1.31	1.20	0.00	0.75	0.74
time (sec)	N/A	0.061	0.217	1.262	0.332	0.458	0.000	0.990	0.128

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	89	53	69	36	0	204	36
normalized size	1	1.00	1.93	1.15	1.50	0.78	0.00	4.43	0.78
time (sec)	N/A	0.056	0.058	1.009	0.337	0.438	0.000	0.808	0.136

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	69	58	53	0	36	36
normalized size	1	1.00	0.83	1.50	1.26	1.15	0.00	0.78	0.78
time (sec)	N/A	0.057	0.101	0.972	0.340	0.446	0.000	0.651	0.067

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	59	35	47	26	0	138	26
normalized size	1	1.00	1.90	1.13	1.52	0.84	0.00	4.45	0.84
time (sec)	N/A	0.052	0.040	0.662	0.341	0.473	0.000	0.562	0.053

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	41	36	33	0	26	26
normalized size	1	1.00	0.90	1.32	1.16	1.06	0.00	0.84	0.84
time (sec)	N/A	0.051	0.049	1.049	0.345	0.439	0.000	0.361	0.108

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	13	0	52	13
normalized size	1	1.00	1.00	0.93	1.53	0.87	0.00	3.47	0.87
time (sec)	N/A	0.037	0.007	0.499	0.344	0.438	0.000	0.372	0.034

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	11	3636	11	11
normalized size	1	1.00	2.09	1.09	1.00	1.00	330.55	1.00	1.00
time (sec)	N/A	0.017	0.009	0.339	0.344	0.473	21.208	0.362	0.024

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	34	233	50	0	484	26
normalized size	1	1.00	1.04	1.21	8.32	1.79	0.00	17.29	0.93
time (sec)	N/A	0.041	0.018	0.673	0.476	0.426	0.000	1.696	0.114

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	974	96	0	1327	49
normalized size	1	1.00	2.92	1.16	19.88	1.96	0.00	27.08	1.00
time (sec)	N/A	0.059	0.262	0.915	0.381	0.438	0.000	1.607	0.129

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	1780	130	0	3033	61
normalized size	1	1.00	0.47	1.15	26.97	1.97	0.00	45.95	0.92
time (sec)	N/A	0.063	0.021	0.765	0.532	0.473	0.000	3.944	0.186

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	3846	148	0	5476	78
normalized size	1	1.00	3.01	1.11	43.21	1.66	0.00	61.53	0.88
time (sec)	N/A	0.072	0.514	0.954	0.505	0.494	0.000	10.216	0.122

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	85	111	87	87	0	95	149
normalized size	1	1.00	0.55	0.72	0.56	0.56	0.00	0.61	0.96
time (sec)	N/A	0.171	0.255	1.411	0.358	0.458	0.000	1.614	2.272

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	48	53	72	36	0	183	35
normalized size	1	1.00	1.09	1.20	1.64	0.82	0.00	4.16	0.80
time (sec)	N/A	0.065	0.175	0.728	0.343	0.424	0.000	0.995	0.149

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	83	65	66	0	75	109
normalized size	1	1.00	0.56	0.75	0.59	0.59	0.00	0.68	0.98
time (sec)	N/A	0.120	0.199	0.972	0.347	0.458	0.000	1.021	1.690

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	48	35	50	26	0	139	25
normalized size	1	1.00	1.66	1.21	1.72	0.90	0.00	4.79	0.86
time (sec)	N/A	0.057	0.125	0.576	0.351	0.418	0.000	0.676	0.052

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	55	43	47	0	55	65
normalized size	1	1.00	0.67	0.92	0.72	0.78	0.00	0.92	1.08
time (sec)	N/A	0.074	0.103	0.912	0.355	0.459	0.000	0.769	0.490

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	26	13	0	69	13
normalized size	1	1.00	1.00	1.08	2.00	1.00	0.00	5.31	1.00
time (sec)	N/A	0.041	0.007	0.569	0.340	0.449	0.000	0.490	0.111

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	28	18	22	0	29	17
normalized size	1	1.00	0.95	1.33	0.86	1.05	0.00	1.38	0.81
time (sec)	N/A	0.034	0.022	0.592	0.340	0.410	0.000	0.494	0.148

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	81	14	0	55	13
normalized size	1	1.00	1.67	1.08	6.75	1.17	0.00	4.58	1.08
time (sec)	N/A	0.020	0.018	0.281	0.352	0.451	0.000	0.986	0.125

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	34	27	656	65	0	813	36
normalized size	1	1.00	1.13	0.90	21.87	2.17	0.00	27.10	1.20
time (sec)	N/A	0.042	0.055	0.728	0.372	0.433	0.000	1.007	0.137

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	54	0	1079	37
normalized size	1	1.00	1.14	1.21	7.33	1.29	0.00	25.69	0.88
time (sec)	N/A	0.059	0.080	1.107	0.360	0.427	0.000	1.806	0.140

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	69	3188	138	0	2808	82
normalized size	1	1.00	0.90	1.15	53.13	2.30	0.00	46.80	1.37
time (sec)	N/A	0.067	0.361	0.862	0.450	0.512	0.000	3.364	0.219

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	90	87	1227	86	0	3445	55
normalized size	1	1.00	1.25	1.21	17.04	1.19	0.00	47.85	0.76
time (sec)	N/A	0.070	0.055	1.235	0.368	0.417	0.000	2.834	0.381

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	111	7650	194	0	6374	114
normalized size	1	1.00	0.84	1.23	85.00	2.16	0.00	70.82	1.27
time (sec)	N/A	0.082	0.425	0.842	0.720	0.566	0.000	7.126	0.294

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	132	123	2710	118	0	7477	83
normalized size	1	1.00	1.29	1.21	26.57	1.16	0.00	73.30	0.81
time (sec)	N/A	0.084	0.073	1.261	0.479	0.511	0.000	12.221	0.281

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	91	46	0	314	46
normalized size	1	1.00	1.95	1.16	1.49	0.75	0.00	5.15	0.75
time (sec)	N/A	0.072	0.152	0.949	0.347	0.652	0.000	3.297	0.132

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	107	91	83	0	56	55
normalized size	1	1.00	0.76	1.41	1.20	1.09	0.00	0.74	0.72
time (sec)	N/A	0.077	0.327	1.330	0.833	0.481	0.000	1.931	0.115

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	104	53	80	36	0	248	36
normalized size	1	1.00	2.26	1.15	1.74	0.78	0.00	5.39	0.78
time (sec)	N/A	0.067	0.101	0.768	1.018	0.445	0.000	1.344	0.137

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	79	69	63	0	46	45
normalized size	1	1.00	0.79	1.30	1.13	1.03	0.00	0.75	0.74
time (sec)	N/A	0.071	0.169	0.986	0.754	0.426	0.000	1.339	0.125

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	35	47	26	0	182	26
normalized size	1	1.00	0.87	1.13	1.52	0.84	0.00	5.87	0.84
time (sec)	N/A	0.061	0.151	0.671	1.046	0.438	0.000	1.443	0.054

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	51	47	43	0	36	36
normalized size	1	1.00	0.80	1.11	1.02	0.93	0.00	0.78	0.78
time (sec)	N/A	0.066	0.111	0.959	0.447	0.690	0.000	0.776	0.124

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	13	0	74	13
normalized size	1	1.00	1.00	0.93	2.27	0.87	0.00	4.93	0.87
time (sec)	N/A	0.044	0.009	0.582	0.819	0.506	0.000	0.979	0.047

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	22	23	21	0	22	24
normalized size	1	1.00	1.04	0.81	0.85	0.78	0.00	0.81	0.89
time (sec)	N/A	0.039	0.011	0.819	1.875	0.644	0.000	0.505	0.102

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	34	92	38	0	57	22
normalized size	1	1.00	1.83	1.42	3.83	1.58	0.00	2.38	0.92
time (sec)	N/A	0.043	0.023	0.568	0.992	0.623	0.000	0.574	0.112

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	84	13	0	13	13
normalized size	1	1.00	1.00	1.27	7.64	1.18	0.00	1.18	1.18
time (sec)	N/A	0.026	0.011	0.294	0.349	0.518	0.000	0.348	0.038

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	47	834	94	0	2096	38
normalized size	1	1.00	0.72	1.09	19.40	2.19	0.00	48.74	0.88
time (sec)	N/A	0.049	0.018	0.893	1.145	0.434	0.000	1.075	0.126

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	129	78	2237	132	0	3775	66
normalized size	1	1.00	1.84	1.11	31.96	1.89	0.00	53.93	0.94
time (sec)	N/A	0.073	4.470	0.738	0.590	0.443	0.000	4.386	0.150

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	31	97	3095	166	0	6318	71
normalized size	1	1.00	0.38	1.20	38.21	2.05	0.00	78.00	0.88
time (sec)	N/A	0.073	0.036	0.706	1.052	0.564	0.000	9.367	0.179

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	278	120	4268	194	0	0	100
normalized size	1	1.00	2.48	1.07	38.11	1.73	0.00	0.00	0.89
time (sec)	N/A	0.087	0.846	0.854	1.565	0.554	0.000	0.000	0.227

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	183661406	0	291	0	0	-1
normalized size	1	1.00	0.72	1350451.52	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.297	89.385	0.000	0.622	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	65166862	0	280	0	0	-1
normalized size	1	1.00	0.78	592426.02	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.199	25.674	0.000	0.481	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	6214674	0	266	0	0	-1
normalized size	1	1.00	0.86	73984.21	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.076	2.853	0.000	0.781	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	15972263	0	240	0	0	-1
normalized size	1	1.00	0.86	275383.84	0.00	4.14	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.053	3.214	0.000	0.494	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	59119746	0	39	0	0	34
normalized size	1	1.00	0.96	2570423.74	0.00	1.70	0.00	0.00	1.48
time (sec)	N/A	0.018	0.020	11.404	0.000	0.473	0.000	0.000	0.284

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	597	0	69	0	0	108
normalized size	1	1.00	0.81	11.26	0.00	1.30	0.00	0.00	2.04
time (sec)	N/A	0.040	0.109	41.618	0.000	0.451	0.000	0.000	3.080

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	88	0	0	131
normalized size	1	1.00	0.66	0.00	0.00	1.11	0.00	0.00	1.66
time (sec)	N/A	0.059	0.195	180.000	0.000	0.432	0.000	0.000	3.391

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	113	0	0	351
normalized size	1	1.00	0.64	0.00	0.00	1.08	0.00	0.00	3.34
time (sec)	N/A	0.083	0.146	180.000	0.000	0.453	0.000	0.000	4.335

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.407	180.000	0.000	0.573	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	278615779	0	0	0	0	-1
normalized size	1	1.00	0.96	4037909.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.220	167.057	0.000	0.521	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	172368074	0	0	0	0	-1
normalized size	1	1.00	1.10	2498088.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.359	71.760	0.000	0.441	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	16549859	0	0	0	0	-1
normalized size	1	1.00	0.85	413746.48	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.087	5.709	0.000	0.468	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	75	53360209	0	0	0	0	-1
normalized size	1	1.00	1.88	1334005.22	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.245	13.507	0.000	0.448	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	83175028	0	0	0	0	-1
normalized size	1	1.00	0.91	1848333.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.098	15.515	0.000	0.449	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	83	123	0	0	0	0	-1
normalized size	1	1.00	1.73	2.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.194	59.888	0.000	0.452	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.810	180.000	0.000	0.432	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	322540527	0	290	0	0	-1
normalized size	1	1.00	0.72	2371621.52	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.347	194.690	0.000	0.469	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	47453403	0	281	0	0	-1
normalized size	1	1.00	0.78	431394.57	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.220	23.188	0.000	0.507	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	74	155804040	0	268	0	0	-1
normalized size	1	1.00	0.88	1854810.00	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.144	46.944	0.000	0.488	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	149378290	0	296	0	0	-1
normalized size	1	1.00	0.89	1844176.42	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.095	52.777	0.000	0.464	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	727	0	48	0	0	85
normalized size	1	1.00	0.96	25.96	0.00	1.71	0.00	0.00	3.04
time (sec)	N/A	0.028	0.055	183.929	0.000	0.450	0.000	0.000	2.194

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	55	0	0	88
normalized size	1	1.00	0.64	0.00	0.00	1.00	0.00	0.00	1.60
time (sec)	N/A	0.049	0.095	180.000	0.000	0.486	0.000	0.000	3.260

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	79	0	0	300
normalized size	1	1.00	0.68	0.00	0.00	0.98	0.00	0.00	3.70
time (sec)	N/A	0.069	0.109	180.000	0.000	0.429	0.000	0.000	3.713

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	98	0	0	383
normalized size	1	1.00	0.58	0.00	0.00	0.92	0.00	0.00	3.58
time (sec)	N/A	0.092	0.167	180.000	0.000	0.447	0.000	0.000	5.161

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	973	0	290	0	0	-1
normalized size	1	1.00	0.72	7.15	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.326	19.859	0.000	0.448	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	243	0	281	0	0	-1
normalized size	1	1.00	0.78	2.21	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.202	20.549	0.000	0.462	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	362	0	266	0	0	-1
normalized size	1	1.00	0.86	4.47	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.086	10.365	0.000	0.474	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	157	0	242	0	0	-1
normalized size	1	1.00	0.98	2.96	0.00	4.57	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.044	8.587	0.000	0.488	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	308	0	39	0	0	24
normalized size	1	1.00	0.96	12.83	0.00	1.62	0.00	0.00	1.00
time (sec)	N/A	0.024	0.046	9.957	0.000	0.448	0.000	0.000	0.308

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	0	103
normalized size	1	1.00	0.81	3.66	0.00	1.40	0.00	0.00	1.94
time (sec)	N/A	0.064	0.106	13.100	0.000	0.522	0.000	0.000	2.962

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	481	0	103	0	0	136
normalized size	1	1.00	0.66	6.09	0.00	1.30	0.00	0.00	1.72
time (sec)	N/A	0.086	0.125	37.234	0.000	0.555	0.000	0.000	3.362

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	118	0	0	350
normalized size	1	1.00	0.64	0.00	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.108	0.142	180.000	0.000	0.460	0.000	0.000	4.130

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	204	0	0	0	0	-1
normalized size	1	1.00	0.62	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.245	29.075	0.000	0.519	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	76	139	0	0	0	0	-1
normalized size	1	1.00	0.72	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.268	25.250	0.000	0.460	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	34	137	0	0	0	0	-1
normalized size	1	1.00	0.45	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.083	22.392	0.000	0.472	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	111	0	0	0	0	-1
normalized size	1	1.00	1.04	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.923	15.645	0.000	0.457	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	176	0	0	0	0	-1
normalized size	1	1.00	0.84	4.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.131	17.731	0.000	0.436	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	0	0	0	0	-1
normalized size	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.993	180.000	0.000	0.497	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	227	0	0	0	0	-1
normalized size	1	1.00	0.83	2.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.591	59.936	0.000	0.467	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	154	0	0	0	0	-1
normalized size	1	1.00	0.86	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.469	90.513	0.000	0.544	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	85	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.827	180.000	0.000	0.523	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	86	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.364	180.000	0.000	0.476	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	100	441	0	291	0	0	-1
normalized size	1	1.00	0.53	2.32	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.438	211.099	0.000	0.490	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	84	973	0	280	0	0	-1
normalized size	1	1.00	0.51	5.93	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.227	81.756	0.000	0.678	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	70	243	0	268	0	0	-1
normalized size	1	1.00	0.55	1.91	0.00	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.139	41.222	0.000	0.574	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	68	542	0	295	0	0	-1
normalized size	1	1.00	0.65	5.21	0.00	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.099	86.501	0.000	0.541	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	0	95
normalized size	1	1.00	0.96	6.86	0.00	1.89	0.00	0.00	3.39
time (sec)	N/A	0.028	0.051	31.018	0.000	0.457	0.000	0.000	1.500

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	76	0	0	93
normalized size	1	1.00	0.64	0.00	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.053	0.093	180.000	0.000	0.475	0.000	0.000	3.162

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	222	0	104	0	0	302
normalized size	1	1.00	0.68	2.74	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.092	0.116	107.667	0.000	0.512	0.000	0.000	3.700

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	131	0	0	383
normalized size	1	1.00	0.58	0.00	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.116	0.097	180.000	0.000	0.607	0.000	0.000	4.989

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0	-1
normalized size	1	1.00	7.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	5.726	10.162	0.000	0.508	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0	-1
normalized size	1	1.00	7.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	3.509	6.229	0.000	0.533	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	152	0	0	0	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.264	5.039	0.000	0.502	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	254	0	0	0	0	0	-1
normalized size	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.866	4.345	0.000	0.575	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	938	0	0	0	0	0	-1
normalized size	1	1.00	11.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	5.416	4.280	0.000	0.506	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2308	0	0	0	0	0	-1
normalized size	1	1.00	27.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	19.220	3.849	0.000	0.476	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	91	46	0	110	46
normalized size	1	1.00	0.77	1.82	1.49	0.75	0.00	1.80	0.75
time (sec)	N/A	0.060	0.439	0.413	0.350	0.468	0.000	2.903	0.035

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	80	73	0	96	45
normalized size	1	1.00	0.77	1.59	1.31	1.20	0.00	1.57	0.74
time (sec)	N/A	0.061	0.291	1.760	0.338	0.510	0.000	1.726	0.132

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	69	36	199	82	36
normalized size	1	1.00	0.80	1.80	1.50	0.78	4.33	1.78	0.78
time (sec)	N/A	0.055	0.254	0.458	0.339	0.463	113.174	0.943	0.146

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	53	162	68	36
normalized size	1	1.00	0.80	1.50	1.26	1.15	3.52	1.48	0.78
time (sec)	N/A	0.054	0.135	1.223	0.336	0.410	38.308	2.925	0.141

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	26	128	54	26
normalized size	1	1.00	0.87	1.77	1.52	0.84	4.13	1.74	0.84
time (sec)	N/A	0.050	0.093	0.409	0.330	0.452	11.275	0.290	0.032

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	36	33	90	40	26
normalized size	1	1.00	0.87	1.32	1.16	1.06	2.90	1.29	0.84
time (sec)	N/A	0.048	0.064	0.622	0.328	0.420	3.117	0.455	0.028

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	13	53	26	43
normalized size	1	1.00	0.50	0.90	0.87	0.43	1.77	0.87	1.43
time (sec)	N/A	0.011	0.006	0.219	0.317	0.480	0.760	0.278	0.192

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	22	84	30	0	16	12
normalized size	1	1.00	3.00	1.57	6.00	2.14	0.00	1.14	0.86
time (sec)	N/A	0.016	0.012	0.551	0.337	0.597	0.000	0.223	0.022

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	34	233	50	0	38	26
normalized size	1	1.00	1.04	1.21	8.32	1.79	0.00	1.36	0.93
time (sec)	N/A	0.038	0.019	0.861	0.465	0.474	0.000	0.271	0.024

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	974	96	0	140	49
normalized size	1	1.00	2.92	1.16	19.88	1.96	0.00	2.86	1.00
time (sec)	N/A	0.055	0.255	0.947	0.352	0.468	0.000	0.412	0.083

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	1780	130	0	72	61
normalized size	1	1.00	0.47	1.15	26.97	1.97	0.00	1.09	0.92
time (sec)	N/A	0.059	0.023	1.113	0.528	0.466	0.000	0.437	0.098

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	3846	148	0	209	78
normalized size	1	1.00	3.01	1.11	43.21	1.66	0.00	2.35	0.88
time (sec)	N/A	0.067	0.476	0.963	0.502	0.477	0.000	0.643	0.119

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	72	36	0	85	35
normalized size	1	1.00	1.55	1.95	1.64	0.82	0.00	1.93	0.80
time (sec)	N/A	0.067	0.412	0.339	0.338	0.459	0.000	0.493	0.175

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	65	66	434	74	109
normalized size	1	1.00	0.82	0.99	0.86	0.87	5.71	0.97	1.43
time (sec)	N/A	0.064	0.185	1.652	0.341	0.465	116.231	0.398	1.727

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	48	58	50	26	359	57	26
normalized size	1	1.00	1.71	2.07	1.79	0.93	12.82	2.04	0.93
time (sec)	N/A	0.057	0.116	0.416	0.337	0.652	39.276	0.320	0.144

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	43	47	231	46	43
normalized size	1	1.00	0.82	0.96	0.88	0.96	4.71	0.94	0.88
time (sec)	N/A	0.054	0.096	0.962	0.339	0.501	11.677	0.191	0.313

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	13	131	29	13
normalized size	1	1.00	1.00	2.00	1.73	0.87	8.73	1.93	0.87
time (sec)	N/A	0.033	0.005	0.240	0.335	0.518	3.241	0.496	0.153

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	22	13	82	14	0	56	14
normalized size	1	1.00	1.57	0.93	5.86	1.00	0.00	4.00	1.00
time (sec)	N/A	0.025	0.016	0.644	0.332	0.518	0.000	0.224	0.157

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	13	11
normalized size	1	1.00	1.00	0.92	4.08	1.46	0.00	1.00	0.85
time (sec)	N/A	0.033	0.013	1.128	0.333	0.515	0.000	0.230	0.142

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	34	27	656	65	0	119	36
normalized size	1	1.00	1.13	0.90	21.87	2.17	0.00	3.97	1.20
time (sec)	N/A	0.047	0.049	1.075	0.351	0.585	0.000	0.361	0.178

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	54	0	35	37
normalized size	1	1.00	1.14	1.21	7.33	1.29	0.00	0.83	0.88
time (sec)	N/A	0.060	0.052	1.625	0.343	0.459	0.000	0.495	0.184

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	69	3188	138	0	232	82
normalized size	1	1.00	0.90	1.15	53.13	2.30	0.00	3.87	1.37
time (sec)	N/A	0.068	0.351	1.265	0.437	0.466	0.000	0.739	0.144

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	80	36	0	96	36
normalized size	1	1.00	0.80	2.11	1.74	0.78	0.00	2.09	0.78
time (sec)	N/A	0.061	0.390	0.397	0.336	0.446	0.000	0.598	0.071

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	69	63	0	82	45
normalized size	1	1.00	0.77	1.36	1.13	1.03	0.00	1.34	0.74
time (sec)	N/A	0.065	0.213	1.251	0.340	0.484	0.000	0.371	0.061

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	26	284	54	26
normalized size	1	1.00	0.87	1.77	1.52	0.84	9.16	1.74	0.84
time (sec)	N/A	0.056	0.137	0.454	0.329	0.423	113.547	0.374	0.152

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	47	43	202	54	36
normalized size	1	1.00	0.80	1.20	1.02	0.93	4.39	1.17	0.78
time (sec)	N/A	0.060	0.093	0.987	0.330	0.414	38.318	0.226	0.160

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	13	117	40	13
normalized size	1	1.00	1.00	2.73	2.27	0.87	7.80	2.67	0.87
time (sec)	N/A	0.033	0.008	0.169	0.326	0.455	11.227	0.178	0.142

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	34	92	38	0	57	22
normalized size	1	1.00	1.64	1.21	3.29	1.36	0.00	2.04	0.79
time (sec)	N/A	0.035	0.020	0.565	0.343	0.495	0.000	0.691	0.053

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	84	13	0	13	13
normalized size	1	1.00	1.00	1.08	6.46	1.00	0.00	1.00	1.00
time (sec)	N/A	0.035	0.012	0.472	0.337	0.463	0.000	0.250	0.023

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	79	40	558	72	0	92	36
normalized size	1	1.00	2.32	1.18	16.41	2.12	0.00	2.71	1.06
time (sec)	N/A	0.041	0.015	0.857	0.345	0.478	0.000	0.657	0.059

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	47	834	94	0	52	38
normalized size	1	1.00	0.72	1.09	19.40	2.19	0.00	1.21	0.88
time (sec)	N/A	0.051	0.018	1.214	0.656	0.447	0.000	0.929	0.064

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	195	78	2237	132	0	163	66
normalized size	1	1.00	2.79	1.11	31.96	1.89	0.00	2.33	0.94
time (sec)	N/A	0.070	0.333	1.120	0.415	0.435	0.000	0.825	0.186

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	199880292	0	290	0	0	-1
normalized size	1	1.00	0.72	1469708.03	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.345	91.921	0.000	0.496	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	74316382	0	281	0	0	-1
normalized size	1	1.00	0.78	675603.47	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.190	24.515	0.000	0.497	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	5342284	0	266	0	0	-1
normalized size	1	1.00	0.83	63598.62	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.098	2.750	0.000	0.533	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	52	16154758	0	242	0	0	-1
normalized size	1	1.00	0.90	278530.31	0.00	4.17	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.049	3.185	0.000	0.441	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	55902198	0	39	0	0	24
normalized size	1	1.00	0.96	2329258.25	0.00	1.62	0.00	0.00	1.00
time (sec)	N/A	0.018	0.023	10.737	0.000	0.503	0.000	0.000	0.192

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	0	104
normalized size	1	1.00	0.81	3.66	0.00	1.40	0.00	0.00	1.96
time (sec)	N/A	0.038	0.108	33.699	0.000	0.592	0.000	0.000	3.168

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	103	0	0	136
normalized size	1	1.00	0.66	0.00	0.00	1.30	0.00	0.00	1.72
time (sec)	N/A	0.059	0.125	180.000	0.000	0.458	0.000	0.000	3.233

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	118	0	0	350
normalized size	1	1.00	0.64	0.00	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.080	0.143	180.000	0.000	0.433	0.000	0.000	3.866

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.390	180.000	0.000	0.555	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	336654856	0	0	0	0	-1
normalized size	1	1.00	0.96	4879055.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.205	196.388	0.000	0.551	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	174364216	0	0	0	0	-1
normalized size	1	1.00	1.10	2527017.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.348	61.350	0.000	0.484	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	24858211	0	0	0	0	-1
normalized size	1	1.00	0.85	621455.28	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.057	7.574	0.000	0.542	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	76	59624230	0	0	0	0	-1
normalized size	1	1.00	1.90	1490605.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.890	15.386	0.000	0.600	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	94273592	0	0	0	0	-1
normalized size	1	1.00	0.85	2049425.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.112	17.328	0.000	0.616	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	123	0	0	0	0	-1
normalized size	1	1.00	1.71	2.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	1.002	59.046	0.000	0.520	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	227	0	0	0	0	-1
normalized size	1	1.00	0.83	2.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.561	212.846	0.000	0.554	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	99	0	0	291	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.336	180.000	0.000	0.459	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	84	88680892	0	280	0	0	-1
normalized size	1	1.00	0.76	806189.93	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.187	32.658	0.000	0.565	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupa
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	191110780	0	268	0	0	-1
normalized size	1	1.00	0.87	2275128.33	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.107	69.299	0.000	0.725	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	179323229	0	295	0	0	-1
normalized size	1	1.00	0.85	2186868.65	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.092	32.122	0.000	0.542	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	0	94
normalized size	1	1.00	0.96	6.86	0.00	1.89	0.00	0.00	3.36
time (sec)	N/A	0.027	0.054	167.451	0.000	0.645	0.000	0.000	1.168

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	76	0	0	93
normalized size	1	1.00	0.64	0.00	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.047	0.092	180.000	0.000	0.470	0.000	0.000	3.075

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	104	0	0	302
normalized size	1	1.00	0.68	0.00	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.068	0.116	180.000	0.000	0.518	0.000	0.000	3.661

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	131	0	0	383
normalized size	1	1.00	0.58	0.00	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.090	0.093	180.000	0.000	0.482	0.000	0.000	4.881

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	-1
normalized size	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.025	0.814	0.000	0.487	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	99	0	137	0	0	-1
normalized size	1	1.00	1.00	3.96	0.00	5.48	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.016	2.461	0.000	0.453	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2472	0	0	0	0	0	-1
normalized size	1	1.00	29.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	13.248	6.181	0.000	0.432	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	890	0	0	0	0	0	-1
normalized size	1	1.00	10.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	7.809	5.900	0.000	0.431	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	149	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.258	5.358	0.000	0.477	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	36	0	68	36
normalized size	1	1.00	0.80	1.50	1.26	0.78	0.00	1.48	0.78
time (sec)	N/A	0.097	0.156	0.426	0.341	0.531	0.000	0.638	0.205

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	209	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.848	0.895	14.316	0.000	0.450	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	916	122	921	84	311
normalized size	1	1.00	0.95	0.92	10.07	1.34	10.12	0.92	3.42
time (sec)	N/A	0.076	0.545	0.856	0.414	0.572	32.888	0.745	1.567

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	414	71	408	56	98
normalized size	1	1.00	1.11	0.92	6.68	1.15	6.58	0.90	1.58
time (sec)	N/A	0.053	0.714	0.200	0.348	0.426	6.696	0.188	0.736

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
normalized size	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.035	0.194	0.418	0.319	0.428	1.486	0.363	1.063

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	325	108	31	333	0	111
normalized size	1	1.00	1.00	12.50	4.15	1.19	12.81	0.00	4.27
time (sec)	N/A	0.027	0.151	1.549	0.396	0.424	8.199	0.000	0.869

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	890	454	71	3266	0	252
normalized size	1	1.00	2.50	24.72	12.61	1.97	90.72	0.00	7.00
time (sec)	N/A	0.033	0.099	2.226	0.363	0.509	101.783	0.000	5.209

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	120	399	47	0	145	-1
normalized size	1	1.00	0.90	3.08	10.23	1.21	0.00	3.72	-0.03
time (sec)	N/A	0.044	0.202	2.343	0.336	0.433	0.000	0.215	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	14880	1773	141	0	0	-1
normalized size	1	1.00	1.00	222.09	26.46	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.576	4.898	0.423	0.651	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	321	1076	75	0	301	-1
normalized size	1	1.00	0.97	5.35	17.93	1.25	0.00	5.02	-0.02
time (sec)	N/A	0.047	0.381	4.635	0.349	0.573	0.000	3.250	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	97954	3879	197	0	0	-1
normalized size	1	1.00	0.84	1042.06	41.27	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.160	14.958	0.569	0.492	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.973	0.403	7.184	0.000	0.432	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	80	63	371	69	408	61	105
normalized size	1	1.00	1.18	0.93	5.46	1.01	6.00	0.90	1.54
time (sec)	N/A	0.054	0.342	0.152	0.351	0.465	6.514	0.202	0.751

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	83	620	118	1027	80	177
normalized size	1	1.00	1.20	0.94	7.05	1.34	11.67	0.91	2.01
time (sec)	N/A	0.065	0.761	1.078	0.378	0.468	22.695	0.184	0.886

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	192	1999	129	469
normalized size	1	1.00	1.10	0.92	9.46	1.33	13.88	0.90	3.26
time (sec)	N/A	0.097	1.568	0.276	0.423	0.612	113.790	2.527	1.873

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.722	0.579	6.415	0.000	0.658	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	90	789	115	933	89	494
normalized size	1	1.00	0.94	0.93	8.13	1.19	9.62	0.92	5.09
time (sec)	N/A	0.075	0.537	0.993	0.403	0.565	31.735	1.460	1.717

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	189	2030	124	437
normalized size	1	1.00	1.11	0.92	9.86	1.37	14.71	0.90	3.17
time (sec)	N/A	0.098	1.598	0.293	0.433	0.473	113.351	0.183	2.127

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	177	184	2612	291	0	181	997
normalized size	1	1.00	0.91	0.94	13.39	1.49	0.00	0.93	5.11
time (sec)	N/A	0.132	1.661	2.019	0.574	0.590	0.000	0.178	4.315

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	202	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.589	1.025	3.262	0.000	0.579	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	87	84	912	106	918	84	297
normalized size	1	1.00	0.96	0.92	10.02	1.16	10.09	0.92	3.26
time (sec)	N/A	0.066	0.505	0.173	0.391	0.416	32.160	1.920	1.615

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	71	57	414	66	405	56	97
normalized size	1	1.00	1.15	0.92	6.68	1.06	6.53	0.90	1.56
time (sec)	N/A	0.047	0.778	0.169	0.348	0.502	6.539	5.904	0.767

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	155	40	85
normalized size	1	1.00	1.00	0.93	0.93	0.98	3.60	0.93	1.98
time (sec)	N/A	0.036	0.187	0.176	0.312	0.463	1.454	0.262	0.842

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	563	73	31	435	158	112
normalized size	1	1.00	1.00	20.85	2.70	1.15	16.11	5.85	4.15
time (sec)	N/A	0.018	0.150	1.558	0.342	0.457	10.125	0.203	0.894

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	88	888	387	69	5552	0	254
normalized size	1	1.00	2.59	26.12	11.38	2.03	163.29	0.00	7.47
time (sec)	N/A	0.027	0.100	1.969	0.519	0.480	165.058	0.000	5.345

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	150	391	42	0	174	-1
normalized size	1	1.00	0.89	3.95	10.29	1.11	0.00	4.58	-0.03
time (sec)	N/A	0.037	0.176	3.140	0.343	0.536	0.000	3.852	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	14825	1424	94	0	0	-1
normalized size	1	1.00	0.96	221.27	21.25	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.443	8.843	0.577	0.526	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	333	1074	53	0	327	-1
normalized size	1	1.00	0.81	5.64	18.20	0.90	0.00	5.54	-0.02
time (sec)	N/A	0.051	0.343	13.273	0.352	0.483	0.000	0.230	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	97703	3096	107	0	0	-1
normalized size	1	1.00	0.83	1039.39	32.94	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.986	14.867	0.686	0.496	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	242	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.700	1.832	5.432	0.000	0.489	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	371	70	408	61	105
normalized size	1	1.00	1.12	0.93	5.46	1.03	6.00	0.90	1.54
time (sec)	N/A	0.051	0.793	0.631	0.350	0.455	6.697	0.196	0.809

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	108	83	620	108	1027	80	177
normalized size	1	1.00	1.23	0.94	7.05	1.23	11.67	0.91	2.01
time (sec)	N/A	0.068	0.730	1.231	0.379	0.435	22.977	0.198	0.929

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	174	1999	129	495
normalized size	1	1.00	1.10	0.92	9.46	1.21	13.88	0.90	3.44
time (sec)	N/A	0.096	1.759	1.194	0.431	0.432	115.103	0.278	1.969

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	329	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.180	23.938	5.522	0.000	0.540	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	785	116	933	89	471
normalized size	1	1.00	0.93	0.93	8.09	1.20	9.62	0.92	4.86
time (sec)	N/A	0.068	0.527	0.266	0.383	0.441	32.622	0.249	1.613

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	179	2020	124	438
normalized size	1	1.00	1.11	0.92	9.86	1.30	14.64	0.90	3.17
time (sec)	N/A	0.096	1.574	0.288	0.420	0.438	117.133	0.195	2.032

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	2612	264	0	181	951
normalized size	1	1.00	0.90	0.94	13.39	1.35	0.00	0.93	4.88
time (sec)	N/A	0.124	1.546	0.312	0.540	0.497	0.000	0.282	4.136

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	58	325	106	30	333	482	115
normalized size	1	1.00	2.15	12.04	3.93	1.11	12.33	17.85	4.26
time (sec)	N/A	0.017	0.179	1.935	0.359	0.478	8.034	1.556	0.703

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	90	1062	450	71	3266	893	252
normalized size	1	1.00	2.57	30.34	12.86	2.03	93.31	25.51	7.20
time (sec)	N/A	0.028	0.102	2.163	0.376	0.473	102.513	0.466	5.210

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	55	395	45	0	327	-1
normalized size	1	1.00	0.92	1.45	10.39	1.18	0.00	8.61	-0.03
time (sec)	N/A	0.043	0.210	12.388	0.345	0.471	0.000	0.285	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	186	1027	376	0	0	-1
normalized size	1	1.00	0.97	2.58	14.26	5.22	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.372	1.005	0.546	0.535	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	143	520	315	0	0	294
normalized size	1	1.00	2.48	3.25	11.82	7.16	0.00	0.00	6.68
time (sec)	N/A	0.036	0.096	0.792	0.530	0.467	0.000	0.000	5.306

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	99	131	188	0	0	227
normalized size	1	1.00	3.24	3.41	4.52	6.48	0.00	0.00	7.83
time (sec)	N/A	0.016	0.048	0.515	0.515	0.451	0.000	0.000	5.487

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	95	105	197	0	0	233
normalized size	1	1.00	3.21	3.28	3.62	6.79	0.00	0.00	8.03
time (sec)	N/A	0.016	0.057	0.799	0.351	0.447	0.000	0.000	4.846

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	143	612	316	0	0	290
normalized size	1	1.00	2.41	3.11	13.30	6.87	0.00	0.00	6.30
time (sec)	N/A	0.040	0.096	1.007	0.371	0.451	0.000	0.000	5.262

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	184	1254	372	0	0	-1
normalized size	1	1.00	0.96	2.49	16.95	5.03	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.374	0.880	0.396	0.508	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	116	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.806	1.465	0.000	0.477	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	260	0	0	0	0	0	-1
normalized size	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	3.615	1.770	0.000	0.540	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	914	109	918	84	313
normalized size	1	1.00	0.93	0.92	10.04	1.20	10.09	0.92	3.44
time (sec)	N/A	0.065	0.518	0.942	0.405	0.431	32.436	5.945	1.949

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	416	63	405	56	98
normalized size	1	1.00	1.11	0.92	6.71	1.02	6.53	0.90	1.58
time (sec)	N/A	0.045	0.756	0.648	0.372	0.454	6.680	0.196	1.017

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
normalized size	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.033	0.184	0.663	0.333	0.448	1.476	0.820	1.295

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	461	74	31	435	440	109
normalized size	1	1.00	1.00	17.73	2.85	1.19	16.73	16.92	4.19
time (sec)	N/A	0.013	0.121	1.572	0.343	0.536	10.435	0.261	1.039

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	89	1049	391	69	5552	1341	246
normalized size	1	1.00	2.54	29.97	11.17	1.97	158.63	38.31	7.03
time (sec)	N/A	0.029	0.090	2.252	0.524	0.458	160.750	1.829	6.525

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	56	382	40	0	315	-1
normalized size	1	1.00	0.92	1.47	10.05	1.05	0.00	8.29	-0.03
time (sec)	N/A	0.039	0.197	3.672	0.333	0.505	0.000	5.752	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	163	2009	129	495
normalized size	1	1.00	1.10	0.92	9.46	1.13	13.95	0.90	3.44
time (sec)	N/A	0.089	1.574	1.286	0.426	0.631	114.287	4.935	2.445

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	105	83	620	105	1027	80	177
normalized size	1	1.00	1.19	0.94	7.05	1.19	11.67	0.91	2.01
time (sec)	N/A	0.069	0.723	1.689	0.371	0.444	23.830	1.985	1.058

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	2614	240	0	181	999
normalized size	1	1.00	0.90	0.94	13.41	1.23	0.00	0.93	5.12
time (sec)	N/A	0.135	1.637	1.999	0.582	0.480	0.000	0.174	4.542

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	181	1027	366	0	0	-1
normalized size	1	1.00	0.97	2.51	14.26	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.372	0.958	0.542	0.490	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	149	526	316	0	0	285
normalized size	1	1.00	2.41	3.24	11.43	6.87	0.00	0.00	6.20
time (sec)	N/A	0.039	0.094	0.859	0.520	0.509	0.000	0.000	5.243

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	97	131	196	0	0	237
normalized size	1	1.00	3.10	3.23	4.37	6.53	0.00	0.00	7.90
time (sec)	N/A	0.018	0.053	0.608	0.510	0.467	0.000	0.000	4.727

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	93	105	190	0	0	231
normalized size	1	1.00	3.24	3.21	3.62	6.55	0.00	0.00	7.97
time (sec)	N/A	0.021	0.052	0.753	0.346	0.459	0.000	0.000	5.386

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	112	145	613	316	0	0	289
normalized size	1	1.00	2.43	3.15	13.33	6.87	0.00	0.00	6.28
time (sec)	N/A	0.041	0.101	0.842	0.359	0.493	0.000	0.000	5.439

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	71	179	1254	385	0	0	-1
normalized size	1	1.00	0.97	2.45	17.18	5.27	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.342	1.006	0.404	0.514	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.660	1.591	0.000	0.455	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	108	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.795	1.766	0.000	0.411	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [201] had the largest ratio of [.5882]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	18	0.167
2	A	4	3	1.00	18	0.167
3	A	4	3	1.00	18	0.167
4	A	4	3	1.00	18	0.167
5	A	4	3	1.00	18	0.167
6	A	4	3	1.00	18	0.167
7	A	1	1	1.00	16	0.062

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	2	2	1.00	16	0.125
9	A	4	4	1.00	18	0.222
10	A	5	5	1.00	18	0.278
11	A	6	5	1.00	18	0.278
12	A	7	5	1.00	18	0.278
13	A	5	4	1.00	20	0.200
14	A	6	5	1.00	20	0.250
15	A	4	3	1.00	20	0.150
16	A	5	5	1.00	20	0.250
17	A	3	3	1.00	18	0.167
18	A	2	2	1.00	18	0.111
19	A	3	3	1.00	20	0.150
20	A	4	3	1.00	20	0.150
21	A	4	3	1.00	20	0.150
22	A	5	4	1.00	20	0.200
23	A	4	3	1.00	20	0.150
24	A	4	3	1.00	20	0.150
25	A	4	3	1.00	20	0.150
26	A	4	3	1.00	20	0.150
27	A	3	3	1.00	18	0.167
28	A	4	4	1.00	18	0.222
29	A	3	3	1.00	20	0.150
30	A	3	3	1.00	20	0.150
31	A	5	4	1.00	20	0.200
32	A	6	5	1.00	20	0.250
33	A	4	3	1.00	18	0.167
34	A	4	3	1.00	18	0.167
35	A	4	3	1.00	18	0.167
36	A	4	3	1.00	18	0.167
37	A	4	3	1.00	18	0.167
38	A	4	3	1.00	18	0.167
39	A	3	3	1.00	18	0.167
40	A	2	2	1.00	16	0.125
41	A	4	4	1.00	16	0.250
42	A	5	5	1.00	18	0.278
43	A	6	5	1.00	18	0.278

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	7	5	1.00	18	0.278
45	A	9	4	1.00	20	0.200
46	A	5	4	1.00	20	0.200
47	A	7	4	1.00	20	0.200
48	A	4	3	1.00	20	0.150
49	A	5	4	1.00	20	0.200
50	A	3	3	1.00	20	0.150
51	A	3	3	1.00	20	0.150
52	A	2	2	1.00	18	0.111
53	A	4	3	1.00	18	0.167
54	A	4	3	1.00	20	0.150
55	A	5	4	1.00	20	0.200
56	A	4	3	1.00	20	0.150
57	A	5	4	1.00	20	0.200
58	A	4	3	1.00	20	0.150
59	A	4	3	1.00	20	0.150
60	A	4	3	1.00	20	0.150
61	A	4	3	1.00	20	0.150
62	A	4	3	1.00	20	0.150
63	A	4	3	1.00	20	0.150
64	A	4	3	1.00	20	0.150
65	A	3	3	1.00	20	0.150
66	A	3	2	1.00	20	0.100
67	A	4	4	1.00	20	0.200
68	A	3	3	1.00	18	0.167
69	A	5	4	1.00	18	0.222
70	A	6	5	1.00	20	0.250
71	A	6	5	1.00	20	0.250
72	A	8	5	1.00	20	0.250
73	A	4	3	1.00	20	0.150
74	A	3	3	1.00	20	0.150
75	A	2	2	1.00	20	0.100
76	A	1	1	1.00	20	0.050
77	A	1	1	1.00	20	0.050
78	A	2	2	1.00	20	0.100
79	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	4	3	1.00	20	0.150
81	A	4	3	1.00	22	0.136
82	A	3	3	1.00	22	0.136
83	A	3	3	1.00	22	0.136
84	A	2	2	1.00	22	0.091
85	A	2	2	1.00	22	0.091
86	A	2	2	1.00	22	0.091
87	A	2	2	1.00	22	0.091
88	A	3	3	1.00	22	0.136
89	A	4	4	1.00	22	0.182
90	A	3	3	1.00	22	0.136
91	A	2	2	1.00	22	0.091
92	A	3	3	1.00	22	0.136
93	A	1	1	1.00	22	0.045
94	A	2	2	1.00	22	0.091
95	A	3	3	1.00	22	0.136
96	A	4	4	1.00	22	0.182
97	A	5	4	1.00	20	0.200
98	A	4	4	1.00	20	0.200
99	A	3	3	1.00	20	0.150
100	A	2	2	1.00	20	0.100
101	A	1	1	1.00	20	0.050
102	A	3	3	1.00	20	0.150
103	A	4	4	1.00	20	0.200
104	A	5	4	1.00	20	0.200
105	A	4	3	1.00	22	0.136
106	A	4	3	1.00	22	0.136
107	A	3	3	1.00	22	0.136
108	A	3	3	1.00	22	0.136
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	3	3	1.00	22	0.136
112	A	3	3	1.00	22	0.136
113	A	4	3	1.00	22	0.136
114	A	4	3	1.00	22	0.136
115	A	7	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	6	5	1.00	22	0.227
117	A	5	5	1.00	22	0.227
118	A	4	4	1.00	22	0.182
119	A	1	1	1.00	22	0.045
120	A	2	2	1.00	22	0.091
121	A	4	4	1.00	22	0.182
122	A	5	5	1.00	22	0.227
123	A	2	2	1.00	20	0.100
124	A	2	2	1.00	20	0.100
125	A	2	2	1.00	18	0.111
126	A	2	2	1.00	18	0.111
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	4	3	1.00	18	0.167
130	A	4	3	1.00	18	0.167
131	A	4	3	1.00	18	0.167
132	A	4	3	1.00	18	0.167
133	A	4	3	1.00	18	0.167
134	A	4	3	1.00	18	0.167
135	A	1	1	1.00	16	0.062
136	A	2	2	1.00	16	0.125
137	A	4	4	1.00	18	0.222
138	A	5	5	1.00	18	0.278
139	A	6	5	1.00	18	0.278
140	A	7	5	1.00	18	0.278
141	A	5	4	1.00	20	0.200
142	A	6	5	1.00	20	0.250
143	A	4	3	1.00	20	0.150
144	A	5	5	1.00	20	0.250
145	A	3	3	1.00	18	0.167
146	A	2	2	1.00	18	0.111
147	A	3	3	1.00	20	0.150
148	A	4	3	1.00	20	0.150
149	A	4	3	1.00	20	0.150
150	A	5	4	1.00	20	0.200
151	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	4	3	1.00	20	0.150
153	A	4	3	1.00	20	0.150
154	A	4	3	1.00	20	0.150
155	A	3	3	1.00	18	0.167
156	A	4	4	1.00	18	0.222
157	A	3	3	1.00	20	0.150
158	A	3	3	1.00	20	0.150
159	A	5	4	1.00	20	0.200
160	A	6	5	1.00	20	0.250
161	A	4	3	1.00	20	0.150
162	A	3	3	1.00	20	0.150
163	A	2	2	1.00	20	0.100
164	A	1	1	1.00	20	0.050
165	A	1	1	1.00	20	0.050
166	A	2	2	1.00	20	0.100
167	A	3	3	1.00	20	0.150
168	A	4	3	1.00	20	0.150
169	A	4	3	1.00	22	0.136
170	A	3	3	1.00	22	0.136
171	A	3	3	1.00	22	0.136
172	A	2	2	1.00	22	0.091
173	A	2	2	1.00	22	0.091
174	A	2	2	1.00	22	0.091
175	A	2	2	1.00	22	0.091
176	A	3	3	1.00	22	0.136
177	A	4	4	1.00	22	0.182
178	A	3	3	1.00	22	0.136
179	A	2	2	1.00	22	0.091
180	A	3	3	1.00	22	0.136
181	A	1	1	1.00	22	0.045
182	A	2	2	1.00	22	0.091
183	A	3	3	1.00	22	0.136
184	A	4	4	1.00	22	0.182
185	A	1	1	1.00	11	0.091
186	A	2	2	1.00	11	0.182
187	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	2	2	1.00	20	0.100
189	A	2	2	1.00	18	0.111
190	A	4	3	1.00	28	0.107
191	A	10	5	1.00	15	0.333
192	A	6	2	1.00	15	0.133
193	A	5	2	1.00	15	0.133
194	A	4	2	1.00	13	0.154
195	A	3	3	1.00	13	0.231
196	A	4	4	1.00	15	0.267
197	A	5	5	1.00	15	0.333
198	A	5	5	1.00	15	0.333
199	A	5	4	1.00	15	0.267
200	A	6	5	1.00	15	0.333
201	A	15	10	1.00	17	0.588
202	A	5	2	1.00	15	0.133
203	A	6	2	1.00	17	0.118
204	A	8	2	1.00	17	0.118
205	A	18	5	1.00	17	0.294
206	A	6	2	1.00	15	0.133
207	A	8	2	1.00	17	0.118
208	A	10	2	1.00	17	0.118
209	A	8	4	1.00	15	0.267
210	A	6	2	1.00	15	0.133
211	A	5	2	1.00	15	0.133
212	A	4	2	1.00	13	0.154
213	A	3	3	1.00	13	0.231
214	A	4	4	1.00	15	0.267
215	A	5	5	1.00	15	0.333
216	A	5	5	1.00	15	0.333
217	A	5	4	1.00	15	0.267
218	A	6	5	1.00	15	0.333
219	A	11	7	1.00	17	0.412
220	A	5	2	1.00	15	0.133
221	A	6	2	1.00	17	0.118
222	A	8	2	1.00	17	0.118
223	A	14	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	6	2	1.00	15	0.133
225	A	8	2	1.00	17	0.118
226	A	10	2	1.00	17	0.118
227	A	3	3	1.00	13	0.231
228	A	4	4	1.00	15	0.267
229	A	5	5	1.00	15	0.333
230	A	9	7	1.00	15	0.467
231	A	6	6	1.00	15	0.400
232	A	3	3	1.00	13	0.231
233	A	3	3	1.00	13	0.231
234	A	6	6	1.00	15	0.400
235	A	9	7	1.00	15	0.467
236	A	6	3	1.00	13	0.231
237	A	6	3	1.00	13	0.231
238	A	6	2	1.00	15	0.133
239	A	5	2	1.00	15	0.133
240	A	4	2	1.00	13	0.154
241	A	3	3	1.00	13	0.231
242	A	4	4	1.00	15	0.267
243	A	5	5	1.00	15	0.333
244	A	8	2	1.00	17	0.118
245	A	6	2	1.00	17	0.118
246	A	10	2	1.00	17	0.118
247	A	9	7	1.00	15	0.467
248	A	6	6	1.00	15	0.400
249	A	3	3	1.00	13	0.231
250	A	3	3	1.00	13	0.231
251	A	6	6	1.00	15	0.400
252	A	9	7	1.00	15	0.467
253	A	6	3	1.00	13	0.231
254	A	6	3	1.00	13	0.231

Chapter 3

Listing of integrals

3.1 $\int \sin(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{15}(a+bx)}{15b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{9b}$$

```
[Out] 128/9*sin(b*x+a)^9/b-384/11*sin(b*x+a)^11/b+384/13*sin(b*x+a)^13/b-128/15*s
in(b*x+a)^15/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4288, 2564, 270\}$

$$-\frac{128\sin^{15}(a+bx)}{15b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^9)/(9*b) - (384*Sin[a + b*x]^11)/(11*b) + (384*Sin[a + b*
x]^13)/(13*b) - (128*Sin[a + b*x]^15)/(15*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^7(a+bx)\sin^8(a+bx)\,dx$$
$$= \frac{128\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\,\text{Subst}\left(\int \left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\sin^9(a+bx)}{9b} - \frac{384\sin^{11}(a+bx)}{11b} + \frac{384\sin^{13}(a+bx)}{13b} - \frac{128\sin^{15}(a+bx)}{15b}$$

Mathematica [A] time = 0.48, size = 47, normalized size = 0.77

$$\frac{4\sin^9(a+bx)(10755\cos(2(a+bx))+3366\cos(4(a+bx))+429\cos(6(a+bx))+8330)}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*(8330 + 10755*Cos[2*(a + b*x)] + 3366*Cos[4*(a + b*x)] + 429*Cos[6*(a +
b*x)])*Sin[a + b*x]^9)/(6435*b)
```

fricas [A] time = 0.48, size = 83, normalized size = 1.36

$$\frac{128\left(429\cos(bx+a)^{14}-1518\cos(bx+a)^{12}+1854\cos(bx+a)^{10}-800\cos(bx+a)^{8}+5\cos(bx+a)^{6}+6\cos\right.}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^14 - 1518*cos(b*x + a)^12 + 1854*cos(b*x + a)^10
 - 800*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a
)^2 + 16)*sin(b*x + a)/b
```

giac [B] time = 3.14, size = 110, normalized size = 1.80

$$\frac{\sin(15\,bx+15\,a)}{1920\,b} - \frac{\sin(13\,bx+13\,a)}{1664\,b} - \frac{7\,\sin(11\,bx+11\,a)}{1408\,b} + \frac{7\,\sin(9\,bx+9\,a)}{1152\,b} + \frac{3\,\sin(7\,bx+7\,a)}{128\,b} - \frac{21\,\sin(5\,bx+}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] 1/1920*sin(15*b*x + 15*a)/b - 1/1664*sin(13*b*x + 13*a)/b - 7/1408*sin(11*b
*x + 11*a)/b + 7/1152*sin(9*b*x + 9*a)/b + 3/128*sin(7*b*x + 7*a)/b - 21/64
0*sin(5*b*x + 5*a)/b - 35/384*sin(3*b*x + 3*a)/b + 35/128*sin(b*x + a)/b
```

maple [B] time = 1.94, size = 111, normalized size = 1.82

$$\frac{35\sin(bx+a)}{128b} - \frac{35\sin(3bx+3a)}{384b} - \frac{21\sin(5bx+5a)}{640b} + \frac{3\sin(7bx+7a)}{128b} + \frac{7\sin(9bx+9a)}{1152b} - \frac{7\sin(11bx+11a)}{1408b} - \text{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] 35/128*sin(b*x+a)/b-35/384*sin(3*b*x+3*a)/b-21/640/b*sin(5*b*x+5*a)+3/128/b
*sin(7*b*x+7*a)+7/1152/b*sin(9*b*x+9*a)-7/1408/b*sin(11*b*x+11*a)-1/1664/b*
sin(13*b*x+13*a)+1/1920/b*sin(15*b*x+15*a)
```

maxima [A] time = 0.37, size = 91, normalized size = 1.49

$$\frac{429 \sin(15\,bx + 15\,a) - 495 \sin(13\,bx + 13\,a) - 4095 \sin(11\,bx + 11\,a) + 5005 \sin(9\,bx + 9\,a) + 19305 \sin(}{823680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*sin(15*b*x + 15*a) - 495*sin(13*b*x + 13*a) - 4095*sin(11*b*x
 + 11*a) + 5005*sin(9*b*x + 9*a) + 19305*sin(7*b*x + 7*a) - 27027*sin(5*b*x
 + 5*a) - 75075*sin(3*b*x + 3*a) + 225225*sin(b*x + a))/b
```

mupad [B] time = 0.08, size = 45, normalized size = 0.74

$$\frac{-\frac{128 \sin(a+b\,x)^{15}}{15} + \frac{384 \sin(a+b\,x)^{13}}{13} - \frac{384 \sin(a+b\,x)^{11}}{11} + \frac{128 \sin(a+b\,x)^{9}}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

```
[Out] ((128*sin(a + b*x)^9)/9 - (384*sin(a + b*x)^11)/11 + (384*sin(a + b*x)^13)/
13 - (128*sin(a + b*x)^15)/15)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

3.2 $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{64\cos^{13}(a+bx)}{13b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{9}(a+bx)}{3b}-\frac{64\cos^{7}(a+bx)}{7b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+64/3*cos(b*x+a)^9/b-192/11*cos(b*x+a)^11/b+64/13*cos(b
*x+a)^13/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$\frac{64\cos^{13}(a+bx)}{13b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{9}(a+bx)}{3b}-\frac{64\cos^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(3*b) - (192*Cos[a + b*x]^
11)/(11*b) + (64*Cos[a + b*x]^13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^6(2a+2bx)\,dx&=64\int \cos^6(a+bx)\sin^7(a+bx)\,dx\\&=-\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\&=-\frac{64\,\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)\,dx,x,\cos(a+bx)\right)}{b}\\&=-\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.33, size = 47, normalized size = 0.77

$$\frac{2\cos^7(a+bx)(6377\cos(2(a+bx))-1890\cos(4(a+bx))+231\cos(6(a+bx))-5230)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (2*Cos[a + b*x]^7*(-5230 + 6377*Cos[2*(a + b*x)] - 1890*Cos[4*(a + b*x)] +
231*Cos[6*(a + b*x)]))/(3003*b)
```

fricas [A] time = 0.43, size = 46, normalized size = 0.75

$$\frac{64\left(231\cos\left(bx+a\right)^{13}-819\cos\left(bx+a\right)^{11}+1001\cos\left(bx+a\right)^{9}-429\cos\left(bx+a\right)^{7}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/3003*(231*cos(b*x + a)^13 - 819*cos(b*x + a)^11 + 1001*cos(b*x + a)^9 -
429*cos(b*x + a)^7)/b
```

giac [A] time = 1.91, size = 96, normalized size = 1.57

$$\frac{\cos\left(13\,bx+13\,a\right)}{832\,b}-\frac{\cos\left(11\,bx+11\,a\right)}{704\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{96\,b}+\frac{3\cos\left(7\,bx+7\,a\right)}{224\,b}+\frac{3\cos\left(5\,bx+5\,a\right)}{64\,b}-\frac{5\cos\left(3\,bx+3\right.}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/832*cos(13*b*x + 13*a)/b - 1/704*cos(11*b*x + 11*a)/b - 1/96*cos(9*b*x +
9*a)/b + 3/224*cos(7*b*x + 7*a)/b + 3/64*cos(5*b*x + 5*a)/b - 5/64*cos(3*b*
x + 3*a)/b - 5/16*cos(b*x + a)/b
```

maple [A] time = 0.26, size = 97, normalized size = 1.59

$$-\frac{5\cos\left(bx+a\right)}{16b}-\frac{5\cos\left(3bx+3a\right)}{64b}+\frac{3\cos\left(5bx+5a\right)}{64b}+\frac{3\cos\left(7bx+7a\right)}{224b}-\frac{\cos\left(9bx+9a\right)}{96b}-\frac{\cos\left(11bx+11a\right)}{704b}+\frac{\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] -5/16*cos(b*x+a)/b-5/64*cos(3*b*x+3*a)/b+3/64*cos(5*b*x+5*a)/b+3/224*cos(7*
b*x+7*a)/b-1/96*cos(9*b*x+9*a)/b-1/704*cos(11*b*x+11*a)/b+1/832*cos(13*b*x+
13*a)/b
```

maxima [A] time = 0.35, size = 80, normalized size = 1.31

$$\frac{231\cos\left(13\,bx+13\,a\right)-273\cos\left(11\,bx+11\,a\right)-2002\cos\left(9\,bx+9\,a\right)+2574\cos\left(7\,bx+7\,a\right)+9009\cos\left(5\,b\right.}{192192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
+ 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x +
3*a) - 60060*cos(b*x + a))/b
```

mupad [B] time = 0.13, size = 46, normalized size = 0.75

$$-\frac{-\frac{64\cos(a+b\,x)^{13}}{13}+\frac{192\cos(a+b\,x)^{11}}{11}-\frac{64\cos(a+b\,x)^{9}}{3}+\frac{64\cos(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

```
[Out] -((64*cos(a + b*x)^7)/7 - (64*cos(a + b*x)^9)/3 + (192*cos(a + b*x)^11)/11
- (64*cos(a + b*x)^13)/13)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

3.3 $\int \sin(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^7(a+bx)}{7b}$$

```
[Out] 32/7*sin(b*x+a)^7/b-64/9*sin(b*x+a)^9/b+32/11*sin(b*x+a)^11/b
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$\frac{32\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(9*b) + (32*Sin[a + b*x]^11
)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{32\,\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{32\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.26, size = 37, normalized size = 0.80

$$\frac{4\sin^7(a+bx)(364\cos(2(a+bx))+63\cos(4(a+bx))+365)}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(365 + 364*Cos[2*(a + b*x)] + 63*Cos[4*(a + b*x)])*Sin[a + b*x]^7)/(693*
b)
```

fricas [A] time = 0.42, size = 63, normalized size = 1.37

$$-\frac{32\left(63\cos\left(bx+a\right)^{10}-161\cos\left(bx+a\right)^{8}+113\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+\right.}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^10 - 161*cos(b*x + a)^8 + 113*cos(b*x + a)^6 - 3*c
os(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b
```

giac [B] time = 0.71, size = 82, normalized size = 1.78

$$-\frac{\sin\left(11\,bx+11\,a\right)}{352\,b}+\frac{\sin\left(9\,bx+9\,a\right)}{288\,b}+\frac{5\,\sin\left(7\,bx+7\,a\right)}{224\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{32\,b}-\frac{5\,\sin\left(3\,bx+3\,a\right)}{48\,b}+\frac{5\,\sin\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/352*sin(11*b*x + 11*a)/b + 1/288*sin(9*b*x + 9*a)/b + 5/224*sin(7*b*x +
7*a)/b - 1/32*sin(5*b*x + 5*a)/b - 5/48*sin(3*b*x + 3*a)/b + 5/16*sin(b*x +
 a)/b
```

maple [B] time = 1.29, size = 83, normalized size = 1.80

$$\frac{5\sin\left(bx+a\right)}{16b}-\frac{5\sin\left(3bx+3a\right)}{48b}-\frac{\sin\left(5bx+5a\right)}{32b}+\frac{5\sin\left(7bx+7a\right)}{224b}+\frac{\sin\left(9bx+9a\right)}{288b}-\frac{\sin\left(11bx+11a\right)}{352b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^5,x)
```

```
[Out] 5/16*sin(b*x+a)/b-5/48*sin(3*b*x+3*a)/b-1/32/b*sin(5*b*x+5*a)+5/224/b*sin(7
*b*x+7*a)+1/288/b*sin(9*b*x+9*a)-1/352/b*sin(11*b*x+11*a)
```

maxima [A] time = 0.34, size = 69, normalized size = 1.50

$$-\frac{63\sin\left(11\,bx+11\,a\right)-77\sin\left(9\,bx+9\,a\right)-495\sin\left(7\,bx+7\,a\right)+693\sin\left(5\,bx+5\,a\right)+2310\sin\left(3\,bx+3\,a\right)-}{22176\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*sin(11*b*x + 11*a) - 77*sin(9*b*x + 9*a) - 495*sin(7*b*x + 7*a
) + 693*sin(5*b*x + 5*a) + 2310*sin(3*b*x + 3*a) - 6930*sin(b*x + a))/b
```

mupad [B] time = 0.09, size = 36, normalized size = 0.78

$$\frac{32\,\left(63\sin\left(a+b\,x\right)^{11}-154\sin\left(a+b\,x\right)^{9}+99\sin\left(a+b\,x\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

```
[Out] (32*(99*sin(a + b*x)^7 - 154*sin(a + b*x)^9 + 63*sin(a + b*x)^11))/(693*b)
```

sympy [A] time = 124.08, size = 197, normalized size = 4.28

$$\begin{cases} -\frac{422\sin(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{693b} - \frac{608\sin(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{693b} - \frac{256\sin(a+bx)\cos^5(2a+2bx)}{693b} + \frac{151\sin^5(2a+2b}{693} \\ x\sin(a)\sin^5(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-422*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(693*b) -
 608*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(693*b) - 256*sin
(a + b*x)*cos(2*a + 2*b*x)**5/(693*b) + 151*sin(2*a + 2*b*x)**5*cos(a + b*x
)/(693*b) + 272*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(693*b
) + 128*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(693*b), Ne(b, 0)
), (x*sin(a)*sin(2*a)**5, True))
```

3.4 $\int \sin(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{16\cos^9(a+bx)}{9b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+32/7*cos(b*x+a)^7/b-16/9*cos(b*x+a)^9/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$-\frac{16\cos^9(a+bx)}{9b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (32*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 37, normalized size = 0.80

$$\frac{2\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (2*Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(315
*b)
```

fricas [A] time = 0.41, size = 36, normalized size = 0.78

$$-\frac{16\left(35\cos\left(bx+a\right)^{9}-90\cos\left(bx+a\right)^{7}+63\cos\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

giac [A] time = 0.45, size = 68, normalized size = 1.48

$$-\frac{\cos\left(9\,bx+9\,a\right)}{144\,b}+\frac{\cos\left(7\,bx+7\,a\right)}{112\,b}+\frac{\cos\left(5\,bx+5\,a\right)}{20\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{12\,b}-\frac{3\cos\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/144*cos(9*b*x + 9*a)/b + 1/112*cos(7*b*x + 7*a)/b + 1/20*cos(5*b*x + 5*a
)/b - 1/12*cos(3*b*x + 3*a)/b - 3/8*cos(b*x + a)/b
```

maple [A] time = 0.24, size = 69, normalized size = 1.50

$$-\frac{3\cos\left(bx+a\right)}{8b}-\frac{\cos\left(3bx+3a\right)}{12b}+\frac{\cos\left(5bx+5a\right)}{20b}+\frac{\cos\left(7bx+7a\right)}{112b}-\frac{\cos\left(9bx+9a\right)}{144b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] -3/8*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b+1/112*cos(7*b
*x+7*a)/b-1/144*cos(9*b*x+9*a)/b
```

maxima [A] time = 0.34, size = 58, normalized size = 1.26

$$-\frac{35\cos\left(9\,bx+9\,a\right)-45\cos\left(7\,bx+7\,a\right)-252\cos\left(5\,bx+5\,a\right)+420\cos\left(3\,bx+3\,a\right)+1890\cos\left(bx+a\right)}{5040\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] -1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a) +
 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

mupad [B] time = 0.14, size = 36, normalized size = 0.78

$$-\frac{16\left(35\cos\left(a+b\,x\right)^9-90\cos\left(a+b\,x\right)^7+63\cos\left(a+b\,x\right)^5\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

```
[Out] -(16*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

sympy [A] time = 40.28, size = 163, normalized size = 3.54

$$\begin{cases} -\frac{104\sin(a+bx)\sin^3(2a+2bx)\cos(2a+2bx)}{315b} - \frac{64\sin(a+bx)\sin(2a+2bx)\cos^3(2a+2bx)}{315b} - \frac{107\sin^4(2a+2bx)\cos(a+bx)}{315b} - \frac{16\sin^2(2a+2bx)\cos(a}{21b} \\ x\sin(a)\sin^4(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((-104*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(315*b) -
 64*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(315*b) - 107*sin(2*a
 + 2*b*x)**4*cos(a + b*x)/(315*b) - 16*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos
(2*a + 2*b*x)**2/(21*b) - 128*cos(a + b*x)*cos(2*a + 2*b*x)**4/(315*b), Ne(
b, 0)), (x*sin(a)*sin(2*a)**4, True))
```

3.5 $\int \sin(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

```
[Out] 8/5*sin(b*x+a)^5/b-8/7*sin(b*x+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 14}

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^5)/(5*b) - (8*Sin[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{8\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 27, normalized size = 0.87

$$\frac{4\sin^5(a+bx)(5\cos(2(a+bx))+9)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*(9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(35*b)
```

fricas [A] time = 0.47, size = 41, normalized size = 1.32

$$\frac{8\left(5\cos\left(bx+a\right)^{6}-8\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

giac [A] time = 0.37, size = 54, normalized size = 1.74

$$\frac{\sin\left(7\,bx+7\,a\right)}{56\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{40\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{8\,b}+\frac{3\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/56*sin(7*b*x + 7*a)/b - 1/40*sin(5*b*x + 5*a)/b - 1/8*sin(3*b*x + 3*a)/b
+ 3/8*sin(b*x + a)/b
```

maple [A] time = 0.93, size = 55, normalized size = 1.77

$$\frac{3\sin\left(bx+a\right)}{8b}-\frac{\sin\left(3bx+3a\right)}{8b}-\frac{\sin\left(5bx+5a\right)}{40b}+\frac{\sin\left(7bx+7a\right)}{56b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^3,x)
```

```
[Out] 3/8*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b-1/40/b*sin(5*b*x+5*a)+1/56/b*sin(7*b*
x+7*a)
```

maxima [A] time = 0.34, size = 47, normalized size = 1.52

$$\frac{5\sin\left(7\,bx+7\,a\right)-7\sin\left(5\,bx+5\,a\right)-35\sin\left(3\,bx+3\,a\right)+105\sin\left(bx+a\right)}{280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(5*sin(7*b*x + 7*a) - 7*sin(5*b*x + 5*a) - 35*sin(3*b*x + 3*a) + 105*
sin(b*x + a))/b
```

mupad [B] time = 0.05, size = 26, normalized size = 0.84

$$\frac{8\,\left(7\sin\left(a+b\,x\right)^5-5\sin\left(a+b\,x\right)^7\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

```
[Out] (8*(7*sin(a + b*x)^5 - 5*sin(a + b*x)^7))/(35*b)
```

sympy [A] time = 11.91, size = 126, normalized size = 4.06

$$\begin{cases} -\frac{22\sin(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{35b} - \frac{16\sin(a+bx)\cos^3(2a+2bx)}{35b} + \frac{9\sin^3(2a+2bx)\cos(a+bx)}{35b} + \frac{8\sin(2a+2bx)\cos(a+bx)\cos^2(2a- \\ x\sin(a)\sin^3(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-22*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(35*b) - 1
6*sin(a + b*x)*cos(2*a + 2*b*x)**3/(35*b) + 9*sin(2*a + 2*b*x)**3*cos(a + b
*x)/(35*b) + 8*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b), Ne
(b, 0)), (x*sin(a)*sin(2*a)**3, True))
```

3.6 $\int \sin(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{4\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] -4/3*cos(b*x+a)^3/b+4/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 14}

$$\frac{4\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (4*Cos[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{4\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 0.87

$$\frac{2\cos^3(a+bx)(3\cos(2(a+bx))-7)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
[Out] (2*Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(15*b)
```

fricas [A] time = 0.51, size = 26, normalized size = 0.84

$$\frac{4\left(3\cos\left(bx+a\right)^{5}-5\cos\left(bx+a\right)^{3}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] 4/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

giac [A] time = 0.35, size = 40, normalized size = 1.29

$$\frac{\cos\left(5\,bx+5\,a\right)}{20\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{12\,b}-\frac{\cos\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
[Out] 1/20*cos(5*b*x + 5*a)/b - 1/12*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

maple [A] time = 0.27, size = 41, normalized size = 1.32

$$-\frac{\cos\left(bx+a\right)}{2b}-\frac{\cos\left(3bx+3a\right)}{12b}+\frac{\cos\left(5bx+5a\right)}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^2,x)
[Out] -1/2*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b
```

maxima [A] time = 0.34, size = 36, normalized size = 1.16

$$\frac{3\cos\left(5\,bx+5\,a\right)-5\cos\left(3\,bx+3\,a\right)-30\cos\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] 1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

mupad [B] time = 0.11, size = 26, normalized size = 0.84

$$-\frac{4\left(5\cos\left(a+b\,x\right)^{3}-3\cos\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^2,x)
[Out] -(4*(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5))/(15*b)
```

sympy [A] time = 3.40, size = 92, normalized size = 2.97

$$\begin{cases} -\frac{4\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(2a+2bx\right)}{15b}-\frac{7\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)}{15b}-\frac{8\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{15b} & \text{for}\ b\neq 0\\ x\sin\left(a\right)\sin^{2}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-4*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(15*b) - 7*sin
(2*a + 2*b*x)**2*cos(a + b*x)/(15*b) - 8*cos(a + b*x)*cos(2*a + 2*b*x)**2/(
15*b), Ne(b, 0)), (x*sin(a)*sin(2*a)**2, True))
```

3.7 $\int \sin(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

```
[Out] 1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4282}

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 0.50

$$\frac{2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^3)/(3*b)
```

fricas [A] time = 0.44, size = 21, normalized size = 0.70

$$-\frac{2\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

giac [A] time = 0.32, size = 26, normalized size = 0.87

$$-\frac{\sin\left(3\,bx+3\,a\right)}{6\,b}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

maple [A] time = 0.48, size = 27, normalized size = 0.90

$$\frac{\sin(bx+a)}{2b} - \frac{\sin(3bx+3a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

maxima [A] time = 0.32, size = 26, normalized size = 0.87

$$-\frac{\sin(3\,bx+3\,a)}{6\,b} + \frac{\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

mupad [B] time = 0.36, size = 44, normalized size = 1.47

$$\begin{cases} 2\,x\,\left(\cos(a)-\cos(a)^3\right) & \text{if } b=0 \\ \frac{3\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x)}{6\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x),x)
```

```
[Out] piecewise(b == 0, 2*x*(cos(a) - cos(a)^3), b ~= 0, (3*sin(a + b*x) - sin(3*
a + 3*b*x))/(6*b))
```

sympy [A] time = 0.81, size = 51, normalized size = 1.70

$$\begin{cases} -\frac{2\sin(a+bx)\cos(2a+2bx)}{3b} + \frac{\sin(2a+2bx)\cos(a+bx)}{3b} & \text{for } b\neq 0 \\ x\sin(a)\sin(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)*cos(2*a + 2*b*x)/(3*b) + sin(2*a + 2*b*x)*cos(a
+ b*x)/(3*b), Ne(b, 0)), (x*sin(a)*sin(2*a), True))
```

3.8 $\int \csc(2a + 2bx) \sin(a + bx)\, dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b}$$

[Out] 1/2*arctanh(sin(b*x+a))/b

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4288, 3770}

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/(2*b)

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2a + 2bx) \sin(a + bx)\, dx &= \frac{1}{2} \int \sec(a + bx)\, dx\\ &= \frac{\tanh^{-1}(\sin(a + bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/(2*b)

fricas [B] time = 0.48, size = 28, normalized size = 2.00

$$\frac{\log\left(\sin\left(bx + a\right) + 1\right) - \log\left(-\sin\left(bx + a\right) + 1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="fricas")

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

giac [B] time = 0.80, size = 174, normalized size = 12.43

$$\log\left(\left|\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{3}+3\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{2}-\tan\left(\frac{1}{2}a\right)^{3}-3\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)+3\text{ ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2
*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - t
an(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) - log(abs(tan(1/2*b*x + 2*a)*tan(1/2
*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x +
2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))
/b
```

maple [A] time = 0.68, size = 20, normalized size = 1.43

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a),x)
```

```
[Out] 1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.46, size = 115, normalized size = 8.21

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
+ 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2))/b
```

mupad [B] time = 0.15, size = 12, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x),x)
```

```
[Out] atanh(sin(a + b*x))/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.9 $\int \csc^2(2a + 2bx)\sin(a + bx)\, dx$

Optimal. Leaf size=28

$$\frac{\sec(a+bx)}{4b} - \frac{\tanh^{-1}(\cos(a+bx))}{4b}$$

```
[Out] -1/4*arctanh(cos(b*x+a))/b+1/4*sec(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2622, 321, 207}

$$\frac{\sec(a+bx)}{4b} - \frac{\tanh^{-1}(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(4*b) + Sec[a + b*x]/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{4}\int \csc(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b} \\
&= \frac{\sec(a+bx)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b} \\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{4b} + \frac{\sec(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.79

$$
\frac{\sec(a+bx)}{4b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{4b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/(4*b) + Sec[a + b*x]/(
4*b)
```

fricas [B] time = 0.45, size = 52, normalized size = 1.86

$$
-\frac{\cos\left(bx+a\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right) - \cos\left(bx+a\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right) - 2}{8\,b\cos\left(bx+a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

giac [B] time = 2.37, size = 412, normalized size = 14.71

$$
-\frac{2\left(6\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{11}-\tan\left(\frac{1}{2}a\right)^{12}-2\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{9}+12\tan\left(\frac{1}{2}a\right)^{10}-36\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{7}+27\tan\left(\frac{1}{2}a\right)^{8}-}{\left(\tan\left(\frac{1}{2}bx+2a\right)^{2}\tan\left(\frac{1}{2}a\right)^{6}-15\tan\left(\frac{1}{2}bx+2a\right)^{2}\tan\left(\frac{1}{2}a\right)^{4}+12\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{5}-\tan\left(\frac{1}{2}a\right)^{6}+15\tan\left(\frac{1}{2}bx+2a\right)^{2}\tan\left(\frac{1}{2}a\right)^{2}-40\tan\left(\frac{1}{2}bx+2a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^
7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x +
2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 12*
tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15
*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 1
5*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1))
 + log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a
```

```
) + 3*tan(1/2*a)^2 - 1)) - log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(
1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

maple [A] time = 0.98, size = 36, normalized size = 1.29

$$\frac{1}{4b\cos(bx+a)}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a),x)
```

```
[Out] 1/4/b/cos(b*x+a)+1/4/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.35, size = 236, normalized size = 8.43

$$4\cos(2bx+2a)\cos(bx+a)-\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)\log\left(\cos(bx)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*b*x + 2*a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^
2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
 + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(2*b*x + 2*
a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a
)^2 + 2*b*cos(2*b*x + 2*a) + b)
```

mupad [B] time = 0.11, size = 26, normalized size = 0.93

$$\frac{1}{4\,b\,\cos\left(a+b\,x\right)}-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

```
[Out] 1/(4*b*cos(a + b*x)) - atanh(cos(a + b*x))/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.10 $\int \csc^3(2a + 2bx) \sin(a + bx)\, dx$

Optimal. Leaf size=49

$$-\frac{3\csc(a+bx)}{16b}+\frac{3\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 3/16*arctanh(sin(b*x+a))/b-3/16*csc(b*x+a)/b+1/16*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 321, 207}

$$-\frac{3\csc(a+bx)}{16b}+\frac{3\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(16*b) - (3*Csc[a + b*x])/(16*b) + (Csc[a + b*x]*
Sec[a + b*x]^2)/(16*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(2a+2bx)\sin(a+bx)\,dx = \frac{1}{8}\int \csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}$$
$$= \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{16b} - \frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 0.59

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\sin^2(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
```

fricas [A] time = 0.49, size = 85, normalized size = 1.73

$$\frac{3\cos\left(bx+a\right)^2\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right) - 3\cos\left(bx+a\right)^2\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right) - 6\cos}{32\,b\cos\left(bx+a\right)^2\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^
2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x +
a)^2*sin(b*x + a))
```

giac [B] time = 4.65, size = 1421, normalized size = 29.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*((tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)
^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9
 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 +
 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) +
 6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*t
an(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*
tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(
1/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) + 2*(tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 - 6*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 756*tan(1/
```

```
2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 - 114
*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23 + 2058*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 + 1932*tan(1/2
*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 7462*tan(1/2*b*x + 2
*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 2331*tan(1/2*b*x + 2*a)*
tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*tan(1/2*a)^15 + 9396*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11
+ 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 + 15588*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 + 6468*
tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 6210*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 5668*tan(1/2*
a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 2178*tan(1/2*a)^7 - 30*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 -
 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2*a)^5 - tan(1/2*b*x + 2
*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(1/2*b*x + 2*a)*tan(1/2*
a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1/2*a))/((tan(1/2*a)^1
2 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6 + 255*tan(1/2*a)
^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^
6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)
^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2
*a) - 15*tan(1/2*a)^2 + 1)^2) - 3*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 +
 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*ta
n(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 3*log
(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 +
tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b
*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.98, size = 55, normalized size = 1.12

$$\frac{1}{16b\sin(bx+a)\cos(bx+a)^2} - \frac{3}{16b\sin(bx+a)} + \frac{3\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a),x)
```

```
[Out] 1/16/b/sin(b*x+a)/cos(b*x+a)^2-3/16/b/sin(b*x+a)+3/16/b*ln(sec(b*x+a)+tan(b
*x+a))
```

maxima [B] time = 0.50, size = 808, normalized size = 16.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(5*b*x + 5*a) + 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b*
x + 6*a) - 12*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 3*(2*(cos(4*b*x + 4*
a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 - 2*(cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2
 + 2*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6
*a)^2 + sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^
```

```
2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^
2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x + 5*a) + 2*cos(3*
b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) + 12*(cos(4*b*x + 4*a) - cos(
2*b*x + 2*a) - 1)*sin(5*b*x + 5*a) - 4*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a)
)*sin(4*b*x + 4*a) - 8*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 12*cos(b*x + a)*sin(2*b*x + 2*a) - 12*cos(2*b*x
 + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x
 + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a
)^2 - 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*c
os(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) - 2*(b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) + 2*(b*sin(4*b*x + 4*a
) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.12, size = 48, normalized size = 0.98

$$\frac{3\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b}+\frac{\frac{3\,{\sin\left(a+b\,x\right)}^2}{16}-\frac{1}{8}}{b\,\left(\sin\left(a+b\,x\right)-{\sin\left(a+b\,x\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

```
[Out] (3*atanh(sin(a + b*x)))/(16*b) + ((3*sin(a + b*x)^2)/16 - 1/8)/(b*(sin(a +
b*x) - sin(a + b*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.11 $\int \csc^4(2a + 2bx)\sin(a + bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sec^3(a+bx)}{96b}+\frac{5\sec(a+bx)}{32b}-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

```
[Out] -5/32*arctanh(cos(b*x+a))/b+5/32*sec(b*x+a)/b+5/96*sec(b*x+a)^3/b-1/32*csc(
b*x+a)^2*sec(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{5\sec^3(a+bx)}{96b}+\frac{5\sec(a+bx)}{32b}-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(32*b) + (5*Sec[a + b*x])/(32*b) + (5*Sec[a + b*
x]^3)/(96*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(32*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \frac{1}{16}\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{16b}$$

$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{32b}$$

$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+b\right.}{32b}$$

$$= \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{5\tanh^{-1}(\cos(a+bx))}{32b} + \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)\,\mathrm{s}}{32b}$$

Mathematica [B] time = 0.48, size = 205, normalized size = 3.11

$$\frac{\csc^8(a+bx)\left(-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos(3(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[4*
(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]
] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[Sin
[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]*(
-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(24*b*(Csc[(a
+ b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.48, size = 112, normalized size = 1.70

$$\frac{30\cos(bx+a)^4 - 20\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 15\left(\cos(bx\right.}{192\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/192*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
 + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)
*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

giac [B] time = 8.64, size = 3028, normalized size = 45.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/384*(3*(6*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^24 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 60*tan(1/2*b*x + 2*a)^2
```

```
*tan(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 + 798*tan(1/2*b*x + 2*a
)^3*tan(1/2*a)^19 - 924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 290*tan(1/2*b*
x + 2*a)*tan(1/2*a)^21 - 18*tan(1/2*a)^22 - 1170*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^17 + 3892*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2310*tan(1/2*b*x + 2*a
)*tan(1/2*a)^19 + 336*tan(1/2*a)^20 - 3188*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
15 + 1467*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 + 3186*tan(1/2*b*x + 2*a)*tan(
1/2*a)^17 - 1190*tan(1/2*a)^18 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 -
12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*
a)^15 - 288*tan(1/2*a)^16 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 10332
*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 4428*tan(1/2*a)^14 - 3188*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^9 + 12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 10332*tan
(1/2*b*x + 2*a)*tan(1/2*a)^11 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 14
67*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^9
 - 4428*tan(1/2*a)^10 + 798*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 3892*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^6 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 288*ta
n(1/2*a)^8 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 924*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^4 - 2310*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 1190*tan(1/2*a)^6 +
 6*tan(1/2*b*x + 2*a)^3*tan(1/2*a) - 60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 +
 290*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 336*tan(1/2*a)^4 + tan(1/2*b*x + 2*a
)^2 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a) + 18*tan(1/2*a)^2)/((9*tan(1/2*a)^10
- 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(3
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*t
an(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*t
an(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan
(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^2) + 16*(5
4*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36
 - 2610*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 1620*tan(1/2*b*x + 2*a)^4*tan(
1/2*a)^34 - 252*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 12*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^36 + 52920*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 - 58455*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^32 + 19956*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33 - 280
8*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 198*tan(1/2*b*x + 2*a)*tan(1/2*a)^35
 - 7*tan(1/2*a)^36 - 573912*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 + 942984*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 503568*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31
 + 112068*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 11298*tan(1/2*b*x + 2*a)*tan
(1/2*a)^33 + 468*tan(1/2*a)^34 + 3241488*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^27
 - 7636140*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 5941584*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^29 - 1893936*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 + 257976*tan(1
/2*b*x + 2*a)*tan(1/2*a)^31 - 12969*tan(1/2*a)^32 - 6862752*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^25 + 27168408*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^26 - 3237094
4*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 15361632*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^28 - 2923992*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 188808*tan(1/2*a)^30 -
 1663560*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 24990420*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^24 + 67201056*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 54063504*tan
(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 16195152*tan(1/2*b*x + 2*a)*tan(1/2*a)^27
 - 1484964*tan(1/2*a)^28 + 20436072*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 57
467160*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^22 + 13860720*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^23 + 50019840*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 33931872*tan(1
/2*b*x + 2*a)*tan(1/2*a)^25 + 5559192*tan(1/2*a)^26 - 14627700*tan(1/2*b*x
+ 2*a)^5*tan(1/2*a)^19 + 112971402*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 204
830640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 114114960*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^22 - 7584840*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 4921020*tan(1/2*
a)^24 - 14627700*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 150157800*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^19 - 227062440*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 10
2261096*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 11686680*tan(1/2*a)^22 + 2043607
2*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 112971402*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^16 + 150157800*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^17 - 74262420*tan(1/2*
b*x + 2*a)*tan(1/2*a)^19 + 22278726*tan(1/2*a)^20 - 1663560*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^13 + 57467160*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 2048306
40*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 227062440*tan(1/2*b*x + 2*a)^2*tan(
```

```
1/2*a)^16 - 74262420*tan(1/2*b*x + 2*a)*tan(1/2*a)^17 - 6862752*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^11 + 24990420*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 138
60720*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 114114960*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^14 + 102261096*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 22278726*tan(1/
2*a)^16 + 3241488*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 - 27168408*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^10 + 67201056*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 5001
9840*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 7584840*tan(1/2*b*x + 2*a)*tan(1/
2*a)^13 + 11686680*tan(1/2*a)^14 - 573912*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7
 + 7636140*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 32370944*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^9 + 54063504*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 33931872*tan
(1/2*b*x + 2*a)*tan(1/2*a)^11 + 4921020*tan(1/2*a)^12 + 52920*tan(1/2*b*x +
 2*a)^5*tan(1/2*a)^5 - 942984*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 5941584*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 15361632*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^8 + 16195152*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 5559192*tan(1/2*a)^10 - 261
0*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 58455*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^4 - 503568*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 + 1893936*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^6 - 2923992*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 1484964*tan(1/2*
a)^8 + 54*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 1620*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^2 + 19956*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 112068*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^4 + 257976*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 188808*tan(1/2
*a)^6 + 9*tan(1/2*b*x + 2*a)^4 - 252*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 2808
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 11298*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
+ 12969*tan(1/2*a)^4 - 12*tan(1/2*b*x + 2*a)^2 + 198*tan(1/2*b*x + 2*a)*tan
(1/2*a) - 468*tan(1/2*a)^2 + 7)/((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*ta
n(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8
 + 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*
b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2
*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^3) + 60*log
(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*
tan(1/2*a)^2 - 1)) - 60*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2
*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

maple [A] time = 0.87, size = 78, normalized size = 1.18

$$\frac{1}{48b\sin(bx+a)^2\cos(bx+a)^3}-\frac{5}{96b\sin(bx+a)^2\cos(bx+a)}+\frac{5}{32b\cos(bx+a)}+\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a),x)
```

```
[Out] 1/48/b/sin(b*x+a)^2/cos(b*x+a)^3-5/96/b/sin(b*x+a)^2/cos(b*x+a)+5/32/b/cos(
b*x+a)+5/32/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.40, size = 2174, normalized size = 32.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*cos(9*b*x + 9*a) + 20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) +
 20*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) + 60*(cos(8*b*x
+ 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*co
s(9*b*x + 9*a) + 4*(20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) + 20*cos(3*b*
x + 3*a) + 15*cos(b*x + a))*cos(8*b*x + 8*a) - 80*(2*cos(6*b*x + 6*a) + 2*c
os(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(7*b*x + 7*a) + 8*(22*cos(5*b*x
+ 5*a) - 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 88*(2*co
```

```
s(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) - 40*(4*cos(3*b*x +
 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 80*(cos(2*b*x + 2*a) + 1)*cos(3*
b*x + 3*a) + 60*cos(2*b*x + 2*a)*cos(b*x + a) - 15*(2*(cos(8*b*x + 8*a) - 2
*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x +
 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a)
- cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 4*(2*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)^2
- 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + cos(2*
b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a
) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2 - 2*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + si
n(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*
a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 +
 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)
+ 15*(2*(cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x
 + 6*a) + 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos
(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x +
 6*a) + 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
4*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin
(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)
^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) + 20*sin(7*b*x + 7*a)
 - 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x
+ 10*a) + 60*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) +
sin(2*b*x + 2*a))*sin(9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) - 22*sin(5*b*x
+ 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(8*b*x + 8*a) - 80*(2*si
n(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(22*sin(5*b*x + 5*a) - 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 88*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*(4
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*a) + 80*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) + 60*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/(b*c
os(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 + b*sin(8
*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 - 4*b*sin(4
*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(8*b*x + 8*a)
 - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) + b)*co
s(10*b*x + 10*a) - 2*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) - b*cos(2
*b*x + 2*a) - b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4*a) - b*cos(2*b*x +
 2*a) - b)*cos(6*b*x + 6*a) - 4*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) +
 2*b*cos(2*b*x + 2*a) + 2*(b*sin(8*b*x + 8*a) - 2*b*sin(6*b*x + 6*a) - 2*b*
sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 2*(2*b*sin(6*b*
x + 6*a) + 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 4*
(2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.10, size = 60, normalized size = 0.91

$$\frac{-\frac{5\cos(a+b\,x)^4}{32}+\frac{5\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

```
[Out] ((5*cos(a + b*x)^2)/48 - (5*cos(a + b*x)^4)/32 + 1/48)/(b*(cos(a + b*x)^3 -
 cos(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(32*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.12 $\int \csc^5(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\csc^3(a+bx)}{768b}-\frac{35\csc(a+bx)}{256b}+\frac{35\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}+\frac{7\csc^3(a+bx)\sec^2(a+b}{256b}$$

```
[Out] 35/256*arctanh(sin(b*x+a))/b-35/256*csc(b*x+a)/b-35/768*csc(b*x+a)^3/b+7/25
6*csc(b*x+a)^3*sec(b*x+a)^2/b+1/128*csc(b*x+a)^3*sec(b*x+a)^4/b
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{35\csc^3(a+bx)}{768b}-\frac{35\csc(a+bx)}{256b}+\frac{35\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}+\frac{7\csc^3(a+bx)\sec^2(a+b}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(256*b) - (35*Csc[a + b*x])/(256*b) - (35*Csc[a
+ b*x]^3)/(768*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*
x]^3*Sec[a + b*x]^4)/(128*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^5(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{32}\int \csc^4(a+bx)\sec^5(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{32b} \\
&= \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{128b} \\
&= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+\cdots}\right.}{} \\
&= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\,\text{Subst}\left(\int \left(1\cdots\right.\right.}{} \\
&= -\frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a\cdots}{} \\
&= \frac{35\tanh^{-1}(\sin(a+bx))}{256b} - \frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+b\cdots}{2\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 31, normalized size = 0.35

$$
-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(a+bx)\right)}{96b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

```
[Out] -1/96*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

fricas [A] time = 0.51, size = 140, normalized size = 1.57

$$
-\frac{210\cos(bx+a)^6 - 280\cos(bx+a)^4 - 105\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(bx+a\cdots}{1536\left(b\cos(bx+a)^6 - b\cos(b\cdots\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1536*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)
/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

giac [B] time = 12.38, size = 5647, normalized size = 63.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/768*((27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^35 + tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 2574*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^32 - 1563*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 234*tan(1/2*b*x
```

+ 2*a)^3*tan(1/2*a)^34 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 74706*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^30 + 88434*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 -
32229*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 3669*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^33 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 598014*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^28 - 1170882*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 730596*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^30 - 178020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 +
14238*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 + 54*tan(1/2*a)^33 - 1958598*tan(1/2
*b*x + 2*a)^5*tan(1/2*a)^26 + 6055062*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 -
5907096*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 + 2329236*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^29 - 370194*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 17442*tan(1/2*a)^3
1 + 1859910*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 12207078*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^25 + 19818828*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 12113616*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 + 2973582*tan(1/2*b*x + 2*a)*tan(1/2*a)^
28 - 240066*tan(1/2*a)^29 + 3993750*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 18
79110*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 18713760*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^24 + 24499800*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^25 - 9845190*tan(1/2
*b*x + 2*a)*tan(1/2*a)^26 + 1208842*tan(1/2*a)^27 - 8882730*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^20 + 38331030*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 4109310
0*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 + 3908700*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^23 + 9319830*tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 2402946*tan(1/2*a)^25 -
 29609100*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^19 + 87376914*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^20 - 76707084*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 + 20211030*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^22 - 319590*tan(1/2*a)^23 + 8882730*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^16 - 29609100*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 588
47130*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 - 44106714*tan(1/2*b*x + 2*a)*tan(
1/2*a)^20 + 7594374*tan(1/2*a)^21 - 3993750*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^14 + 38331030*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 - 87376914*tan(1/2*b*x +
2*a)^3*tan(1/2*a)^16 + 58847130*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 618579
0*tan(1/2*a)^19 - 1859910*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^12 - 1879110*tan(
1/2*b*x + 2*a)^4*tan(1/2*a)^13 + 41093100*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^1
4 - 76707084*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 + 44106714*tan(1/2*b*x + 2*
a)*tan(1/2*a)^16 - 6185790*tan(1/2*a)^17 + 1958598*tan(1/2*b*x + 2*a)^5*tan
(1/2*a)^10 - 12207078*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 18713760*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^12 + 3908700*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 -
20211030*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 + 7594374*tan(1/2*a)^15 - 598014*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 6055062*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^9 - 19818828*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 + 24499800*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^11 - 9319830*tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 319590*tan
(1/2*a)^13 + 74706*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 1170882*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^7 + 5907096*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 1211361
6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 + 9845190*tan(1/2*b*x + 2*a)*tan(1/2*a)
^10 - 2402946*tan(1/2*a)^11 - 2574*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 8843
4*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 730596*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^6 + 2329236*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 2973582*tan(1/2*b*x + 2*a
)*tan(1/2*a)^8 + 1208842*tan(1/2*a)^9 - 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^
2 - 1563*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 32229*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^4 - 178020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 370194*tan(1/2*b*x +
 2*a)*tan(1/2*a)^6 - 240066*tan(1/2*a)^7 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a
) - 234*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 3669*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^3 - 14238*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 17442*tan(1/2*a)^5 - tan(
1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 27*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^13 +
981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2*a)^5
 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*
a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2
*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*
tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*
tan(1/2*a))^3) + 6*(13*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^48 - 174*tan(1/2*b*x
 + 2*a)^7*tan(1/2*a)^46 + 162*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^47 - 5*tan(1/

2*b*x + 2*a)^5*tan(1/2*a)^48 - 21522*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^44 + 1
3014*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^45 - 2514*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^46 + 234*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^47 - 5*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^48 + 942746*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^42 - 1100574*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^43 + 453762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^44 - 872
34*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^45 + 8718*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^46 - 474*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^47 + 13*tan(1/2*b*x + 2*a)*tan(1
/2*a)^48 - 17981508*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^40 + 30574566*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^41 - 18985626*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 5
680314*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^43 - 906750*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^44 + 80834*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^45 - 3918*tan(1/2*b*x +
2*a)*tan(1/2*a)^46 + 78*tan(1/2*a)^47 + 193013886*tan(1/2*b*x + 2*a)^7*tan(
1/2*a)^38 - 441566530*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^39 + 374555556*tan(1/
2*b*x + 2*a)^5*tan(1/2*a)^40 - 153629730*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41
 + 33577926*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^42 - 4014234*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^43 + 252270*tan(1/2*b*x + 2*a)*tan(1/2*a)^44 - 6614*tan(1/2*a
)^45 - 1202084806*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^36 + 3626590890*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^37 - 4076849790*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 +
 2214510870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^39 - 633112668*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^40 + 97266546*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^41 - 7629702*
tan(1/2*b*x + 2*a)*tan(1/2*a)^42 + 243198*tan(1/2*a)^43 + 4119245478*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^34 - 16824081618*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^
35 + 25189696662*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^36 - 18102012318*tan(1/2*b
*x + 2*a)^4*tan(1/2*a)^37 + 6776146338*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 -
 1334518022*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^39 + 130749372*tan(1/2*b*x + 2*
a)*tan(1/2*a)^40 - 5045814*tan(1/2*a)^41 - 6836731551*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^32 + 41573675178*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^33 - 8662916611
8*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 + 84079436982*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^35 - 41858006762*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 10786903998*t
an(1/2*b*x + 2*a)^2*tan(1/2*a)^37 - 1346794722*tan(1/2*b*x + 2*a)*tan(1/2*a
)^38 + 64179762*tan(1/2*a)^39 + 267313972*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^3
0 - 37264510188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^31 + 143316996039*tan(1/2*b
*x + 2*a)^5*tan(1/2*a)^32 - 208138130542*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33
 + 144563703930*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 - 50381554230*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^35 + 8311517370*tan(1/2*b*x + 2*a)*tan(1/2*a)^36 - 50
7068874*tan(1/2*a)^37 + 17351751084*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^28 - 56
502600420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^29 - 5710349876*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^30 + 186100588644*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 23940
5365305*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 125386031934*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^33 - 29001178170*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 23928889
14*tan(1/2*a)^35 - 22757749308*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^26 + 1644199
19188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27 - 363804236364*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^28 + 282793219500*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 8521880
716*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 - 111637405764*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^31 + 48133667169*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 - 6017063802*t
an(1/2*a)^33 - 98616913668*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^25 + 47878440044
4*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 - 821763522748*tan(1/2*b*x + 2*a)^4*ta
n(1/2*a)^27 + 607022398644*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 - 17113731793
2*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 1237805836*tan(1/2*b*x + 2*a)*tan(1/
2*a)^30 + 5308170444*tan(1/2*a)^31 + 22757749308*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^22 - 98616913668*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^23 + 492829728300*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^25 - 795918156804*tan(1/2*b*x + 2*a)^3*tan(1/
2*a)^26 + 492162548604*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 - 121726145364*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^28 + 8269813092*tan(1/2*a)^29 - 17351751084*tan
(1/2*b*x + 2*a)^7*tan(1/2*a)^20 + 164419919188*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^21 - 478784400444*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 + 492829728300*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 293938021260*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^25 + 158215581828*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 - 23358680596*tan(1/
2*a)^27 - 267313972*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^18 - 56502600420*tan(1/

```
2*b*x + 2*a)^6*tan(1/2*a)^19 + 363804236364*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^20 - 821763522748*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 + 795918156804*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^22 - 293938021260*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^23 + 13852570212*tan(1/2*a)^25 + 6836731551*tan(1/2*b*x + 2*a)^7*tan(1/2*a
)^16 - 37264510188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^17 + 5710349876*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^18 + 282793219500*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^1
9 - 607022398644*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 492162548604*tan(1/2*
b*x + 2*a)^2*tan(1/2*a)^21 - 158215581828*tan(1/2*b*x + 2*a)*tan(1/2*a)^22
+ 13852570212*tan(1/2*a)^23 - 4119245478*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^14
 + 41573675178*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^15 - 143316996039*tan(1/2*b*
x + 2*a)^5*tan(1/2*a)^16 + 186100588644*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17
- 8521880716*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 171137317932*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^19 + 121726145364*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 23
358680596*tan(1/2*a)^21 + 1202084806*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^12 - 1
6824081618*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 86629166118*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^14 - 208138130542*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 + 239
405365305*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 111637405764*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^17 + 1237805836*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 8269813
092*tan(1/2*a)^19 - 193013886*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^10 + 36265908
90*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 25189696662*tan(1/2*b*x + 2*a)^5*ta
n(1/2*a)^12 + 84079436982*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^13 - 144563703930
*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 125386031934*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^15 - 48133667169*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 + 5308170444*tan(
1/2*a)^17 + 17981508*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^8 - 441566530*tan(1/2*
b*x + 2*a)^6*tan(1/2*a)^9 + 4076849790*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 -
 18102012318*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 41858006762*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^12 - 50381554230*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 + 29
001178170*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 6017063802*tan(1/2*a)^15 - 942
746*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^6 + 30574566*tan(1/2*b*x + 2*a)^6*tan(1
/2*a)^7 - 374555556*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 2214510870*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^9 - 6776146338*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 +
 10786903998*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 8311517370*tan(1/2*b*x +
2*a)*tan(1/2*a)^12 + 2392888914*tan(1/2*a)^13 + 21522*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^4 - 1100574*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^5 + 18985626*tan(1/2
*b*x + 2*a)^5*tan(1/2*a)^6 - 153629730*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 +
633112668*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 1334518022*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^9 + 1346794722*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 507068874*t
an(1/2*a)^11 + 174*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2 + 13014*tan(1/2*b*x +
2*a)^6*tan(1/2*a)^3 - 453762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 5680314*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 33577926*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
6 + 97266546*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 130749372*tan(1/2*b*x + 2*
a)*tan(1/2*a)^8 + 64179762*tan(1/2*a)^9 - 13*tan(1/2*b*x + 2*a)^7 + 162*tan
(1/2*b*x + 2*a)^6*tan(1/2*a) + 2514*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 872
34*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 906750*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^4 - 4014234*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7629702*tan(1/2*b*x + 2*
a)*tan(1/2*a)^6 - 5045814*tan(1/2*a)^7 + 5*tan(1/2*b*x + 2*a)^5 + 234*tan(1
/2*b*x + 2*a)^4*tan(1/2*a) - 8718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 80834
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 252270*tan(1/2*b*x + 2*a)*tan(1/2*a)^4
 + 243198*tan(1/2*a)^5 + 5*tan(1/2*b*x + 2*a)^3 - 474*tan(1/2*b*x + 2*a)^2*
tan(1/2*a) + 3918*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 6614*tan(1/2*a)^3 - 13*
tan(1/2*b*x + 2*a) + 78*tan(1/2*a))/((tan(1/2*a)^24 - 60*tan(1/2*a)^22 + 14
10*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 92655*tan(1/2*a)^16 - 245880*tan(1
/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*tan(1/2*a)^10 + 92655*tan(1/2*a)^8
 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4 - 60*tan(1/2*a)^2 + 1)*(tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/
2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*
a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x +
 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 105*
log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
```

```
 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/
2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 105*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*
a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2
*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/
b
```

maple [**A**] time = 0.98, size = 97, normalized size = 1.09

$$\frac{1}{128b\sin(bx+a)^3\cos(bx+a)^4}-\frac{7}{384b\sin(bx+a)^3\cos(bx+a)^2}+\frac{35}{768b\sin(bx+a)\cos(bx+a)^2}-\frac{35}{256b\sin(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a),x)
```

```
[Out] 1/128/b/sin(b*x+a)^3/cos(b*x+a)^4-7/384/b/sin(b*x+a)^3/cos(b*x+a)^2+35/768/
b/sin(b*x+a)/cos(b*x+a)^2-35/256/b/sin(b*x+a)+35/256/b*ln(sec(b*x+a)+tan(b*
x+a))
```

maxima [**B**] time = 0.62, size = 3088, normalized size = 34.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*sin(13*b*x + 13*a) + 70*sin(11*b*x + 11*a) - 329*sin(9*b*x +
 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) +
 105*sin(b*x + a))*cos(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) - 3*sin(10*
b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a)
- sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) - 329*sin
(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x
+ 3*a) + 105*sin(b*x + a))*cos(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) +
 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x +
 2*a))*cos(11*b*x + 11*a) + 12*(329*sin(9*b*x + 9*a) + 204*sin(7*b*x + 7*a)
 + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(10*b*
x + 10*a) - 1316*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4
*a) + sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) + 329*s
in(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(8*b*x + 8*a)
+ 816*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*
x + 7*a) - 84*(47*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))
*cos(6*b*x + 6*a) + 1316*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x
+ 5*a) + 420*(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 105*(
2*(cos(12*b*x + 12*a) - 3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6
*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a)
 + cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) + 3*cos(8*b*x + 8*a) - 3*
cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(12*b*x +
12*a) + cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) -
 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + 9*cos(10*b
*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x
+ 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 +
2*(sin(12*b*x + 12*a) - 3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6
*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(14*b*x + 14*a) + s
in(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) + 3*sin(8*b*x + 8*a) - 3*sin(
6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 9*sin(10*b*x + 10*a)^2
 - 6*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x
```

```
 + 8*a) + 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + 9*sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) - 4*(105*cos(13*b*x + 13*a) + 70*cos(11*b*x + 11*a) - 329*cos(9*b*x + 9*a
) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) + 105
*cos(b*x + a))*sin(14*b*x + 14*a) + 420*(cos(12*b*x + 12*a) - 3*cos(10*b*x
+ 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*sin(13*b*x + 13*a) - 4*(70*cos(11*b*x + 11*a) - 329*cos
(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x
+ 3*a) + 105*cos(b*x + a))*sin(12*b*x + 12*a) - 280*(3*cos(10*b*x + 10*a) +
 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x +
 2*a) + 1)*sin(11*b*x + 11*a) - 12*(329*cos(9*b*x + 9*a) + 204*cos(7*b*x +
7*a) + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(1
0*b*x + 10*a) + 1316*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x
 + 4*a) + cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 12*(204*cos(7*b*x + 7*a)
 + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(8*b*x
 + 8*a) - 816*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) -
 1)*sin(7*b*x + 7*a) + 84*(47*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 1316*(3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 - 1)*sin(5*b*x + 5*a) - 420*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*
x + 4*a) - 280*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 280*cos(3*b*x + 3*
a)*sin(2*b*x + 2*a) + 420*cos(b*x + a)*sin(2*b*x + 2*a) - 420*cos(2*b*x + 2
*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x + 14*a)^2 + b*cos(12*b*x
 + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 9*b*cos(6*
b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(14*b*x
 + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*sin(10*b*x + 10*a)^2 + 9*b*sin(8*
b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 6*b*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(12*b*x + 12*a
) - 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) +
3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(14*b*x + 14*a) - 2*(3*b*
cos(10*b*x + 10*a) + 3*b*cos(8*b*x + 8*a) - 3*b*cos(6*b*x + 6*a) - 3*b*cos(
4*b*x + 4*a) + b*cos(2*b*x + 2*a) + b)*cos(12*b*x + 12*a) + 6*(3*b*cos(8*b*
x + 8*a) - 3*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a)
 + b)*cos(10*b*x + 10*a) - 6*(3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) -
 b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) + 6*(3*b*cos(4*b*x + 4*a) - b*cos
(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) - 6*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x
+ 4*a) + 2*b*cos(2*b*x + 2*a) + 2*(b*sin(12*b*x + 12*a) - 3*b*sin(10*b*x +
10*a) - 3*b*sin(8*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a)
- b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 2*(3*b*sin(10*b*x + 10*a) + 3*b*
sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) + b*sin(2*b*
x + 2*a))*sin(12*b*x + 12*a) + 6*(3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*
a) - 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 6*(3*b
*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 6*(3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [**B**] time = 0.17, size = 79, normalized size = 0.89

$$\frac{35\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{256\,b}-\frac{\frac{35\sin(a+b\,x)^6}{256}-\frac{175\sin(a+b\,x)^4}{768}+\frac{7\sin(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\sin\left(a+b\,x\right)^7-2\sin\left(a+b\,x\right)^5+\sin\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (35*atanh(sin(a + b*x)))/(256*b) - ((7*sin(a + b*x)^2)/96 - (175*sin(a + b*
x)^4)/768 + (35*sin(a + b*x)^6)/256 + 1/96)/(b*(sin(a + b*x)^3 - 2*sin(a +
b*x)^5 + sin(a + b*x)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.13 $\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$\frac{8\sin^{12}(a+bx)}{3b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{4\sin^{8}(a+bx)}{b}$$

```
[Out] 4*sin(b*x+a)^8/b-32/5*sin(b*x+a)^10/b+8/3*sin(b*x+a)^12/b
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2564, 266, 43}

$$\frac{8\sin^{12}(a+bx)}{3b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{4\sin^{8}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Sin[a + b*x]^8)/b - (32*Sin[a + b*x]^10)/(5*b) + (8*Sin[a + b*x]^12)/(3*
b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^7(a+bx)\,dx$$
$$= \frac{32\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{16\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \sin^2(a+bx)\right)}{b}$$
$$= \frac{16\,\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx, x, \sin^2(a+bx)\right)}{b}$$
$$= \frac{4\sin^8(a+bx)}{b} - \frac{32\sin^{10}(a+bx)}{5b} + \frac{8\sin^{12}(a+bx)}{3b}$$

Mathematica [**A**] time = 0.36, size = 68, normalized size = 1.55

$$\frac{-600\cos(2(a+bx))+75\cos(4(a+bx))+100\cos(6(a+bx))-30\cos(8(a+bx))-12\cos(10(a+bx))+5\cos}{3840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] + 100*Cos[6*(a + b*x)] - 30*Co
s[8*(a + b*x)] - 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

fricas [**A**] time = 0.48, size = 46, normalized size = 1.05

$$\frac{4\left(10\,\cos\left(bx+a\right)^{12}-36\,\cos\left(bx+a\right)^{10}+45\,\cos\left(bx+a\right)^{8}-20\,\cos\left(bx+a\right)^{6}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 4/15*(10*cos(b*x + a)^12 - 36*cos(b*x + a)^10 + 45*cos(b*x + a)^8 - 20*cos(
b*x + a)^6)/b
```

giac [**B**] time = 0.76, size = 85, normalized size = 1.93

$$\frac{\cos\left(12\,bx+12\,a\right)}{768\,b}-\frac{\cos\left(10\,bx+10\,a\right)}{320\,b}-\frac{\cos\left(8\,bx+8\,a\right)}{128\,b}+\frac{5\,\cos\left(6\,bx+6\,a\right)}{192\,b}+\frac{5\,\cos\left(4\,bx+4\,a\right)}{256\,b}-\frac{5\,\cos\left(2\,bx+2\right.}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/768*cos(12*b*x + 12*a)/b - 1/320*cos(10*b*x + 10*a)/b - 1/128*cos(8*b*x +
 8*a)/b + 5/192*cos(6*b*x + 6*a)/b + 5/256*cos(4*b*x + 4*a)/b - 5/32*cos(2*
b*x + 2*a)/b
```

maple [**B**] time = 0.39, size = 86, normalized size = 1.95

$$-\frac{5\cos\left(2bx+2a\right)}{32b}+\frac{5\cos\left(4bx+4a\right)}{256b}+\frac{5\cos\left(6bx+6a\right)}{192b}-\frac{\cos\left(8bx+8a\right)}{128b}-\frac{\cos\left(10bx+10a\right)}{320b}+\frac{\cos\left(12bx+12a\right)}{768b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b+5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b-1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b+1/768*cos(12*b*x+12*a)/b
```

maxima [**A**] time = 0.33, size = 72, normalized size = 1.64

$$\frac{5\cos(12\,bx+12\,a)-12\cos(10\,bx+10\,a)-30\cos(8\,bx+8\,a)+100\cos(6\,bx+6\,a)+75\cos(4\,bx+4\,a)-60}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/3840*(5*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
+ 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) - 600*cos(2*b*x + 2*a))/b
```

mupad [**B**] time = 0.13, size = 46, normalized size = 1.05

$$-\frac{-\frac{8\cos(a+b\,x)^{12}}{3}+\frac{48\cos(a+b\,x)^{10}}{5}-12\cos\left(a+b\,x\right)^{8}+\frac{16\cos(a+b\,x)^{6}}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -((16*cos(a + b*x)^6)/3 - 12*cos(a + b*x)^8 + (48*cos(a + b*x)^10)/5 - (8*c
os(a + b*x)^12)/3)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.14 $\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$-\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

```
[Out] 3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^3/b-1/20*sin(2*b*x+2*a)^5/b
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4286, 2635, 8, 2564, 30}

$$-\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) - Sin[2*a + 2*b*x]^5/(20*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4286

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{1}{2}\int \sin^4(2a+2bx)\,dx - \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx$$

$$= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx - \frac{\text{Subst}\left(\int x^4\,dx, x\right.}{4b}$$

$$= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(2a+}{20b}$$

$$= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(}{}$$

Mathematica [A] time = 0.20, size = 62, normalized size = 0.82

$$\frac{-20\sin(2(a+bx)) - 40\sin(4(a+bx)) + 10\sin(6(a+bx)) + 5\sin(8(a+bx)) - 2\sin(10(a+bx)) + 120bx}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (120*b*x - 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] + 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] - 2*Sin[10*(a + b*x)])/(640*b)
```

fricas [A] time = 0.62, size = 67, normalized size = 0.88

$$\frac{15\,bx - \left(128\,\cos(bx+a)^9 - 336\,\cos(bx+a)^7 + 248\,\cos(bx+a)^5 - 10\,\cos(bx+a)^3 - 15\,\cos(bx+a)\right)\sin(bx+}{80\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x - (128*cos(b*x + a)^9 - 336*cos(b*x + a)^7 + 248*cos(b*x + a)^
5 - 10*cos(b*x + a)^3 - 15*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.60, size = 74, normalized size = 0.97

$$\frac{3}{16}\,x - \frac{\sin(10\,bx+10\,a)}{320\,b} + \frac{\sin(8\,bx+8\,a)}{128\,b} + \frac{\sin(6\,bx+6\,a)}{64\,b} - \frac{\sin(4\,bx+4\,a)}{16\,b} - \frac{\sin(2\,bx+2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x - 1/320*sin(10*b*x + 10*a)/b + 1/128*sin(8*b*x + 8*a)/b + 1/64*sin(6
*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b - 1/32*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.96, size = 75, normalized size = 0.99

$$\frac{3x}{16} - \frac{\sin(2bx+2a)}{32b} - \frac{\sin(4bx+4a)}{16b} + \frac{\sin(6bx+6a)}{64b} + \frac{\sin(8bx+8a)}{128b} - \frac{\sin(10bx+10a)}{320b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/16*x-1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)-1/320/b*sin(10*b*x+10*a)
```

maxima [A] time = 0.33, size = 65, normalized size = 0.86

$$\frac{120\,bx - 2\,\sin(10\,bx+10\,a) + 5\,\sin(8\,bx+8\,a) + 10\,\sin(6\,bx+6\,a) - 40\,\sin(4\,bx+4\,a) - 20\,\sin(2\,bx+2\,a)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

mupad [B] time = 1.68, size = 110, normalized size = 1.45

$$\frac{3x}{16}-\frac{-\frac{3\tan(a+bx)^9}{16}-\frac{7\tan(a+bx)^7}{8}+\frac{8\tan(a+bx)^5}{5}+\frac{7\tan(a+bx)^3}{8}+\frac{3\tan(a+bx)}{16}}{b\left(\tan\left(a+bx\right)^{10}+5\tan\left(a+bx\right)^8+10\tan\left(a+bx\right)^6+10\tan\left(a+bx\right)^4+5\tan\left(a+bx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

```
[Out] (3*x)/16 - ((3*tan(a + b*x))/16 + (7*tan(a + b*x)^3)/8 + (8*tan(a + b*x)^5)
/5 - (7*tan(a + b*x)^7)/8 - (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 +
 1))
```

sympy [A] time = 117.94, size = 434, normalized size = 5.71

$$\begin{cases}\frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16}+\frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8}+\frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16}+\frac{3x\sin^4(2a+2bx)\cos^2(a+bx)}{16}+ \\ x\sin^2(a)\sin^4(2a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a +
2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a + 2
*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(2*
a + 2*b*x)**4/16 - 57*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/
(160*b) - 109*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480*b)
- sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) - 2*sin(a + b*x)*sin
(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) - 4*sin(a + b*x)*co
s(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) + 7*sin(2*a + 2*b*x)**3*cos(a + b*x)*
*2*cos(2*a + 2*b*x)/(160*b) + 19*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a +
 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**4, True))
```

3.15 $\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=29

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

```
[Out] 4/3*sin(b*x+a)^6/b-sin(b*x+a)^8/b
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 14}

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Sin[a + b*x]^6)/(3*b) - Sin[a + b*x]^8/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^5(a+bx)\,dx\\
&= \frac{8\,\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{8\,\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 48, normalized size = 1.66

$$\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(384*b)
```

fricas [A] time = 0.46, size = 36, normalized size = 1.24

$$-\frac{3 \cos(bx+a)^8 - 8 \cos(bx+a)^6 + 6 \cos(bx+a)^4}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

giac [B] time = 0.50, size = 57, normalized size = 1.97

$$-\frac{\cos(8bx+8a)}{128 b} + \frac{\cos(6bx+6a)}{48 b} + \frac{\cos(4bx+4a)}{32 b} - \frac{3 \cos(2bx+2a)}{16 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/128*cos(8*b*x + 8*a)/b + 1/48*cos(6*b*x + 6*a)/b + 1/32*cos(4*b*x + 4*a)
/b - 3/16*cos(2*b*x + 2*a)/b
```

maple [B] time = 0.29, size = 58, normalized size = 2.00

$$-\frac{3 \cos(2bx+2a)}{16b} + \frac{\cos(4bx+4a)}{32b} + \frac{\cos(6bx+6a)}{48b} - \frac{\cos(8bx+8a)}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b+1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b-1/128*co
s(8*b*x+8*a)/b
```

maxima [A] time = 0.33, size = 50, normalized size = 1.72

$$-\frac{3 \cos(8bx+8a) - 8 \cos(6bx+6a) - 12 \cos(4bx+4a) + 72 \cos(2bx+2a)}{384 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(3*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) + 72*
cos(2*b*x + 2*a))/b
```

mupad [B] time = 0.11, size = 33, normalized size = 1.14

$$-\frac{\cos(a+b\,x)^4 \left(\cos(a+b\,x)^4 - \frac{8\cos(a+b\,x)^2}{3} + 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

```
[Out] -(cos(a + b*x)^4*(cos(a + b*x)^4 - (8*cos(a + b*x)^2)/3 + 2))/b
```

sympy [A] time = 40.10, size = 362, normalized size = 12.48

$$\begin{cases} \frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} + \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} + \frac{3x\sin(a+bx)}{} \\ x\sin^2(a)\sin^3(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 + 3*x*sin(a + b*x)*sin(2*a + 2*b*x
)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)*cos(2*
a + 2*b*x)**3/8 - 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 - 3*x*sin(2*a
+ 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 + 17*sin(a + b*x)**2*cos(2*
a + 2*b*x)**3/(96*b) - 13*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16
*b) - 7*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b
) - sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(2*b) - 49*cos(a +
 b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**3, T
rue))
```

3.16 $\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$-\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

```
[Out] 1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/12*sin(2*b*x+2*a)^3/b
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4286, 2635, 8, 2564, 30}

$$-\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) - Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4286

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx &= \frac{1}{2}\int \sin^2(2a+2bx)\,dx-\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx\\
&= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\int 1\,dx}{4}-\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\sin(2a+2bx\right.}{4b}\\
&= \frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}-\frac{\sin^3(2a+2bx)}{12b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 0.82

$$\frac{-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))+12bx}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48*b
)
```

fricas [A] time = 0.49, size = 46, normalized size = 0.94

$$\frac{3\,bx+\left(8\,\cos\left(bx+a\right)^{5}-14\,\cos\left(bx+a\right)^{3}+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b
```

giac [A] time = 0.42, size = 46, normalized size = 0.94

$$\frac{1}{4}\,x+\frac{\sin\left(6\,bx+6\,a\right)}{48\,b}-\frac{\sin\left(4\,bx+4\,a\right)}{16\,b}-\frac{\sin\left(2\,bx+2\,a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/48*sin(6*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b - 1/16*sin(2*b*x
+ 2*a)/b
```

maple [A] time = 0.67, size = 47, normalized size = 0.96

$$\frac{x}{4}-\frac{\sin\left(2bx+2a\right)}{16b}-\frac{\sin\left(4bx+4a\right)}{16b}+\frac{\sin\left(6bx+6a\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

```
[Out] 1/4*x-1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/48/b*sin(6*b*x+6*a)
```

maxima [A] time = 0.33, size = 41, normalized size = 0.84

$$\frac{12\,bx+\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)-3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))/
b
```

mupad [B] time = 0.30, size = 43, normalized size = 0.88

$$\frac{x}{4}-\frac{\frac{\sin(2\,a+2\,b\,x)}{16}+\frac{\sin(4\,a+4\,b\,x)}{16}-\frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

```
[Out] x/4 - (sin(2*a + 2*b*x)/16 + sin(4*a + 4*b*x)/16 - sin(6*a + 6*b*x)/48)/b
```

sympy [**A**] time = 11.96, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} - \frac{7\sin^2(a+bx)\sin}{} \\ x\sin^2(a)\sin^2(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 - 7*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2
*b*x)/(24*b) - sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) - sin(a
+ b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) + sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**2, True))
```

3.17 $\int \sin^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^4(a+bx)}{2b}$$

```
[Out] 1/2*sin(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 30}

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] Sin[a + b*x]^4/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\sin^3(a+bx)\,dx\\ &= \frac{2\,\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

[Out] Sin[a + b*x]^4/(2*b)

fricas [A]   time = 0.52, size = 24, normalized size = 1.60

$$\frac{\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

giac [B]   time = 0.46, size = 29, normalized size = 1.93

$$\frac{\cos\left(4\,bx+4\,a\right)}{16\,b}-\frac{\cos\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
[Out] 1/16*cos(4*b*x + 4*a)/b - 1/4*cos(2*b*x + 2*a)/b
```

maple [B]   time = 0.14, size = 30, normalized size = 2.00

$$-\frac{\cos\left(2bx+2a\right)}{4b}+\frac{\cos\left(4bx+4a\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a),x)
[Out] -1/4*cos(2*b*x+2*a)/b+1/16*cos(4*b*x+4*a)/b
```

maxima [A]   time = 0.33, size = 26, normalized size = 1.73

$$\frac{\cos\left(4\,bx+4\,a\right)-4\,\cos\left(2\,bx+2\,a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
[Out] 1/16*(cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a))/b
```

mupad [B]   time = 0.10, size = 13, normalized size = 0.87

$$\frac{{\sin\left(a+b\,x\right)}^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x),x)
[Out] sin(a + b*x)^4/(2*b)
```

sympy [A]   time = 3.30, size = 133, normalized size = 8.87

$$\begin{cases} \frac{x\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)}{4}+\frac{x\sin\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{2}-\frac{x\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{4}-\frac{3\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{4b}-\text{co} \\ x\sin^{2}\left(a\right)\sin\left(2a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 + x*sin(a + b*x)*cos(a + b*
x)*cos(2*a + 2*b*x)/2 - x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - 3*sin(a + b*
x)*sin(2*a + 2*b*x)*cos(a + b*x)/(4*b) - cos(a + b*x)**2*cos(2*a + 2*b*x)/(
2*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a), True))
```

3.18 $\int \csc(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\log(\cos(a+bx))}{2b}$$

```
[Out] -1/2*ln(cos(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4288, 3475}

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -Log[Cos[a + b*x]]/(2*b)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{2}\int \tan(a+bx)\,dx\\ &= -\frac{\log(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*Log[Cos[a + b*x]]/b
```

fricas [A] time = 0.44, size = 14, normalized size = 1.00

$$-\frac{\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*log(-cos(b*x + a))/b
```

giac [B] time = 0.77, size = 95, normalized size = 6.79

$$\frac{\log\left(\tan\left(bx+4a\right)^{2}+1\right)-2\log\left(\left|6\tan\left(bx+4a\right)\tan\left(\frac{1}{2}a\right)^{5}-\tan\left(\frac{1}{2}a\right)^{6}-20\tan\left(bx+4a\right)\tan\left(\frac{1}{2}a\right)^{3}+15\tan\right.\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(log(tan(b*x + 4*a)^2 + 1) - 2*log(abs(6*tan(b*x + 4*a)*tan(1/2*a)^5 -
tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x
 + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

maple [A] time = 0.40, size = 13, normalized size = 0.93

$$-\frac{\ln\left(\cos\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^2,x)
```

```
[Out] -1/2*ln(cos(b*x+a))/b
```

maxima [B] time = 0.33, size = 55, normalized size = 3.93

$$-\frac{\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 -
 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)/b
```

mupad [B] time = 0.08, size = 12, normalized size = 0.86

$$-\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x),x)
```

```
[Out] -log(cos(a + b*x))/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.19 $\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\tan(a+bx)}{4b}$$

[Out] 1/4*tan(b*x+a)/b

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4288, 3767, 8}

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]

[Out] Tan[a + b*x]/(4*b)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{4}\int \sec^2(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{4b}\\ &= \frac{\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]

[Out] Tan[a + b*x]/(4*b)

fricas [**A**] time = 0.40, size = 19, normalized size = 1.46

$$\frac{\sin\left(bx+a\right)}{4\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sin(b*x + a)/(b*cos(b*x + a))
```

giac [**B**] time = 1.52, size = 152, normalized size = 11.69

$$-\frac{\tan\left(\frac{1}{2}\,a\right)^{12}+6\,\tan\left(\frac{1}{2}\,a\right)^{10}+15\,\tan\left(\frac{1}{2}\,a\right)^{8}+20\,\tan\left(\frac{1}{2}\,a\right)^{6}+15\,\mathrm{ta}}{8\left(6\,\tan\left(bx+4\,a\right)\tan\left(\frac{1}{2}\,a\right)^{5}-\tan\left(\frac{1}{2}\,a\right)^{6}-20\,\tan\left(bx+4\,a\right)\tan\left(\frac{1}{2}\,a\right)^{3}+15\,\tan\left(\frac{1}{2}\,a\right)^{4}+6\,\tan\left(bx+4\,a\right)\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*a)^6 +
 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((6*tan(b*x + 4*a)*tan(1/2*a)^5 - ta
n(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x +
 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 +
 3*tan(1/2*a))*b)
```

maple [**A**] time = 1.07, size = 12, normalized size = 0.92

$$\frac{\tan\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/4*tan(b*x+a)/b
```

maxima [**B**] time = 0.33, size = 53, normalized size = 4.08

$$\frac{\sin\left(2\,bx+2\,a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}+2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos
(2*b*x + 2*a) + b)
```

mupad [**B**] time = 0.11, size = 11, normalized size = 0.85

$$\frac{\tan\left(a+b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

```
[Out] tan(a + b*x)/(4*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\tan^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

```
[Out] 1/8*ln(tan(b*x+a))/b+1/16*tan(b*x+a)^2/b
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2620, 14}

$$\frac{\tan^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] Log[Tan[a + b*x]]/(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{8}\int \csc(a+bx)\sec^3(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 1.20

$$-\frac{-\sec^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2)/b
```

fricas [B] time = 0.43, size = 56, normalized size = 1.87

$$-\frac{\cos\left(bx+a\right)^{2}\log\left(\cos\left(bx+a\right)^{2}\right)-\cos\left(bx+a\right)^{2}\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-1}{16\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
 + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

giac [B] time = 3.55, size = 734, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/64*((12*tan(b*x + 4*a)*tan(1/2*a)^23 - tan(1/2*a)^24 + 3888*tan(b*x + 4*a
)^2*tan(1/2*a)^20 - 1444*tan(b*x + 4*a)*tan(1/2*a)^21 + 168*tan(1/2*a)^22 -
 51840*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 32052*tan(b*x + 4*a)*tan(1/2*a)^19
- 4074*tan(1/2*a)^20 + 274752*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 260028*tan(b
*x + 4*a)*tan(1/2*a)^17 + 49480*tan(1/2*a)^18 - 731520*tan(b*x + 4*a)^2*tan
(1/2*a)^14 + 979064*tan(b*x + 4*a)*tan(1/2*a)^15 - 276687*tan(1/2*a)^16 + 1
021728*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 1873128*tan(b*x + 4*a)*tan(1/2*a)^1
3 + 737808*tan(1/2*a)^14 - 731520*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 1873128*
tan(b*x + 4*a)*tan(1/2*a)^11 - 1009292*tan(1/2*a)^12 + 274752*tan(b*x + 4*a
)^2*tan(1/2*a)^8 - 979064*tan(b*x + 4*a)*tan(1/2*a)^9 + 737808*tan(1/2*a)^1
0 - 51840*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 260028*tan(b*x + 4*a)*tan(1/2*a)^
7 - 276687*tan(1/2*a)^8 + 3888*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 32052*tan(b*
x + 4*a)*tan(1/2*a)^5 + 49480*tan(1/2*a)^6 + 1444*tan(b*x + 4*a)*tan(1/2*a)
^3 - 4074*tan(1/2*a)^4 - 12*tan(b*x + 4*a)*tan(1/2*a) + 168*tan(1/2*a)^2 -
1)/((9*tan(1/2*a)^10 - 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4
 + 9*tan(1/2*a)^2)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b
*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15
*tan(1/2*a)^2 + 1)^2) + 8*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x
+ 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*
tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 8*log(abs(6*tan(b*x + 4*a)
*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*
a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

maple [A] time = 0.87, size = 27, normalized size = 0.90

$$\frac{1}{16b\cos\left(bx+a\right)^{2}}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/16/b/cos(b*x+a)^2+1/8*ln(tan(b*x+a))/b
```

maxima [B] time = 0.35, size = 641, normalized size = 21.37

$$\frac{4\ \cos\left(4\,bx+4\,a\right)\cos\left(2\,bx+2\,a\right)+8\ \cos\left(2\,bx+2\,a\right)^{2}-\left(2\left(2\ \cos\left(2\,bx+2\,a\right)+1\right)\cos\left(4\,bx+4\,a\right)+\cos\left(4\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + 8*cos(2*b*x + 2*a)^2 - (2*(2*co
s(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2
*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (2
*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b
*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*
cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

mupad [B] time = 0.15, size = 35, normalized size = 1.17

$$\frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}-\frac{\ln(\cos(a+b\,x))}{8}+\frac{1}{16\cos(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

```
[Out] (log(sin(a + b*x)^2)/16 - log(cos(a + b*x))/8 + 1/(16*cos(a + b*x)^2))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.21 $\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{8b}-\frac{\cot(a+bx)}{16b}$$

```
[Out] -1/16*cot(b*x+a)/b+1/8*tan(b*x+a)/b+1/48*tan(b*x+a)^3/b
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2620, 270}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{8b}-\frac{\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(16*b) + Tan[a + b*x]/(8*b) + Tan[a + b*x]^3/(48*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{16}\int \csc^2(a+bx)\sec^4(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(a+bx)\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{16b}\\ &= -\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.14

$$\frac{5\tan(a+bx)}{48b}-\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)\sec^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*Cot[a + b*x]/b + (5*Tan[a + b*x])/(48*b) + (Sec[a + b*x]^2*Tan[a + b*
x])/(48*b)
```

fricas [A] time = 0.39, size = 43, normalized size = 1.02

$$-\frac{8 \cos \left(bx+a\right)^{4}-4 \cos \left(bx+a\right)^{2}-1}{48 \, b \cos \left(bx+a\right)^{3} \sin \left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
 a))
```

giac [B] time = 2.26, size = 1034, normalized size = 24.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*
a)^6 + 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((tan(b*x + 4*a)*tan(1/2*a)^6
- 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1
/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))*(tan(1/2*a)^6 -
15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1)) + (108*tan(b*x + 4*a)^2*tan(1/2*a)^
34 - 18*tan(b*x + 4*a)*tan(1/2*a)^35 + tan(1/2*a)^36 + 4464*tan(b*x + 4*a)^
2*tan(1/2*a)^32 - 1182*tan(b*x + 4*a)*tan(1/2*a)^33 + 126*tan(1/2*a)^34 - 2
8608*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 26208*tan(b*x + 4*a)*tan(1/2*a)^31 -
3159*tan(1/2*a)^32 + 14544*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 81216*tan(b*x +
 4*a)*tan(1/2*a)^29 + 29232*tan(1/2*a)^30 + 197136*tan(b*x + 4*a)^2*tan(1/2
*a)^26 - 130344*tan(b*x + 4*a)*tan(1/2*a)^27 - 26460*tan(1/2*a)^28 - 230160
*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 657288*tan(b*x + 4*a)*tan(1/2*a)^25 - 228
600*tan(1/2*a)^26 - 753984*tan(b*x + 4*a)^2*tan(1/2*a)^22 + 419328*tan(b*x
+ 4*a)*tan(1/2*a)^23 + 237588*tan(1/2*a)^24 + 604368*tan(b*x + 4*a)^2*tan(1
/2*a)^20 - 2188128*tan(b*x + 4*a)*tan(1/2*a)^21 + 944208*tan(1/2*a)^22 + 19
57128*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 1928412*tan(b*x + 4*a)*tan(1/2*a)^19
 - 142434*tan(1/2*a)^20 + 604368*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 1928412*t
an(b*x + 4*a)*tan(1/2*a)^17 - 1358860*tan(1/2*a)^18 - 753984*tan(b*x + 4*a)
^2*tan(1/2*a)^14 + 2188128*tan(b*x + 4*a)*tan(1/2*a)^15 - 142434*tan(1/2*a)
^16 - 230160*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 419328*tan(b*x + 4*a)*tan(1/2
*a)^13 + 944208*tan(1/2*a)^14 + 197136*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 657
288*tan(b*x + 4*a)*tan(1/2*a)^11 + 237588*tan(1/2*a)^12 + 14544*tan(b*x + 4
*a)^2*tan(1/2*a)^8 + 130344*tan(b*x + 4*a)*tan(1/2*a)^9 - 228600*tan(1/2*a)
^10 - 28608*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 81216*tan(b*x + 4*a)*tan(1/2*a)
^7 - 26460*tan(1/2*a)^8 + 4464*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 26208*tan(b*
x + 4*a)*tan(1/2*a)^5 + 29232*tan(1/2*a)^6 + 108*tan(b*x + 4*a)^2*tan(1/2*a
)^2 + 1182*tan(b*x + 4*a)*tan(1/2*a)^3 - 3159*tan(1/2*a)^4 + 18*tan(b*x + 4
*a)*tan(1/2*a) + 126*tan(1/2*a)^2 + 1)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^
13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2
*a)^5 + 27*tan(1/2*a)^3)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20
*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a
) - 15*tan(1/2*a)^2 + 1)^3))/b
```

maple [A] time = 1.22, size = 51, normalized size = 1.21

$$\frac{\frac{1}{3 \sin(bx+a) \cos(bx+a)^{3}}+\frac{4}{3 \sin(bx+a) \cos(bx+a)}-\frac{8 \cot(bx+a)}{3}}{16 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/16/b*(1/3/sin(b*x+a)/cos(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

maxima [B] time = 0.35, size = 308, normalized size = 7.33

$$-\frac{(2\,\mathrm{co}}{3\left(b\cos\left(8\,bx+8\,a\right)^2+4\,b\cos\left(6\,bx+6\,a\right)^2+4\,b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(8\,bx+8\,a\right)^2+4\,b\sin\left(6\,bx+6\,a\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*((2*cos(2*b*x + 2*a) + 1)*sin(8*b*x + 8*a) + 2*(2*cos(2*b*x + 2*a) + 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) - 4*cos(6*b*x + 6*
a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin
(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(6*b*x
+ 6*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*
a) + b)*cos(6*b*x + 6*a) + 4*b*cos(2*b*x + 2*a) + 4*(b*sin(6*b*x + 6*a) - b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

mupad [B] time = 0.13, size = 33, normalized size = 0.79

$$\frac{\tan\left(a+b\,x\right)^4+6\,\tan\left(a+b\,x\right)^2-3}{48\,b\,\tan\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

```
[Out] (6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(48*b*tan(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.22 $\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^4(a+bx)}{128b}+\frac{3\tan^2(a+bx)}{64b}-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

```
[Out] -1/64*cot(b*x+a)^2/b+3/32*ln(tan(b*x+a))/b+3/64*tan(b*x+a)^2/b+1/128*tan(b*
x+a)^4/b
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^4(a+bx)}{128b}+\frac{3\tan^2(a+bx)}{64b}-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^2/(64*b) + (3*Log[Tan[a + b*x]])/(32*b) + (3*Tan[a + b*x]^2)/
(64*b) + Tan[a + b*x]^4/(128*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \frac{1}{32}\int \csc^3(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{3\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

Mathematica [A] time = 0.25, size = 56, normalized size = 0.93

$$-\frac{2\csc^2(a+bx)-\sec^4(a+bx)-4\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -1/128*(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4*
Sec[a + b*x]^2 - Sec[a + b*x]^4)/b
```

fricas [B] time = 0.43, size = 112, normalized size = 1.87

$$\frac{6\cos(bx+a)^4-3\cos(bx+a)^2-6\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right)+6\left(\cos(bx+a)^6-\text{c}\right.}{128\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x +
a)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*co
s(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

giac [B] time = 2.35, size = 2657, normalized size = 44.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2048*(32*(9*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 540*tan(b*x + 4*a)^2*tan(1/
2*a)^22 + 96*tan(b*x + 4*a)*tan(1/2*a)^23 + tan(1/2*a)^24 + 12690*tan(b*x +
 4*a)^2*tan(1/2*a)^20 - 5072*tan(b*x + 4*a)*tan(1/2*a)^21 + 264*tan(1/2*a)^
22 - 145836*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 94416*tan(b*x + 4*a)*tan(1/2*a
)^19 - 11766*tan(1/2*a)^20 + 833895*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 775584
*tan(b*x + 4*a)*tan(1/2*a)^17 + 152744*tan(1/2*a)^18 - 2212920*tan(b*x + 4*
a)^2*tan(1/2*a)^14 + 2952832*tan(b*x + 4*a)*tan(1/2*a)^15 - 825777*tan(1/2*
a)^16 + 3025404*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 5609184*tan(b*x + 4*a)*tan
(1/2*a)^13 + 2205264*tan(1/2*a)^14 - 2212920*tan(b*x + 4*a)^2*tan(1/2*a)^10
 + 5609184*tan(b*x + 4*a)*tan(1/2*a)^11 - 3045556*tan(1/2*a)^12 + 833895*ta
```

```
n(b*x + 4*a)^2*tan(1/2*a)^8 - 2952832*tan(b*x + 4*a)*tan(1/2*a)^9 + 2205264
*tan(1/2*a)^10 - 145836*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 775584*tan(b*x + 4*
a)*tan(1/2*a)^7 - 825777*tan(1/2*a)^8 + 12690*tan(b*x + 4*a)^2*tan(1/2*a)^4
 - 94416*tan(b*x + 4*a)*tan(1/2*a)^5 + 152744*tan(1/2*a)^6 - 540*tan(b*x +
4*a)^2*tan(1/2*a)^2 + 5072*tan(b*x + 4*a)*tan(1/2*a)^3 - 11766*tan(1/2*a)^4
 + 9*tan(b*x + 4*a)^2 - 96*tan(b*x + 4*a)*tan(1/2*a) + 264*tan(1/2*a)^2 + 1
)/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6
+ 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15
*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a
)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^2) - (864*tan(b*x +
4*a)^3*tan(1/2*a)^45 - 216*tan(b*x + 4*a)^2*tan(1/2*a)^46 + 24*tan(b*x + 4*
a)*tan(1/2*a)^47 - tan(1/2*a)^48 + 50976*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 1
8000*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 2584*tan(b*x + 4*a)*tan(1/2*a)^45 - 9
6*tan(1/2*a)^46 + 41990400*tan(b*x + 4*a)^4*tan(1/2*a)^40 - 29618496*tan(b*
x + 4*a)^3*tan(1/2*a)^41 + 8128920*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 1000584
*tan(b*x + 4*a)*tan(1/2*a)^43 + 46860*tan(1/2*a)^44 - 1119744000*tan(b*x +
4*a)^4*tan(1/2*a)^38 + 1086111040*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 36523900
8*tan(b*x + 4*a)^2*tan(1/2*a)^40 + 52636536*tan(b*x + 4*a)*tan(1/2*a)^41 -
2775328*tan(1/2*a)^42 + 13399603200*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 169820
62752*tan(b*x + 4*a)^3*tan(1/2*a)^37 + 7267900536*tan(b*x + 4*a)^2*tan(1/2*
a)^38 - 1271292760*tan(b*x + 4*a)*tan(1/2*a)^39 + 78743742*tan(1/2*a)^40 -
94929408000*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 150936080928*tan(b*x + 4*a)^3*
tan(1/2*a)^35 - 81088683024*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 17523815592*ta
n(b*x + 4*a)*tan(1/2*a)^37 - 1302867360*tan(1/2*a)^38 + 442437811200*tan(b*
x + 4*a)^4*tan(1/2*a)^32 - 858638750464*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 56
7089250120*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 150754238328*tan(b*x + 4*a)*tan
(1/2*a)^35 + 13639996380*tan(1/2*a)^36 - 1426650624000*tan(b*x + 4*a)^4*tan
(1/2*a)^30 + 3319597849344*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 2647648010880*t
an(b*x + 4*a)^2*tan(1/2*a)^32 + 854992222856*tan(b*x + 4*a)*tan(1/2*a)^33 -
 94087910880*tan(1/2*a)^34 + 3262626662400*tan(b*x + 4*a)^4*tan(1/2*a)^28 -
 9004992609600*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 8563235603472*tan(b*x + 4*a
)^2*tan(1/2*a)^30 - 3316579144464*tan(b*x + 4*a)*tan(1/2*a)^31 + 4400725427
37*tan(1/2*a)^32 - 5349143040000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 174203278
27008*tan(b*x + 4*a)^3*tan(1/2*a)^27 - 19596982709664*tan(b*x + 4*a)^2*tan(
1/2*a)^28 + 9014744094960*tan(b*x + 4*a)*tan(1/2*a)^29 - 1427870886848*tan(
1/2*a)^30 + 6307092928000*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 24202990429056*t
an(b*x + 4*a)^3*tan(1/2*a)^25 + 32094110324592*tan(b*x + 4*a)^2*tan(1/2*a)^
26 - 17430945047632*tan(b*x + 4*a)*tan(1/2*a)^27 + 3269665274136*tan(1/2*a)
^28 - 5349143040000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 24202990429056*tan(b*x
 + 4*a)^3*tan(1/2*a)^23 - 37811152430400*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 2
4188717892528*tan(b*x + 4*a)*tan(1/2*a)^25 - 5348679038784*tan(1/2*a)^26 +
3262626662400*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 17420327827008*tan(b*x + 4*a
)^3*tan(1/2*a)^21 + 32094110324592*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 2418871
7892528*tan(b*x + 4*a)*tan(1/2*a)^23 + 6296990528100*tan(1/2*a)^24 - 142665
0624000*tan(b*x + 4*a)^4*tan(1/2*a)^18 + 9004992609600*tan(b*x + 4*a)^3*tan
(1/2*a)^19 - 19596982709664*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17430945047632
*tan(b*x + 4*a)*tan(1/2*a)^21 - 5348679038784*tan(1/2*a)^22 + 442437811200*
tan(b*x + 4*a)^4*tan(1/2*a)^16 - 3319597849344*tan(b*x + 4*a)^3*tan(1/2*a)^
17 + 8563235603472*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 9014744094960*tan(b*x +
 4*a)*tan(1/2*a)^19 + 3269665274136*tan(1/2*a)^20 - 94929408000*tan(b*x + 4
*a)^4*tan(1/2*a)^14 + 858638750464*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 2647648
010880*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 3316579144464*tan(b*x + 4*a)*tan(1/
2*a)^17 - 1427870886848*tan(1/2*a)^18 + 13399603200*tan(b*x + 4*a)^4*tan(1/
2*a)^12 - 150936080928*tan(b*x + 4*a)^3*tan(1/2*a)^13 + 567089250120*tan(b*
x + 4*a)^2*tan(1/2*a)^14 - 854992222856*tan(b*x + 4*a)*tan(1/2*a)^15 + 4400
72542737*tan(1/2*a)^16 - 1119744000*tan(b*x + 4*a)^4*tan(1/2*a)^10 + 169820
62752*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 81088683024*tan(b*x + 4*a)^2*tan(1/2
*a)^12 + 150754238328*tan(b*x + 4*a)*tan(1/2*a)^13 - 94087910880*tan(1/2*a)
^14 + 41990400*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 1086111040*tan(b*x + 4*a)^3*
```

```
tan(1/2*a)^9 + 7267900536*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 17523815592*tan(
b*x + 4*a)*tan(1/2*a)^11 + 13639996380*tan(1/2*a)^12 + 29618496*tan(b*x + 4
*a)^3*tan(1/2*a)^7 - 365239008*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1271292760*t
an(b*x + 4*a)*tan(1/2*a)^9 - 1302867360*tan(1/2*a)^10 - 50976*tan(b*x + 4*a
)^3*tan(1/2*a)^5 + 8128920*tan(b*x + 4*a)^2*tan(1/2*a)^6 - 52636536*tan(b*x
 + 4*a)*tan(1/2*a)^7 + 78743742*tan(1/2*a)^8 - 864*tan(b*x + 4*a)^3*tan(1/2
*a)^3 - 18000*tan(b*x + 4*a)^2*tan(1/2*a)^4 + 1000584*tan(b*x + 4*a)*tan(1/
2*a)^5 - 2775328*tan(1/2*a)^6 - 216*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 2584*ta
n(b*x + 4*a)*tan(1/2*a)^3 + 46860*tan(1/2*a)^4 - 24*tan(b*x + 4*a)*tan(1/2*
a) - 96*tan(1/2*a)^2 - 1)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*ta
n(1/2*a)^16 - 15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^
10 + 5724*tan(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(6*tan(b*x +
4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(
1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 192*log(
abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/
2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a)
+ 6*tan(1/2*a))) + 192*log(abs(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6
 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(
1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

maple [A] time = 0.92, size = 69, normalized size = 1.15

$$\frac{1}{128b\sin(bx+a)^2\cos(bx+a)^4}+\frac{3}{128b\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{64b\sin(bx+a)^2}+\frac{3\ln(\tan(bx+a))}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/128/b/sin(b*x+a)^2/cos(b*x+a)^4+3/128/b/sin(b*x+a)^2/cos(b*x+a)^2-3/64/b/
sin(b*x+a)^2+3/32*ln(tan(b*x+a))/b
```

maxima [B] time = 0.44, size = 3164, normalized size = 52.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) + 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) - 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) + 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) + 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) +
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) - 6)*cos(8*b*x + 8*a) - 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) + 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) + 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) + 2)*cos(4*b*x + 4*a)
 - 24*cos(4*b*x + 4*a)^2 + 24*cos(2*b*x + 2*a)^2 - 3*(2*(2*cos(10*b*x + 10*
a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
```

```
(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) + 3*(2*(2*cos(10*b*x + 10*a) - cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b
*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*si
n(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*(2*(2
*cos(10*b*x + 10*a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + co
s(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) + 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) - 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 + 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) - 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + 32*sin(6*b*x + 6*a)^2 - 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(10*b*x + 10*a) - b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) + b)*cos(12*b*x + 12*a) - 4*(b*cos(8*b*x + 8*a) + 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) + 2*(4
*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*
b*x + 8*a) + 8*(b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) - 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a)
 + 2*(2*b*sin(10*b*x + 10*a) - b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) -
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 4*(b*sin(8*
b*x + 8*a) + 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) + 2*(4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 2*b
```

```
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 8*(b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [**B**] time = 0.16, size = 74, normalized size = 1.23

$$\frac{3\ln\left(\sin(a+bx)^2\right)}{64\,b}-\frac{3\ln(\cos(a+bx))}{32\,b}+\frac{-\frac{3\cos(a+bx)^4}{64}+\frac{3\cos(a+bx)^2}{128}+\frac{1}{128}}{b\left(\cos(a+bx)^4-\cos(a+bx)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a +
 b*x)^2)/128 - (3*cos(a + b*x)^4)/64 + 1/128)/(b*(cos(a + b*x)^4 - cos(a +
b*x)^6))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.23 $\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^{13}(a+bx)}{13b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^9(a+bx)}{9b}$$

```
[Out] 32/9*sin(b*x+a)^9/b-64/11*sin(b*x+a)^11/b+32/13*sin(b*x+a)^13/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 270}

$$\frac{32\sin^{13}(a+bx)}{13b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^9)/(9*b) - (64*Sin[a + b*x]^11)/(11*b) + (32*Sin[a + b*x]^
13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^8(a+bx)\,dx\\ &= \frac{32\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.37, size = 37, normalized size = 0.80

$$\frac{4\sin^9(a+bx)(540\cos(2(a+bx))+99\cos(4(a+bx))+505)}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(505 + 540*Cos[2*(a + b*x)] + 99*Cos[4*(a + b*x)])*Sin[a + b*x]^9)/(1287
*b)
```

fricas [A] time = 0.42, size = 73, normalized size = 1.59

$$\frac{32\left(99\,\cos\left(bx+a\right)^{12}-360\,\cos\left(bx+a\right)^{10}+458\,\cos\left(bx+a\right)^{8}-212\,\cos\left(bx+a\right)^{6}+3\,\cos\left(bx+a\right)^{4}+4\,\cos\left(\right.\right.}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 32/1287*(99*cos(b*x + a)^12 - 360*cos(b*x + a)^10 + 458*cos(b*x + a)^8 - 21
2*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

giac [B] time = 0.62, size = 96, normalized size = 2.09

$$\frac{\sin\left(13\,bx+13\,a\right)}{1664\,b}-\frac{3\,\sin\left(11\,bx+11\,a\right)}{1408\,b}-\frac{\sin\left(9\,bx+9\,a\right)}{576\,b}+\frac{\sin\left(7\,bx+7\,a\right)}{64\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{128\,b}-\frac{25\,\sin\left(3\,bx+3\,a\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/1664*sin(13*b*x + 13*a)/b - 3/1408*sin(11*b*x + 11*a)/b - 1/576*sin(9*b*x
 + 9*a)/b + 1/64*sin(7*b*x + 7*a)/b - 1/128*sin(5*b*x + 5*a)/b - 25/384*sin
(3*b*x + 3*a)/b + 5/32*sin(b*x + a)/b
```

maple [B] time = 1.48, size = 97, normalized size = 2.11

$$\frac{5\sin\left(bx+a\right)}{32b}-\frac{25\sin\left(3bx+3a\right)}{384b}-\frac{\sin\left(5bx+5a\right)}{128b}+\frac{\sin\left(7bx+7a\right)}{64b}-\frac{\sin\left(9bx+9a\right)}{576b}-\frac{3\sin\left(11bx+11a\right)}{1408b}+\frac{\sin(13}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] 5/32*sin(b*x+a)/b-25/384*sin(3*b*x+3*a)/b-1/128/b*sin(5*b*x+5*a)+1/64/b*sin
(7*b*x+7*a)-1/576/b*sin(9*b*x+9*a)-3/1408/b*sin(11*b*x+11*a)+1/1664/b*sin(1
3*b*x+13*a)
```

maxima [A] time = 0.35, size = 80, normalized size = 1.74

$$\frac{99\,\sin\left(13\,bx+13\,a\right)-351\,\sin\left(11\,bx+11\,a\right)-286\,\sin\left(9\,bx+9\,a\right)+2574\,\sin\left(7\,bx+7\,a\right)-1287\,\sin\left(5\,bx+\right.}{164736\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/164736*(99*sin(13*b*x + 13*a) - 351*sin(11*b*x + 11*a) - 286*sin(9*b*x +
9*a) + 2574*sin(7*b*x + 7*a) - 1287*sin(5*b*x + 5*a) - 10725*sin(3*b*x + 3*
a) + 25740*sin(b*x + a))/b
```

mupad [B] time = 0.07, size = 36, normalized size = 0.78

$$\frac{32\,\left(99\,\sin\left(a+b\,x\right)^{13}-234\,\sin\left(a+b\,x\right)^{11}+143\,\sin\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

```
[Out] (32*(143*sin(a + b*x)^9 - 234*sin(a + b*x)^11 + 99*sin(a + b*x)^13))/(1287*
b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.24 $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{16\cos^{11}(a+bx)}{11b}-\frac{16\cos^9(a+bx)}{3b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+48/7*cos(b*x+a)^7/b-16/3*cos(b*x+a)^9/b+16/11*cos(b*x+
a)^11/b
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 270}

$$\frac{16\cos^{11}(a+bx)}{11b}-\frac{16\cos^9(a+bx)}{3b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (48*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(3*b) + (16*Cos[a + b*x]^11)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^7(a+bx)\,dx\\
&= -\frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{16\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 47, normalized size = 0.77

$$\frac{\cos^5(a+bx)(3335\cos(2(a+bx))-910\cos(4(a+bx))+105\cos(6(a+bx))-3042)}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (Cos[a + b*x]^5*(-3042 + 3335*Cos[2*(a + b*x)] - 910*Cos[4*(a + b*x)] + 105
*Cos[6*(a + b*x)]))/(2310*b)
```

fricas [A] time = 0.42, size = 46, normalized size = 0.75

$$\frac{16\left(105\cos\left(bx+a\right)^{11}-385\cos\left(bx+a\right)^{9}+495\cos\left(bx+a\right)^{7}-231\cos\left(bx+a\right)^{5}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/1155*(105*cos(b*x + a)^11 - 385*cos(b*x + a)^9 + 495*cos(b*x + a)^7 - 23
1*cos(b*x + a)^5)/b
```

giac [A] time = 0.80, size = 82, normalized size = 1.34

$$\frac{\cos\left(11\,bx+11\,a\right)}{704\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{192\,b}-\frac{\cos\left(7\,bx+7\,a\right)}{448\,b}+\frac{11\cos\left(5\,bx+5\,a\right)}{320\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{32\,b}-\frac{7\cos\left(bx+a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/704*cos(11*b*x + 11*a)/b - 1/192*cos(9*b*x + 9*a)/b - 1/448*cos(7*b*x + 7
*a)/b + 11/320*cos(5*b*x + 5*a)/b - 1/32*cos(3*b*x + 3*a)/b - 7/32*cos(b*x
+ a)/b
```

maple [A] time = 0.30, size = 83, normalized size = 1.36

$$-\frac{7\cos\left(bx+a\right)}{32b}-\frac{\cos\left(3bx+3a\right)}{32b}+\frac{11\cos\left(5bx+5a\right)}{320b}-\frac{\cos\left(7bx+7a\right)}{448b}-\frac{\cos\left(9bx+9a\right)}{192b}+\frac{\cos\left(11bx+11a\right)}{704b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] -7/32*cos(b*x+a)/b-1/32*cos(3*b*x+3*a)/b+11/320*cos(5*b*x+5*a)/b-1/448*cos(
7*b*x+7*a)/b-1/192*cos(9*b*x+9*a)/b+1/704*cos(11*b*x+11*a)/b
```

maxima [A] time = 0.34, size = 69, normalized size = 1.13

$$\frac{105\cos\left(11\,bx+11\,a\right)-385\cos\left(9\,bx+9\,a\right)-165\cos\left(7\,bx+7\,a\right)+2541\cos\left(5\,bx+5\,a\right)-2310\cos\left(3\,bx+3\,a\right.}{73920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7*
a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/b
```

mupad [B] time = 0.12, size = 46, normalized size = 0.75

$$-\frac{-\frac{16\cos\left(a+b\,x\right)^{11}}{11}+\frac{16\cos\left(a+b\,x\right)^{9}}{3}-\frac{48\cos\left(a+b\,x\right)^{7}}{7}+\frac{16\cos\left(a+b\,x\right)^{5}}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

```
[Out] -((16*cos(a + b*x)^5)/5 - (48*cos(a + b*x)^7)/7 + (16*cos(a + b*x)^9)/3 - (
16*cos(a + b*x)^11)/11)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.25 $\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

```
[Out] 8/7*sin(b*x+a)^7/b-8/9*sin(b*x+a)^9/b
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 14}

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^7)/(7*b) - (8*Sin[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{8\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.19, size = 27, normalized size = 0.87

$$\frac{4\sin^7(a+bx)(7\cos(2(a+bx))+11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*(11 + 7*Cos[2*(a + b*x)])*Sin[a + b*x]^7)/(63*b)
```

fricas [A] time = 0.41, size = 53, normalized size = 1.71

$$-\frac{8\left(7\cos\left(bx+a\right)^{8}-19\cos\left(bx+a\right)^{6}+15\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -8/63*(7*cos(b*x + a)^8 - 19*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - cos(b*x +
 a)^2 - 2)*sin(b*x + a)/b
```

giac [A] time = 0.45, size = 54, normalized size = 1.74

$$-\frac{\sin\left(9\,bx+9\,a\right)}{288\,b}+\frac{3\,\sin\left(7\,bx+7\,a\right)}{224\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{12\,b}+\frac{3\,\sin\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/288*sin(9*b*x + 9*a)/b + 3/224*sin(7*b*x + 7*a)/b - 1/12*sin(3*b*x + 3*a
)/b + 3/16*sin(b*x + a)/b
```

maple [A] time = 0.88, size = 55, normalized size = 1.77

$$\frac{3\sin\left(bx+a\right)}{16b}-\frac{\sin\left(3bx+3a\right)}{12b}+\frac{3\sin\left(7bx+7a\right)}{224b}-\frac{\sin\left(9bx+9a\right)}{288b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] 3/16*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b+3/224/b*sin(7*b*x+7*a)-1/288/b*sin(
9*b*x+9*a)
```

maxima [A] time = 0.35, size = 47, normalized size = 1.52

$$-\frac{7\,\sin\left(9\,bx+9\,a\right)-27\,\sin\left(7\,bx+7\,a\right)+168\,\sin\left(3\,bx+3\,a\right)-378\,\sin\left(bx+a\right)}{2016\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/2016*(7*sin(9*b*x + 9*a) - 27*sin(7*b*x + 7*a) + 168*sin(3*b*x + 3*a) -
378*sin(b*x + a))/b
```

mupad [B] time = 0.05, size = 26, normalized size = 0.84

$$\frac{8\,\left(9\sin\left(a+b\,x\right)^{7}-7\sin\left(a+b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

```
[Out] (8*(9*sin(a + b*x)^7 - 7*sin(a + b*x)^9))/(63*b)
```

sympy [A] time = 115.46, size = 284, normalized size = 9.16

$$\begin{cases} -\frac{46\sin^3(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{105b} - \frac{16\sin^3(a+bx)\cos^3(2a+2bx)}{63b} - \frac{13\sin^2(a+bx)\sin^3(2a+2bx)\cos(a+bx)}{105b} - \frac{8\sin^2(a+bx)\sin(2a+2b}{} \\ x\sin^3(a)\sin^3(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-46*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(105*b)
 - 16*sin(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b) - 13*sin(a + b*x)**2*sin(2
*a + 2*b*x)**3*cos(a + b*x)/(105*b) - 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*co
s(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) - 4*sin(a + b*x)*sin(2*a + 2*b*x)**2*
cos(a + b*x)**2*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)*cos(a + b*x)**2*co
s(2*a + 2*b*x)**3/(105*b) + 94*sin(2*a + 2*b*x)**3*cos(a + b*x)**3/(315*b)
+ 32*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(b, 0)
), (x*sin(a)**3*sin(2*a)**3, True))
```

3.26 $\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^7(a+bx)}{7b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] -4/3*cos(b*x+a)^3/b+8/5*cos(b*x+a)^5/b-4/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 270}

$$-\frac{4\cos^7(a+bx)}{7b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (8*Cos[a + b*x]^5)/(5*b) - (4*Cos[a + b*x]^7)/(
7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 37, normalized size = 0.80

$$\frac{\cos^3(a+bx)(108\cos(2(a+bx))-15\cos(4(a+bx))-157)}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(210*b
)
```

fricas [A] time = 0.45, size = 36, normalized size = 0.78

$$-\frac{4\left(15\cos\left(bx+a\right)^{7}-42\cos\left(bx+a\right)^{5}+35\cos\left(bx+a\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

giac [A] time = 0.33, size = 54, normalized size = 1.17

$$-\frac{\cos\left(7\,bx+7\,a\right)}{112\,b}+\frac{3\cos\left(5\,bx+5\,a\right)}{80\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{48\,b}-\frac{5\cos\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/112*cos(7*b*x + 7*a)/b + 3/80*cos(5*b*x + 5*a)/b - 1/48*cos(3*b*x + 3*a)
/b - 5/16*cos(b*x + a)/b
```

maple [A] time = 0.30, size = 55, normalized size = 1.20

$$-\frac{5\cos\left(bx+a\right)}{16b}-\frac{\cos\left(3bx+3a\right)}{48b}+\frac{3\cos\left(5bx+5a\right)}{80b}-\frac{\cos\left(7bx+7a\right)}{112b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] -5/16*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+3/80*cos(5*b*x+5*a)/b-1/112*cos(7*
b*x+7*a)/b
```

maxima [A] time = 0.34, size = 47, normalized size = 1.02

$$-\frac{15\cos\left(7\,bx+7\,a\right)-63\cos\left(5\,bx+5\,a\right)+35\cos\left(3\,bx+3\,a\right)+525\cos\left(bx+a\right)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
525*cos(b*x + a))/b
```

mupad [B] time = 0.12, size = 36, normalized size = 0.78

$$-\frac{4\left(15\cos\left(a+b\,x\right)^{7}-42\cos\left(a+b\,x\right)^{5}+35\cos\left(a+b\,x\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

```
[Out] -(4*(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7))/(105*b)
```

sympy [A] time = 38.63, size = 202, normalized size = 4.39

$$\begin{cases} -\frac{12\sin^3(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{35b} - \frac{11\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{35b} - \frac{24\sin^2(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{35b} + \frac{8\sin(a+b}{} \\ x\sin^3(a)\sin^2(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-12*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(35*b) - 1
1*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)/(35*b) - 24*sin(a + b*x)
**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) + 8*sin(a + b*x)*sin(2*a + 2*b*
x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(35*b) - 38*sin(2*a + 2*b*x)**2*cos(a +
 b*x)**3/(105*b) - 32*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(b, 0)
), (x*sin(a)**3*sin(2*a)**2, True))
```

3.27 $\int \sin^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{2\sin^5(a+bx)}{5b}$$

```
[Out] 2/5*sin(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 30}

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{2\,\mathrm{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{2\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

fricas [B] time = 0.46, size = 31, normalized size = 2.07

$$\frac{2\left(\cos\left(bx+a\right)^{4}-2\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

giac [B] time = 0.33, size = 40, normalized size = 2.67

$$\frac{\sin\left(5\,bx+5\,a\right)}{40\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{8\,b}+\frac{\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
[Out] 1/40*sin(5*b*x + 5*a)/b - 1/8*sin(3*b*x + 3*a)/b + 1/4*sin(b*x + a)/b
```

maple [B] time = 0.58, size = 41, normalized size = 2.73

$$\frac{\sin\left(bx+a\right)}{4b}-\frac{\sin\left(3bx+3a\right)}{8b}+\frac{\sin\left(5bx+5a\right)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a),x)
[Out] 1/4*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b+1/40/b*sin(5*b*x+5*a)
```

maxima [B] time = 0.33, size = 34, normalized size = 2.27

$$\frac{\sin\left(5\,bx+5\,a\right)-5\,\sin\left(3\,bx+3\,a\right)+10\,\sin\left(bx+a\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
[Out] 1/40*(sin(5*b*x + 5*a) - 5*sin(3*b*x + 3*a) + 10*sin(b*x + a))/b
```

mupad [B] time = 0.10, size = 13, normalized size = 0.87

$$\frac{2\sin\left(a+b\,x\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x),x)
[Out] (2*sin(a + b*x)^5)/(5*b)
```

sympy [A] time = 11.22, size = 117, normalized size = 7.80

$$\begin{cases}-\frac{2\sin^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{5b}-\frac{\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{5b}-\frac{4\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)\cos\left(2a+2bx\right)}{5b}+\frac{2\sin\left(2a+2bx\right)\cos^{3}\left(a+bx\right)}{5b}\\ x\sin^{3}\left(a\right)\sin\left(2a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*cos(2*a + 2*b*x)/(5*b) - sin(a + b*x)**2*sin(
2*a + 2*b*x)*cos(a + b*x)/(5*b) - 4*sin(a + b*x)*cos(a + b*x)**2*cos(2*a +
2*b*x)/(5*b) + 2*sin(2*a + 2*b*x)*cos(a + b*x)**3/(5*b), Ne(b, 0)), (x*sin(
a)**3*sin(2*a), True))
```

3.28 $\int \csc(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*sin(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2592, 321, 206}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Sin[a + b*x]/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{2}\int \sin(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= -\frac{\sin(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\sin(a+bx)}{2b}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.96

$$\frac{1}{2}\left(\frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b)/2
```

fricas [A] time = 0.49, size = 36, normalized size = 1.29

$$\frac{\log\left(\sin\left(bx+a\right)+1\right) - \log\left(-\sin\left(bx+a\right)+1\right) - 2\,\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

giac [B] time = 0.99, size = 618, normalized size = 22.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^15 + 3*tan(1/2*a)^14 + 3*tan(1/2*a)^13 + 17*tan(1/2*a)^12
 - 3*tan(1/2*a)^11 + 39*tan(1/2*a)^10 - 25*tan(1/2*a)^9 + 45*tan(1/2*a)^8 -
45*tan(1/2*a)^7 + 25*tan(1/2*a)^6 - 39*tan(1/2*a)^5 + 3*tan(1/2*a)^4 - 17*t
an(1/2*a)^3 - 3*tan(1/2*a)^2 - 3*tan(1/2*a) - 1)*log(abs(tan(1/2*b*x + 2*a)
*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/
2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a
) - 1))/(tan(1/2*a)^3 + 3*tan(1/2*a)^2 - 3*tan(1/2*a) - 1) - (tan(1/2*a)^15
 - 3*tan(1/2*a)^14 + 3*tan(1/2*a)^13 - 17*tan(1/2*a)^12 - 3*tan(1/2*a)^11 -
 39*tan(1/2*a)^10 - 25*tan(1/2*a)^9 - 45*tan(1/2*a)^8 - 45*tan(1/2*a)^7 - 2
5*tan(1/2*a)^6 - 39*tan(1/2*a)^5 - 3*tan(1/2*a)^4 - 17*tan(1/2*a)^3 + 3*tan
(1/2*a)^2 - 3*tan(1/2*a) + 1)*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/
2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1))/(tan(1/2*a)
^3 - 3*tan(1/2*a)^2 - 3*tan(1/2*a) + 1) + 2*(tan(1/2*b*x + 2*a)*tan(1/2*a)^
12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x +
2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 -
```

```
2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1/2*a))/(tan(1/2*b*x + 2*a)^2 +
 1))/b
```

maple [A] time = 0.52, size = 32, normalized size = 1.14

$$-\frac{\sin(bx+a)}{2b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^3,x)
```

```
[Out] -1/2*sin(b*x+a)/b+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.47, size = 124, normalized size = 4.43

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)+2\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
+ 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + s
in(a)^2)) + 2*sin(b*x + a))/b
```

mupad [B] time = 0.12, size = 23, normalized size = 0.82

$$-\frac{\frac{\sin(a+bx)}{2}-\frac{\operatorname{atanh}(\sin(a+bx))}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x),x)
```

```
[Out] -(sin(a + b*x)/2 - atanh(sin(a + b*x))/2)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.29 $\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\sec(a+bx)}{4b}$$

```
[Out] 1/4*sec(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{4288, 2606, 8\}$

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] Sec[a + b*x]/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{4}\int \sec(a+bx)\tan(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(a+bx)\right)}{4b}\\ &= \frac{\sec(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] Sec[a + b*x]/(4*b)
```

fricas [A] time = 0.44, size = 13, normalized size = 1.00

$$\frac{1}{4\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b*cos(b*x + a))
```

giac [B] time = 3.87, size = 319, normalized size = 24.54

$$-\frac{6\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{11}-\tan\left(\frac{1}{2}\,a\right)^{12}-2\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{9}+12\,\tan\left(\frac{1}{2}\right.}{2\left(\tan\left(\frac{1}{2}\,bx+2\,a\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{6}-15\,\tan\left(\frac{1}{2}\,bx+2\,a\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{4}+12\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{5}-\tan\left(\frac{1}{2}\,a\right)^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b*x +
2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 +
 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 12*tan
(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan
(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*ta
n(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*t
an(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1)*b)
```

maple [A] time = 0.68, size = 14, normalized size = 1.08

$$\frac{1}{4b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/4/b/cos(b*x+a)
```

maxima [B] time = 0.33, size = 83, normalized size = 6.38

$$\frac{\cos\left(2\,bx+2\,a\right)\cos\left(bx+a\right)+\sin\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}+2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*
x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a)
 + b)
```

mupad [B] time = 0.09, size = 13, normalized size = 1.00

$$\frac{1}{4\,b\,\cos\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

```
[Out] 1/(4*b*cos(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.30 $\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{8}\int \sec^3(a+bx)\,dx\\ &= \frac{\sec(a+bx)\tan(a+bx)}{16b}+\frac{1}{16}\int \sec(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$\frac{1}{8}\left(\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
[Out] (ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

fricas [B] time = 0.45, size = 61, normalized size = 1.79

$$\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\sin(bx+a)}{32\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/32*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

giac [B] time = 2.95, size = 1111, normalized size = 32.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/16*(2*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*tan(
1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan(1
/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23 +
2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 -
 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/2*
b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 233
1*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*tan
(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a)*
tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6
 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1/2
*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 21
78*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2*a
)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(1/
2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1
/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*
a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1
/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*
b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1
/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) - log(abs(tan(1/2*b*x +
2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*ta
n(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1
/2*a) - 1)) + log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2
*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 10.77, size = 38, normalized size = 1.12

$$\frac{\sec(bx+a)\tan(bx+a)}{16b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/16*sec(b*x+a)*tan(b*x+a)/b+1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.47, size = 480, normalized size = 14.12

$$\frac{4\left(\sin\left(3\,bx+3\,a\right)-\sin\left(bx+a\right)\right)\cos\left(4\,bx+4\,a\right)-\left(2\left(2\,\cos\left(2\,bx+2\,a\right)+1\right)\cos\left(4\,bx+4\,a\right)+\cos\left(4\,bx+4\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(sin(3*b*x + 3*a) - sin(b*x + a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*
x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2
+ sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a
)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(
cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 -
2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(cos(3*b*x + 3*a) - cos(b*x + a))*
sin(4*b*x + 4*a) + 4*(2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 8*cos(b*x + a)*sin(2*b*x + 2*a) - 8*cos(2*b*x +
 2*a)*sin(b*x + a) - 4*sin(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x
+ 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b
*cos(2*b*x + 2*a) + b)
```

mupad [B] time = 0.17, size = 36, normalized size = 1.06

$$\frac{\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b}-\frac{\sin\left(a+b\,x\right)}{16\,b\,\left({\sin\left(a+b\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

```
[Out] atanh(sin(a + b*x))/(16*b) - sin(a + b*x)/(16*b*(sin(a + b*x)^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.31 $\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sec^3(a+bx)}{48b}+\frac{\sec(a+bx)}{16b}-\frac{\tanh^{-1}(\cos(a+bx))}{16b}$$

```
[Out] -1/16*arctanh(cos(b*x+a))/b+1/16*sec(b*x+a)/b+1/48*sec(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2622, 302, 207}

$$\frac{\sec^3(a+bx)}{48b}+\frac{\sec(a+bx)}{16b}-\frac{\tanh^{-1}(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(16*b) + Sec[a + b*x]/(16*b) + Sec[a + b*x]^3/(48*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{16}\int \csc(a+bx)\sec^4(a+bx)\,dx\\ &= \frac{\mathrm{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}\\ &= \frac{\mathrm{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{16b}\\ &= \frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{16b}+\frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.42

$$\frac{1}{16}\left(\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] (-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Se
c[a + b*x]^3/(3*b))/16
```

fricas [A] time = 0.48, size = 67, normalized size = 1.56

$$-\frac{3\cos(bx+a)^3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\cos(bx+a)^3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)^2-2}{96\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(
-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

giac [B] time = 6.79, size = 2114, normalized size = 49.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*(4*(18*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 3*tan(1/2*b*x + 2*a)^4*ta
n(1/2*a)^36 - 762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 486*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^34 - 72*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 3*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^36 + 13644*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 - 15561*t
an(1/2*b*x + 2*a)^4*tan(1/2*a)^32 + 5424*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33
 - 756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 54*tan(1/2*b*x + 2*a)*tan(1/2*a
)^35 - 2*tan(1/2*a)^36 - 140076*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 + 233916
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 126936*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^31 + 28701*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 2934*tan(1/2*b*x + 2*a)*t
an(1/2*a)^33 + 126*tan(1/2*a)^34 + 811140*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2
```

7 - 1893744*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 1474776*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^29 - 471816*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 + 64908*tan(1/
2*b*x + 2*a)*tan(1/2*a)^31 - 3330*tan(1/2*a)^32 - 1739556*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^25 + 6839316*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^26 - 8095576*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 3832164*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
28 - 727596*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 47148*tan(1/2*a)^30 - 461700
*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 6247800*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^24 + 16873272*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 13541976*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^26 + 4049556*tan(1/2*b*x + 2*a)*tan(1/2*a)^27 - 368892*
tan(1/2*a)^28 + 5103972*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 14529780*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^22 + 3596040*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23
+ 12515100*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 8506836*tan(1/2*b*x + 2*a)*
tan(1/2*a)^25 + 1396836*tan(1/2*a)^26 - 3586680*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^19 + 28026162*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 51234312*tan(1/2*b*
x + 2*a)^3*tan(1/2*a)^21 + 28649880*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 19
42020*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 1226940*tan(1/2*a)^24 - 3586680*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 37245240*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^19 - 56612142*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 25560228*tan(1/2*b*x +
2*a)*tan(1/2*a)^21 - 2935620*tan(1/2*a)^22 + 5103972*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^15 - 28026162*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 37245240*tan(1
/2*b*x + 2*a)^3*tan(1/2*a)^17 - 18495360*tan(1/2*b*x + 2*a)*tan(1/2*a)^19 +
5548608*tan(1/2*a)^20 - 461700*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 145297
80*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 51234312*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^15 + 56612142*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 18495360*tan(1/2*b
*x + 2*a)*tan(1/2*a)^17 - 1739556*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^11 + 6247
800*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 3596040*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^13 - 28649880*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 25560228*tan(1/2*b
*x + 2*a)*tan(1/2*a)^15 - 5548608*tan(1/2*a)^16 + 811140*tan(1/2*b*x + 2*a)
^5*tan(1/2*a)^9 - 6839316*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 + 16873272*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 12515100*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
12 - 1942020*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 2935620*tan(1/2*a)^14 - 140
076*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 + 1893744*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^8 - 8095576*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 13541976*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^10 - 8506836*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + 1226940*
tan(1/2*a)^12 + 13644*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^5 - 233916*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^6 + 1474776*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 38321
64*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 4049556*tan(1/2*b*x + 2*a)*tan(1/2*a
)^9 - 1396836*tan(1/2*a)^10 - 762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 15561
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 126936*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^5 + 471816*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 727596*tan(1/2*b*x + 2*a)*t
an(1/2*a)^7 + 368892*tan(1/2*a)^8 + 18*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 48
6*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 + 5424*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
3 - 28701*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 64908*tan(1/2*b*x + 2*a)*tan(
1/2*a)^5 - 47148*tan(1/2*a)^6 + 3*tan(1/2*b*x + 2*a)^4 - 72*tan(1/2*b*x + 2
*a)^3*tan(1/2*a) + 756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 2934*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 3330*tan(1/2*a)^4 - 3*tan(1/2*b*x + 2*a)^2 + 54*tan(
1/2*b*x + 2*a)*tan(1/2*a) - 126*tan(1/2*a)^2 + 2)/((tan(1/2*a)^18 - 45*tan(
1/2*a)^16 + 720*tan(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 -
10890*tan(1/2*a)^8 + 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2
- 1)*(tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*
a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4
- tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^
2 + 1)^3) + 3*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a
)*tan(1/2*a) + 3*tan(1/2*a)^2 - 1)) - 3*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/
2*a)^2 - tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b

maple [A] time = 0.92, size = 49, normalized size = 1.14

$$\frac{1}{48b\cos(bx+a)^3}+\frac{1}{16b\cos(bx+a)}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/48/b/cos(b*x+a)^3+1/16/b/cos(b*x+a)+1/16/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.36, size = 987, normalized size = 22.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) + 12*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*
a) + 12*(10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 40*(3*cos
(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) + 36*cos(2*b*x + 2*a)*cos(b*x + a) - 3*
(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + cos(6*b
*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4
*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(
6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4
*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) + 1)*log(c
os(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) +
 sin(a)^2) + 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
 6*a) + cos(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
9*cos(4*b*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 1
8*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x +
2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) + 3
*sin(b*x + a))*sin(6*b*x + 6*a) + 36*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(5*b*x + 5*a) + 12*(10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*
a) + 120*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 36*sin(2*b*x + 2*a)*sin(b*x +
a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*
cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 + 18*b*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 + 2*(3*b*cos(4*b*
x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 6*(3*b*cos(2*b*x +
2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 6*(b*sin(4*b*x + 4*a) +
 b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.07, size = 37, normalized size = 0.86

$$\frac{\frac{\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\cos\left(a+b\,x\right)^3}-\frac{\operatorname{atanh}\left(\cos\left(a+b\,x\right)\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

```
[Out] (cos(a + b*x)^2/16 + 1/48)/(b*cos(a + b*x)^3) - atanh(cos(a + b*x))/(16*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.32 $\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\csc(a+bx)}{256b}+\frac{15\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}$$

```
[Out] 15/256*arctanh(sin(b*x+a))/b-15/256*csc(b*x+a)/b+5/256*csc(b*x+a)*sec(b*x+a
)^2/b+1/128*csc(b*x+a)*sec(b*x+a)^4/b
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4288, 2621, 288, 321, 207}

$$-\frac{15\csc(a+bx)}{256b}+\frac{15\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(256*b) - (15*Csc[a + b*x])/(256*b) + (5*Csc[a +
 b*x]*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(128*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{32}\int \csc^2(a+bx)\sec^5(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \csc(a+bx)\right)}{32b} \\
&= \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{128b} \\
&= \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,d\right.}{256} \\
&= -\frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{1}{} \\
&= \frac{15\tanh^{-1}(\sin(a+bx))}{256b} - \frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \csc(
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 29, normalized size = 0.41

$$
-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(a+bx)\right)}{32b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] -1/32*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b
```

fricas [A] time = 0.59, size = 95, normalized size = 1.36

$$
\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a) - 15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a) - 30\,\text{co}}{512\,b\cos(bx+a)^4\sin(bx+a)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
 a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

giac [B] time = 14.59, size = 4032, normalized size = 57.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(4*(tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2
*a)^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a
)^9 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^
5 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a
) + 6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a
)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*
```

```
a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*t
an(1/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) + 2*(9*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^48 - 54*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^46 + 90*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^5*tan(1/2*a)^48 - 16
938*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^44 + 10878*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^45 - 2058*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^46 + 162*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^3*tan(1/2*a)^48 + 648690*tan(1/2*b*x +
2*a)^7*tan(1/2*a)^42 - 772902*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^43 + 320922*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^44 - 59994*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4
5 + 5718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^46 - 306*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^47 + 9*tan(1/2*b*x + 2*a)*tan(1/2*a)^48 - 11649780*tan(1/2*b*x + 2*a
)^7*tan(1/2*a)^40 + 20073870*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^41 - 12606642*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 3790962*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^43 - 601830*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^44 + 53562*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^45 - 2646*tan(1/2*b*x + 2*a)*tan(1/2*a)^46 + 54*tan(1/2*a)^47
 + 122301894*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^38 - 281501690*tan(1/2*b*x + 2
*a)^6*tan(1/2*a)^39 + 240373332*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^40 - 992325
06*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41 + 21805614*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^42 - 2617650*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^43 + 166230*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^44 - 4446*tan(1/2*a)^45 - 762446542*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^36 + 2299471746*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^37 - 2589365766*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 + 1410179550*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^39 - 404516268*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^40 + 62438634*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^41 - 4933038*tan(1/2*b*x + 2*a)*tan(1/2*a)^42 + 1594
62*tan(1/2*a)^43 + 2624819022*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^34 - 10699256
970*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^35 + 16002520222*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^36 - 11500345350*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^37 + 4307714970*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 - 849637038*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^39 + 83542284*tan(1/2*b*x + 2*a)*tan(1/2*a)^40 - 3248766*tan(1/2*a)^41
- 4363726131*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^32 + 26510577474*tan(1/2*b*x +
 2*a)^6*tan(1/2*a)^33 - 55167038478*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 + 53
489761470*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 - 26616485090*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^36 + 6857208774*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^37 - 856321
050*tan(1/2*b*x + 2*a)*tan(1/2*a)^38 + 40894922*tan(1/2*a)^39 + 148294628*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^30 - 23718089916*tan(1/2*b*x + 2*a)^6*tan(1/
2*a)^31 + 91329113691*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^32 - 132573864918*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 92023852050*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^34 - 32055103710*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 + 5284344754*tan(1/2
*b*x + 2*a)*tan(1/2*a)^36 - 322093122*tan(1/2*a)^37 + 11061475644*tan(1/2*b
*x + 2*a)^7*tan(1/2*a)^28 - 36130112340*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^29
- 3422040164*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^30 + 118445882004*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^31 - 152452905381*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 +
79837550214*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^33 - 18462316050*tan(1/2*b*x +
2*a)*tan(1/2*a)^34 + 1522201770*tan(1/2*a)^35 - 14430929868*tan(1/2*b*x + 2
*a)^7*tan(1/2*a)^26 + 104533629188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27 - 231
729549276*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 + 180365650620*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^29 + 5258310492*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 - 71047
415988*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 30651378381*tan(1/2*b*x + 2*a)*
tan(1/2*a)^32 - 3833122290*tan(1/2*a)^33 - 62534029428*tan(1/2*b*x + 2*a)^6
*tan(1/2*a)^25 + 304139029068*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 - 52273462
2092*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 + 386477003556*tan(1/2*b*x + 2*a)^3
*tan(1/2*a)^28 - 109057608828*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 75017558
0*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 3377366748*tan(1/2*a)^31 + 14430929868
*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^22 - 62534029428*tan(1/2*b*x + 2*a)^6*tan(
1/2*a)^23 + 313058642364*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^25 - 506043183348*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 + 313110921612*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^27 - 77490791364*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 + 5273542932*tan(1
/2*a)^29 - 11061475644*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^20 + 104533629188*ta
```

n(1/2*b*x + 2*a)^6*tan(1/2*a)^21 - 304139029068*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^22 + 313058642364*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 186855789276*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^25 + 100600780788*tan(1/2*b*x + 2*a)*tan(1/2*
a)^26 - 14857714436*tan(1/2*a)^27 - 148294628*tan(1/2*b*x + 2*a)^7*tan(1/2*
a)^18 - 36130112340*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^19 + 231729549276*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^20 - 522734622092*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^21 + 506043183348*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 - 186855789276*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^23 + 8802017172*tan(1/2*a)^25 + 4363726131*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^16 - 23718089916*tan(1/2*b*x + 2*a)^6*tan(1/2*a)
^17 + 3422040164*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^18 + 180365650620*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^19 - 386477003556*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2
0 + 313110921612*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 - 100600780788*tan(1/2*
b*x + 2*a)*tan(1/2*a)^22 + 8802017172*tan(1/2*a)^23 - 2624819022*tan(1/2*b*
x + 2*a)^7*tan(1/2*a)^14 + 26510577474*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^15 -
 91329113691*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 + 118445882004*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^17 - 5258310492*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 10
9057608828*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 + 77490791364*tan(1/2*b*x + 2
*a)*tan(1/2*a)^20 - 14857714436*tan(1/2*a)^21 + 762446542*tan(1/2*b*x + 2*a
)^7*tan(1/2*a)^12 - 10699256970*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 551670
38478*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 - 132573864918*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^15 + 152452905381*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 7104741
5988*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 + 750175580*tan(1/2*b*x + 2*a)*tan(
1/2*a)^18 + 5273542932*tan(1/2*a)^19 - 122301894*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^10 + 2299471746*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 16002520222*tan(
1/2*b*x + 2*a)^5*tan(1/2*a)^12 + 53489761470*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^13 - 92023852050*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 79837550214*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^15 - 30651378381*tan(1/2*b*x + 2*a)*tan(1/2*a)^16
+ 3377366748*tan(1/2*a)^17 + 11649780*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^8 - 2
81501690*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^9 + 2589365766*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^10 - 11500345350*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 26616485
090*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 - 32055103710*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^13 + 18462316050*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 3833122290*ta
n(1/2*a)^15 - 648690*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^6 + 20073870*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^7 - 240373332*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 14
10179550*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 4307714970*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^10 + 6857208774*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 528434475
4*tan(1/2*b*x + 2*a)*tan(1/2*a)^12 + 1522201770*tan(1/2*a)^13 + 16938*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^4 - 772902*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^5 + 1
2606642*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 99232506*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^7 + 404516268*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 849637038*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^9 + 856321050*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 -
 322093122*tan(1/2*a)^11 + 54*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2 + 10878*tan
(1/2*b*x + 2*a)^6*tan(1/2*a)^3 - 320922*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 +
 3790962*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 21805614*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^6 + 62438634*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 83542284*tan(1/
2*b*x + 2*a)*tan(1/2*a)^8 + 40894922*tan(1/2*a)^9 - 9*tan(1/2*b*x + 2*a)^7
+ 90*tan(1/2*b*x + 2*a)^6*tan(1/2*a) + 2058*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^2 - 59994*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 601830*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^4 - 2617650*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 4933038*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 - 3248766*tan(1/2*a)^7 + tan(1/2*b*x + 2*a)^5 + 162
*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 5718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 +
 53562*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 166230*tan(1/2*b*x + 2*a)*tan(1/
2*a)^4 + 159462*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)^3 - 306*tan(1/2*b*x + 2*a
)^2*tan(1/2*a) + 2646*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 4446*tan(1/2*a)^3 -
 9*tan(1/2*b*x + 2*a) + 54*tan(1/2*a))/((tan(1/2*a)^24 - 60*tan(1/2*a)^22 +
 1410*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 92655*tan(1/2*a)^16 - 245880*ta
n(1/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*tan(1/2*a)^10 + 92655*tan(1/2*a
)^8 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4 - 60*tan(1/2*a)^2 + 1)*(tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan

```
(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*
x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 1
5*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(
1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 15*log(abs(tan(1/2*b*x + 2*a)*tan(1/2
*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x +
2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))
/b
```

maple [A] time = 0.92, size = 76, normalized size = 1.09

$$\frac{1}{128b\sin(bx+a)\cos(bx+a)^4}+\frac{5}{256b\sin(bx+a)\cos(bx+a)^2}-\frac{15}{256b\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(}{256b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x)

[Out] 1/128/b/sin(b*x+a)/cos(b*x+a)^4+5/256/b/sin(b*x+a)/cos(b*x+a)^2-15/256/b/si
n(b*x+a)+15/256/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.52, size = 1805, normalized size = 25.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/512*(4*(15*sin(9*b*x + 9*a) + 40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) +
 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) - 60*(3*sin(8*b*
x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*co
s(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) + 40*sin(3*b
*x + 3*a) + 15*sin(b*x + a))*cos(8*b*x + 8*a) - 160*(2*sin(6*b*x + 6*a) - 2
*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(18*sin(5*b*x
+ 5*a) + 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 72*(2*si
n(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(8*sin(3*b*x + 3
*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 15*(2*(3*cos(8*b*x + 8*a) + 2*cos(
6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10
*a) + cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3
*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4
*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)
^2 + 4*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + 9
*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2
+ 6*(2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*
x + 8*a) + 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 + 12*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) +
 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - 4*(15*cos(9*b*x + 9*a) + 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a
) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(10*b*x + 10*a) + 60*(3*cos(8
*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a)
- 1)*sin(9*b*x + 9*a) - 12*(40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) + 40*
cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(8*b*x + 8*a) + 160*(2*cos(6*b*x + 6
*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*sin(7*b*x + 7*a) - 8*(18
*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a)
 - 72*(2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) + 40*(
```

```
8*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*x + 4*a) - 160*(3*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a) + 480*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 180*co
s(b*x + a)*sin(2*b*x + 2*a) - 180*cos(2*b*x + 2*a)*sin(b*x + a) - 60*sin(b*
x + a))/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x +
6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 1
0*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x +
4*a)^2 + 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 +
2*(3*b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 3*b
*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) + 6*(2*b*cos(6*b*x + 6*a) - 2*b*c
os(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(4
*b*x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 4*(3*b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 2*(3*b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*s
in(10*b*x + 10*a) + 6*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*si
n(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x
+ 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.19, size = 67, normalized size = 0.96

$$\frac{15\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{256\,b}-\frac{\frac{15\,{\sin\left(a+b\,x\right)}^4}{256}-\frac{25\,{\sin\left(a+b\,x\right)}^2}{256}+\frac{1}{32}}{b\,\left({\sin\left(a+b\,x\right)}^5-2\,{\sin\left(a+b\,x\right)}^3+\sin\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (15*atanh(sin(a + b*x)))/(256*b) - ((15*sin(a + b*x)^4)/256 - (25*sin(a + b
*x)^2)/256 + 1/32)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.33 $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{256\cos^{15}(a+bx)}{15b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

```
[Out] -256/9*cos(b*x+a)^9/b+768/11*cos(b*x+a)^11/b-768/13*cos(b*x+a)^13/b+256/15*
cos(b*x+a)^15/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$\frac{256\cos^{15}(a+bx)}{15b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (768*Cos[a + b*x]^11)/(11*b) - (768*Cos[a + b
*x]^13)/(13*b) + (256*Cos[a + b*x]^15)/(15*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^8(2a+2bx)\,dx &= 256\int \cos^8(a+bx)\sin^7(a+bx)\,dx\\
&= -\frac{256\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\,\text{Subst}\left(\int \left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\cos^9(a+bx)}{9b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{256\cos^{15}(a+bx)}{15b}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{64b}-\frac{35\cos(3(a+bx))}{192b}+\frac{21\cos(5(a+bx))}{320b}+\frac{3\cos(7(a+bx))}{64b}-\frac{7\cos(9(a+bx))}{576b}-\frac{7\cos(11(a+bx))}{704b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-35*Cos[a + b*x])/(64*b) - (35*Cos[3*(a + b*x)])/(192*b) + (21*Cos[5*(a +
b*x)])/(320*b) + (3*Cos[7*(a + b*x)])/(64*b) - (7*Cos[9*(a + b*x)])/(576*b)
 - (7*Cos[11*(a + b*x)])/(704*b) + Cos[13*(a + b*x)]/(832*b) + Cos[15*(a +
b*x)]/(960*b)
```

fricas [A] time = 0.51, size = 46, normalized size = 0.75

$$\frac{256 \left(429 \cos \left(bx + a\right)^{15} - 1485 \cos \left(bx + a\right)^{13} + 1755 \cos \left(bx + a\right)^{11} - 715 \cos \left(bx + a\right)^{9}\right)}{6435 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 256/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

giac [B] time = 1.18, size = 270, normalized size = 4.43

$$-\frac{8192 \left(\frac{15 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1} - \frac{105 \left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}} + \frac{455 \left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}} + \frac{5070 \left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}} + \frac{30030 \left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}} + \frac{70070 \left(\cos(bx+a)-\right.}{\left(\cos(bx+a)+1\right)^{6}}\right.}{6435 \, b \left(\frac{\cos(bx-}{\cos(bx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -8192/6435*(15*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 105*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 455*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 +
 5070*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 30030*(cos(b*x + a) - 1)^
5/(cos(b*x + a) + 1)^5 + 70070*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 +
115830*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 109395*(cos(b*x + a) - 1
)^8/(cos(b*x + a) + 1)^8 + 75075*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9
+ 27027*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 6435*(cos(b*x + a) -
1)^11/(cos(b*x + a) + 1)^11 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
- 1)^15)
```

maple [A] time = 0.83, size = 71, normalized size = 1.16

$$\frac{-\frac{256\left(\sin^{6}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{15} - \frac{512\left(\sin^{4}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{65} - \frac{2048\left(\sin^{2}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{715} - \frac{4096\left(\cos^{9}(bx+a)\right)}{6435}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/15*sin(b*x+a)^6*cos(b*x+a)^9-2/65*sin(b*x+a)^4*cos(b*x+a)^9-8/715
*sin(b*x+a)^2*cos(b*x+a)^9-16/6435*cos(b*x+a)^9)
```

maxima [A] time = 0.33, size = 91, normalized size = 1.49

$$\frac{429 \cos \left(15 \, bx + 15 \, a\right) + 495 \cos \left(13 \, bx + 13 \, a\right) - 4095 \cos \left(11 \, bx + 11 \, a\right) - 5005 \cos \left(9 \, bx + 9 \, a\right) + 19305 \cos (7}{411840 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/411840*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

mupad [B] time = 0.15, size = 46, normalized size = 0.75

$$-\frac{-\frac{256\cos(a+b\,x)^{15}}{15}+\frac{768\cos(a+b\,x)^{13}}{13}-\frac{768\cos(a+b\,x)^{11}}{11}+\frac{256\cos(a+b\,x)^{9}}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x),x)
```

```
[Out] -((256*cos(a + b*x)^9)/9 - (768*cos(a + b*x)^11)/11 + (768*cos(a + b*x)^13)
/13 - (256*cos(a + b*x)^15)/15)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

3.34 $\int \csc(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{13}(a+bx)}{13b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^9(a+bx)}{3b}+\frac{128\sin^7(a+bx)}{7b}$$

```
[Out] 128/7*sin(b*x+a)^7/b-128/3*sin(b*x+a)^9/b+384/11*sin(b*x+a)^11/b-128/13*sin
(b*x+a)^13/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4288, 2564, 270}

$$-\frac{128\sin^{13}(a+bx)}{13b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^9(a+bx)}{3b}+\frac{128\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x]
^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^6(a+bx)\,dx\\
&= \frac{128\,\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{128\,\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{128\sin^7(a+bx)}{7b}-\frac{128\sin^9(a+bx)}{3b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{13}(a+bx)}{13b}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 48, normalized size = 0.79

$$\frac{128\left(-231\sin^{13}(a+bx)+819\sin^{11}(a+bx)-1001\sin^9(a+bx)+429\sin^7(a+bx)\right)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(429*Sin[a + b*x]^7 - 1001*Sin[a + b*x]^9 + 819*Sin[a + b*x]^11 - 231*
Sin[a + b*x]^13))/(3003*b)
```

fricas [A] time = 0.46, size = 73, normalized size = 1.20

$$-\frac{128\left(231\,\cos\left(bx+a\right)^{12}-567\,\cos\left(bx+a\right)^{10}+371\,\cos\left(bx+a\right)^{8}-5\,\cos\left(bx+a\right)^{6}-6\,\cos\left(bx+a\right)^{4}-8\,\cos\right.}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] -128/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/
b
```

giac [A] time = 0.99, size = 46, normalized size = 0.75

$$-\frac{128\left(231\,\sin\left(bx+a\right)^{13}-819\,\sin\left(bx+a\right)^{11}+1001\,\sin\left(bx+a\right)^{9}-429\,\sin\left(bx+a\right)^{7}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/3003*(231*sin(b*x + a)^13 - 819*sin(b*x + a)^11 + 1001*sin(b*x + a)^9
- 429*sin(b*x + a)^7)/b
```

maple [A] time = 1.26, size = 97, normalized size = 1.59

$$\frac{-\frac{128\left(\sin^{5}(bx+a)\right)\left(\cos^{8}(bx+a)\right)}{13}-\frac{640\left(\sin^{3}(bx+a)\right)\left(\cos^{8}(bx+a)\right)}{143}-\frac{640\sin(bx+a)\left(\cos^{8}(bx+a)\right)}{429}+\frac{640\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx\right.}{5}\right.}{3003}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/13*sin(b*x+a)^5*cos(b*x+a)^8-5/143*sin(b*x+a)^3*cos(b*x+a)^8-5/42
9*sin(b*x+a)*cos(b*x+a)^8+5/3003*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*co
s(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.33, size = 80, normalized size = 1.31

$$-\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\sin\left(5\,b\right.}{96096\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] -1/96096*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
+ 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
3*a) - 60060*sin(b*x + a))/b
```

mupad [B] time = 0.13, size = 45, normalized size = 0.74

$$\frac{-\frac{128\sin(a+b\,x)^{13}}{13}+\frac{384\sin(a+b\,x)^{11}}{11}-\frac{128\sin(a+b\,x)^{9}}{3}+\frac{128\sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x),x)
```

```
[Out] ((128*sin(a + b*x)^7)/7 - (128*sin(a + b*x)^9)/3 + (384*sin(a + b*x)^11)/11
 - (128*sin(a + b*x)^13)/13)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

3.35 $\int \csc(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{64\cos^{11}(a+bx)}{11b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+128/9*cos(b*x+a)^9/b-64/11*cos(b*x+a)^11/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$-\frac{64\cos^{11}(a+bx)}{11b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (128*Cos[a + b*x]^9)/(9*b) - (64*Cos[a + b*x]^
11)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{64\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.93

$$-\frac{5\cos(a+bx)}{8b}-\frac{5\cos(3(a+bx))}{24b}+\frac{\cos(5(a+bx))}{16b}+\frac{5\cos(7(a+bx))}{112b}-\frac{\cos(9(a+bx))}{144b}-\frac{\cos(11(a+bx))}{176b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cos[a + b*x])/(8*b) - (5*Cos[3*(a + b*x)])/(24*b) + Cos[5*(a + b*x)]/(1
6*b) + (5*Cos[7*(a + b*x)])/(112*b) - Cos[9*(a + b*x)]/(144*b) - Cos[11*(a
+ b*x)]/(176*b)
```

fricas [A] time = 0.44, size = 36, normalized size = 0.78

$$-\frac{64\left(63\cos\left(bx+a\right)^{11}-154\cos\left(bx+a\right)^{9}+99\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

giac [B] time = 0.81, size = 204, normalized size = 4.43

$$-\frac{1024\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{55\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{297\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{1485\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{2079\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{2541\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}\right.}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1024/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 55*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(
cos(b*x + a) + 1)^5 - 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155
*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 462*(cos(b*x + a) - 1)^8/(cos(
b*x + a) + 1)^8 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^11)
```

maple [A] time = 1.01, size = 53, normalized size = 1.15

$$\frac{-\frac{64\left(\sin^4(bx+a)\right)\left(\cos^7(bx+a)\right)}{11}-\frac{256\left(\sin^2(bx+a)\right)\left(\cos^7(bx+a)\right)}{99}-\frac{512\left(\cos^7(bx+a)\right)}{693}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/11*sin(b*x+a)^4*cos(b*x+a)^7-4/99*sin(b*x+a)^2*cos(b*x+a)^7-8/693*
cos(b*x+a)^7)
```

maxima [A] time = 0.34, size = 69, normalized size = 1.50

$$-\frac{63\cos\left(11\,bx+11\,a\right)+77\cos\left(9\,bx+9\,a\right)-495\cos\left(7\,bx+7\,a\right)-693\cos\left(5\,bx+5\,a\right)+2310\cos\left(3\,bx+3\,a\right)}{11088\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/11088*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

mupad [B] time = 0.14, size = 36, normalized size = 0.78

$$-\frac{64\,\left(63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^{9}+99\cos\left(a+b\,x\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x),x)
```

```
[Out] -(64*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

3.36 $\int \csc(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^5(a+bx)}{5b}$$

```
[Out] 32/5*sin(b*x+a)^5/b-64/7*sin(b*x+a)^7/b+32/9*sin(b*x+a)^9/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9)
/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{32\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\sin^5(a+bx)}{5b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 38, normalized size = 0.83

$$\frac{32\left(35\sin^9(a+bx)-90\sin^7(a+bx)+63\sin^5(a+bx)\right)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
[Out] (32*(63*Sin[a + b*x]^5 - 90*Sin[a + b*x]^7 + 35*Sin[a + b*x]^9))/(315*b)
```

fricas [A] time = 0.45, size = 53, normalized size = 1.15

$$\frac{32 \left(35 \cos \left(bx+a\right)^{8}-50 \cos \left(bx+a\right)^{6}+3 \cos \left(bx+a\right)^{4}+4 \cos \left(bx+a\right)^{2}+8\right) \sin \left(bx+a\right)}{315 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
[Out] 32/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

giac [A] time = 0.65, size = 36, normalized size = 0.78

$$\frac{32 \left(35 \sin \left(bx+a\right)^{9}-90 \sin \left(bx+a\right)^{7}+63 \sin \left(bx+a\right)^{5}\right)}{315 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
[Out] 32/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

maple [A] time = 0.97, size = 69, normalized size = 1.50

$$\frac{-\frac{32\left(\sin^{3}(bx+a)\right)\left(\cos^{6}(bx+a)\right)}{9}-\frac{32\sin(bx+a)\left(\cos^{6}(bx+a)\right)}{21}+\frac{32\left(\frac{8}{3}+\cos^{4}(bx+a)+\frac{4\left(\cos^{2}(bx+a)\right)}{3}\right)\sin(bx+a)}{105}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^5,x)
[Out] 32/b*(-1/9*sin(b*x+a)^3*cos(b*x+a)^6-1/21*sin(b*x+a)*cos(b*x+a)^6+1/105*(8/
3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.34, size = 58, normalized size = 1.26

$$\frac{35 \sin \left(9 \, bx+9 \, a\right)+45 \sin \left(7 \, bx+7 \, a\right)-252 \sin \left(5 \, bx+5 \, a\right)-420 \sin \left(3 \, bx+3 \, a\right)+1890 \sin \left(bx+a\right)}{2520 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
[Out] 1/2520*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

mupad [B] time = 0.07, size = 36, normalized size = 0.78

$$\frac{32\,\left(35\sin\left(a+b\,x\right)^{9}-90\sin\left(a+b\,x\right)^{7}+63\sin\left(a+b\,x\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x),x)
[Out] (32*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.37 $\int \csc(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{16\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+16/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4288, 2565, 14\}$

$$\frac{16\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (16*Cos[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{16\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.90

$$-\frac{3\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{4b}+\frac{\cos(5(a+bx))}{20b}+\frac{\cos(7(a+bx))}{28b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(4*b) + Cos[5*(a + b*x)]/(20*b)
+ Cos[7*(a + b*x)]/(28*b)
```

fricas [A] time = 0.47, size = 26, normalized size = 0.84

$$\frac{16\left(5\,\cos\left(bx+a\right)^{7}-7\,\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

giac [B] time = 0.56, size = 138, normalized size = 4.45

$$-\frac{64\left(\frac{7\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+\frac{14\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}+\frac{70\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}}+\frac{35\left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}}+\frac{35\left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}}-1\right)}{35\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -64/35*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 14*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 70*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 35*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 35*(cos(b*x + a) - 1)^5/(cos(b*x +
 a) + 1)^5 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^7)
```

maple [A] time = 0.66, size = 35, normalized size = 1.13

$$\frac{-\frac{16\left(\sin^{2}(bx+a)\right)\left(\cos^{5}(bx+a)\right)}{7}-\frac{32\left(\cos^{5}(bx+a)\right)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] 16/b*(-1/7*sin(b*x+a)^2*cos(b*x+a)^5-2/35*cos(b*x+a)^5)
```

maxima [A] time = 0.34, size = 47, normalized size = 1.52

$$\frac{5\,\cos\left(7\,bx+7\,a\right)+7\,\cos\left(5\,bx+5\,a\right)-35\,\cos\left(3\,bx+3\,a\right)-105\,\cos\left(bx+a\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/140*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*
cos(b*x + a))/b
```

mupad [B] time = 0.05, size = 26, normalized size = 0.84

$$-\frac{16\,\left(7\cos\left(a+b\,x\right)^{5}-5\cos\left(a+b\,x\right)^{7}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x),x)
```

```
[Out] -(16*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.38 $\int \csc(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

```
[Out] 8/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 14}

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^2(a+bx)\,dx\\
&= \frac{8\,\text{Subst}\left(\int x^2\left(1-x^2\right)dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{8\,\text{Subst}\left(\int \left(x^2-x^4\right)dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 28, normalized size = 0.90

$$\frac{8\left(5\sin^3(a+bx)-3\sin^5(a+bx)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
[Out] (8*(5*Sin[a + b*x]^3 - 3*Sin[a + b*x]^5))/(15*b)
```

fricas [A] time = 0.44, size = 33, normalized size = 1.06

$$-\frac{8\left(3\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -8/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

giac [A] time = 0.36, size = 26, normalized size = 0.84

$$-\frac{8\left(3\sin\left(bx+a\right)^{5}-5\sin\left(bx+a\right)^{3}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -8/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

maple [A] time = 1.05, size = 41, normalized size = 1.32

$$\frac{-\frac{8\sin(bx+a)\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(2+\cos^{2}(bx+a)\right)\sin(bx+a)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^3,x)
[Out] 8/b*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.34, size = 36, normalized size = 1.16

$$-\frac{3\sin\left(5\,bx+5\,a\right)+5\sin\left(3\,bx+3\,a\right)-30\sin\left(bx+a\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
[Out] -1/30*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

mupad [B] time = 0.11, size = 26, normalized size = 0.84

$$\frac{8\left(5\sin\left(a+b\,x\right)^{3}-3\sin\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x),x)
[Out] (8*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

3.39 $\int \csc(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] -4/3*cos(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4288, 2565, 30}

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin(a+bx)\,dx\\ &= -\frac{4\,\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{4\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

fricas [A] time = 0.44, size = 13, normalized size = 0.87

$$-\frac{4 \cos (bx+a)^3}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/3*cos(b*x + a)^3/b
```

giac [B] time = 0.37, size = 52, normalized size = 3.47

$$\frac{8 \left(\frac{3 (\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{3 b \left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 8/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)^3)
```

maple [A] time = 0.50, size = 14, normalized size = 0.93

$$-\frac{4 \left(\cos^3 (bx+a)\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^2,x)
```

```
[Out] -4/3*cos(b*x+a)^3/b
```

maxima [A] time = 0.34, size = 23, normalized size = 1.53

$$-\frac{\cos (3 bx+3 a)+3 \cos (bx+a)}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(cos(3*b*x + 3*a) + 3*cos(b*x + a))/b
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{4 \cos (a+b x)^3}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x),x)
```

```
[Out] -(4*cos(a + b*x)^3)/(3*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

3.40 $\int \csc(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$\frac{2\sin(a+bx)}{b}$$

```
[Out] 2*sin(b*x+a)/b
```

Rubi **[A]** time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4288, 2637}

$$\frac{2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x])/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\,dx\\ &= \frac{2\sin(a+bx)}{b}\end{aligned}$$

Mathematica **[B]** time = 0.01, size = 23, normalized size = 2.09

$$2\left(\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] 2*((Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b)
```

fricas **[A]** time = 0.47, size = 11, normalized size = 1.00

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*sin(b*x + a)/b
```

giac [A] time = 0.36, size = 11, normalized size = 1.00

$$\frac{2 \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 2*sin(b*x + a)/b
```

maple [A] time = 0.34, size = 12, normalized size = 1.09

$$\frac{2 \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 2*sin(b*x+a)/b
```

maxima [A] time = 0.34, size = 11, normalized size = 1.00

$$\frac{2 \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*sin(b*x + a)/b
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$\frac{2 \sin(a + b x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x),x)
```

```
[Out] (2*sin(a + b*x))/b
```

sympy [B] time = 21.21, size = 3636, normalized size = 330.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 4*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (0, Eq(b, 0)), (sin(b*x)/b, Eq(a, 0))
, (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3/(b*tan
(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*
b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2
)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)
**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2)
 + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*
tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2
```

```
)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*
tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3/(b*tan(a/2)
**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan
(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)*t
an(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2
*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2)
- 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*tan(a
/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/
2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)**
3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)
**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t
an(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)
**2 + b), True))*cos(a)**2 - 2*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (0, Eq(b
, 0)), (sin(b*x)/b, Eq(a, 0)), (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*ta
n(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + t
an(b*x/2))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(t
an(a/2) + tan(b*x/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 +
 b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*
x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x
/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 +
b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2
)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x
/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(
a/2))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/
2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*
x/2) - 1/tan(a/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*
tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2
)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/
2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b
*tan(b*x/2)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2
 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(
b*x/2)**2 + b) - 4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4
 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) -
 4*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*ta
n(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2
*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b), True)) - 2*Piecewise((zoo*x, Eq(a, 0
) & Eq(b, 0)), (x/sin(a), Eq(b, 0)), (log(tan(b*x/2))/b, Eq(a, 0)), (log(ta
n(a/2) + tan(b*x/2))/b - log(tan(b*x/2) - 1/tan(a/2))/b, True))*sin(a)*cos(
a) + 4*Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/sin(a), Eq(b, 0)), (log(t
an(b*x/2))*tan(b*x/2)**2/(b*tan(b*x/2)**2 + b) + log(tan(b*x/2))/(b*tan(b*x
/2)**2 + b) + 2/(b*tan(b*x/2)**2 + b), Eq(a, 0)), (log(tan(a/2) + tan(b*x/2
))*tan(a/2)**4*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + l
og(tan(a/2) + tan(b*x/2))*tan(a/2)**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)*
*4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(
a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**2/(
b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2
 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + log(tan(a/2) + tan(b*x/2))*tan(
b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*ta
n(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + log(tan(a/2) + tan(b
*x/2))/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b
```

```
*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - log(tan(b*x/2) - 1/tan(
a/2))*tan(a/2)**4*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**
4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b)
- log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b
*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/
2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**2*tan(b*x/2)**2/(b*ta
n(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2
*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(
a/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(
b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - log(tan(b*x/2) - 1/tan
(a/2))*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan
(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - log(tan(b
*x/2) - 1/tan(a/2))/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(
a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*tan(a/2)
**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/
2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 4*tan(a/2)**3*tan(b*x/2)/(
b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2
 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 4*tan(a/2)*tan(b*x/2)/(b*tan(a/
2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*t
an(a/2)**2 + b*tan(b*x/2)**2 + b) + 2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b), True))*sin(a)*cos(a)
```

3.41 $\int \csc(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4288, 2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc^2(a+bx)\sec(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b} \\
&= -\frac{\csc(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b} \\
&= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{2b}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 29, normalized size = 1.04

$$
-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/2*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.43, size = 50, normalized size = 1.79

$$
\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{4\,b\sin\left(bx+a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

giac [B] time = 1.70, size = 484, normalized size = 17.29

$$
-\frac{\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{12}-12\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{10}+6\,\tan\left(\frac{1}{2}\,a\right)^{11}-27\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{8}-2\,\tan\left(\frac{1}{2}\,a\right)^{9}-36\,\tan\left(\frac{1}{2}\,a\right)^{7}+27\,\mathrm{ta}}{\left(3\,\tan\left(\frac{1}{2}\,bx+2\,a\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{5}-\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{6}-10\,\tan\left(\frac{1}{2}\,bx+2\,a\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{3}+15\,\tan\left(\frac{1}{2}\,bx+2\,a\right)\tan\left(\frac{1}{2}\,a\right)^{4}-3\,\tan\left(\frac{1}{2}\,a\right)^{5}+3\,\tan\left(\frac{1}{2}\,bx+2\,a\right)^{2}\,\mathrm{t}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*((tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^
10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9
- 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 +
12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) +
6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*ta
n(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*t
an(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1
/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) - 2*log(abs(tan(1
/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)
^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
+ 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/
```

```
2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
+ 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.67, size = 34, normalized size = 1.21

$$-\frac{1}{2b\sin(bx+a)}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a),x)
```

```
[Out] -1/2/b/sin(b*x+a)+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.48, size = 233, normalized size = 8.32

$$-\frac{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\,\cos(2\,bx+2\,a)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a}\right.}{4\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

mupad [B] time = 0.11, size = 26, normalized size = 0.93

$$\frac{\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{2\,b}-\frac{1}{2\,b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)),x)
```

```
[Out] atanh(sin(a + b*x))/(2*b) - 1/(2*b*sin(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\csc(a+bx)\csc(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x), x)
```

3.42 $\int \csc(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{8b}-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[Out] -3/8*arctanh(cos(b*x+a))/b+3/8*sec(b*x+a)/b-1/8*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4288, 2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{8b}-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(8*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^3(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{\csc^2(a+bx)\sec(a+bx)}{8b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b}$$
$$= \frac{3\sec(a+bx)}{8b} - \frac{\csc^2(a+bx)\sec(a+bx)}{8b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+\right.}{8b}$$
$$= -\frac{3\tanh^{-1}(\cos(a+bx))}{8b} + \frac{3\sec(a+bx)}{8b} - \frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

Mathematica [B] time = 0.26, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx)) + 2\cos(3(a+bx)) + 3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right) - 3\cos(3(a+bx)\right.}{8b\left(\csc^2\left(\frac{1}{2}(a+bx)\right) - \sec^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(8*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

fricas [B] time = 0.44, size = 96, normalized size = 1.96

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\text{lo}}{16\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

giac [B] time = 1.61, size = 1327, normalized size = 27.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(16*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b
*x + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a
)^7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 1
2*tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 1
5*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 +
```

```
15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) -
 15*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1
)) + (6*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - tan(1/2*b*x + 2*a)^2*tan(1/2*a
)^24 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 60*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^22 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 + 798*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^19 - 924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 290*tan(1/2*b*x + 2
*a)*tan(1/2*a)^21 - 18*tan(1/2*a)^22 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^17 + 3892*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2310*tan(1/2*b*x + 2*a)*tan
(1/2*a)^19 + 336*tan(1/2*a)^20 - 3188*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 +
1467*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a
)^17 - 1190*tan(1/2*a)^18 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 12744
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^15
 - 288*tan(1/2*a)^16 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 10332*tan(
1/2*b*x + 2*a)*tan(1/2*a)^13 + 4428*tan(1/2*a)^14 - 3188*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^9 + 12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 10332*tan(1/2*
b*x + 2*a)*tan(1/2*a)^11 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 1467*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 44
28*tan(1/2*a)^10 + 798*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 3892*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^6 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 288*tan(1/2
*a)^8 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 924*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^4 - 2310*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 1190*tan(1/2*a)^6 + 6*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a) - 60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 + 290*
tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 336*tan(1/2*a)^4 + tan(1/2*b*x + 2*a)^2 -
 6*tan(1/2*b*x + 2*a)*tan(1/2*a) + 18*tan(1/2*a)^2)/((9*tan(1/2*a)^10 - 60*
tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(3*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/
2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*
a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^2) + 12*log(abs
(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(
1/2*a)^2 - 1)) - 12*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^
3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

maple [A] time = 0.92, size = 57, normalized size = 1.16

$$-\frac{1}{8b\sin(bx+a)^2\cos(bx+a)}+\frac{3}{8b\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^2,x)
```

```
[Out] -1/8/b/sin(b*x+a)^2/cos(b*x+a)+3/8/b/cos(b*x+a)+3/8/b*ln(csc(b*x+a)-cot(b*x
+a))
```

maxima [B] time = 0.38, size = 974, normalized size = 19.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
```

```
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.13, size = 49, normalized size = 1.00

$$-\frac{3\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{8\,b}-\frac{\frac{3\cos\left(a+b\,x\right)^2}{8}-\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^2),x)
```

```
[Out] - (3*atanh(cos(a + b*x)))/(8*b) - ((3*cos(a + b*x)^2)/8 - 1/4)/(b*(cos(a +
b*x) - cos(a + b*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(a+bx\right)\csc^2\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**2, x)
```

3.43 $\int \csc(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{48b}-\frac{5\csc(a+bx)}{16b}+\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 5/16*arctanh(sin(b*x+a))/b-5/16*csc(b*x+a)/b-5/48*csc(b*x+a)^3/b+1/16*csc(b
*x+a)^3*sec(b*x+a)^2/b
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{48b}-\frac{5\csc(a+bx)}{16b}+\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) - (5*Csc[a + b*x])/(16*b) - (5*Csc[a + b*x
]^3)/(48*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(16*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{8b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right.}{16b}$$
$$= -\frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{5\tanh^{-1}(\sin(a+bx))}{16b} - \frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\,\text{sec}}{16b}$$

Mathematica [C] time = 0.02, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
[Out] -1/24*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.47, size = 130, normalized size = 1.97

$$-\frac{30\cos(bx+a)^4 - 15\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a) + 15\left(\cos(bx+a)^4 -\right.}{96\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)\sin(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -1/96*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

giac [B] time = 3.94, size = 3033, normalized size = 45.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -1/192*(24*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan
(1/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23
+ 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17
```

```
 - 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 2
331*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*t
an(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a
)*tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^6 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1
/2*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 -
2178*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2
*a)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan
(1/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/
2*a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan
(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/
2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan
(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) + (27*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 + tan(1/2*b*x + 2
*a)^3*tan(1/2*a)^36 + 1602*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^32 - 915*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^33 + 126*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 + 9*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 50082*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^30
 + 58626*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 21141*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^32 + 2373*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^33 + 27*tan(1/2*b*x + 2*
a)*tan(1/2*a)^34 + 400050*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 - 783810*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^29 + 487692*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 -
 118404*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 9378*tan(1/2*b*x + 2*a)*tan(1/
2*a)^32 + 54*tan(1/2*a)^33 - 1301382*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 + 4
035870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 - 3943944*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^28 + 1555092*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 247074*tan(1/2*b
*x + 2*a)*tan(1/2*a)^30 + 11610*tan(1/2*a)^31 + 1238250*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^24 - 8115822*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^25 + 13194468*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 8075232*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2
7 + 1983762*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 - 160362*tan(1/2*a)^29 + 26423
10*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 1212390*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^23 - 12476880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 16317288*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^25 - 6559110*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 + 80578
6*tan(1/2*a)^27 - 5947398*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20 + 25548198*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 27336420*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
22 + 2575260*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 6211530*tan(1/2*b*x + 2*a
)*tan(1/2*a)^24 - 1599858*tan(1/2*a)^25 - 19824660*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^19 + 58330530*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 51141420*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^21 + 13453830*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 - 2
09790*tan(1/2*a)^23 + 5947398*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 - 19824660
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 39278250*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^19 - 29430054*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 5059854*tan(1/2*a)^21
 - 2642310*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 + 25548198*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^15 - 58330530*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 39278250*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 4136670*tan(1/2*a)^19 - 1238250*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^12 - 1212390*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^13 + 2
7336420*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 - 51141420*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^15 + 29430054*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 4136670*tan(1/2
*a)^17 + 1301382*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 8115822*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^11 + 12476880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 + 25752
60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 - 13453830*tan(1/2*b*x + 2*a)*tan(1/2
*a)^14 + 5059854*tan(1/2*a)^15 - 400050*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 +
```

```
4035870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 13194468*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^10 + 16317288*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 6211530*tan(1
/2*b*x + 2*a)*tan(1/2*a)^12 - 209790*tan(1/2*a)^13 + 50082*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^6 - 783810*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 + 3943944*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 8075232*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 +
 6559110*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 1599858*tan(1/2*a)^11 - 1602*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 58626*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 -
 487692*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 1555092*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^7 - 1983762*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 805786*tan(1/2*a)^9
- 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 915*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^3 + 21141*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 118404*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^5 + 247074*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 160362*tan(1/2*a)^
7 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 126*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^2 + 2373*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 9378*tan(1/2*b*x + 2*a)*tan(1
/2*a)^4 + 11610*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^
2*tan(1/2*a) - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*t
an(1/2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 +
981*tan(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan
(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^3) - 60*log(abs(tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*
tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan
(1/2*a) - 1)) + 60*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*ta
n(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.76, size = 76, normalized size = 1.15

$$-\frac{1}{24b\sin(bx+a)^3\cos(bx+a)^2}+\frac{5}{48b\sin(bx+a)\cos(bx+a)^2}-\frac{5}{16b\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^3,x)
```

```
[Out] -1/24/b/sin(b*x+a)^3/cos(b*x+a)^2+5/48/b/sin(b*x+a)/cos(b*x+a)^2-5/16/b/sin
(b*x+a)+5/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.53, size = 1780, normalized size = 26.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x +
 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*b
*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3
*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*sin
(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
- 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b*
x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2
```

```
*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(
sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*s
in(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 +
 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x +
 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 -
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2
*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20*
cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x +
 a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos
(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*b
*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
+ 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) -
 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x +
 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*si
n(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*sin
(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 +
 b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4
*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
 + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a)
 - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b
*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*b
*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin
(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a)
)*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.19, size = 61, normalized size = 0.92

$$\frac{5\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b}-\frac{-\frac{5\,{\sin\left(a+b\,x\right)}^4}{16}+\frac{5\,{\sin\left(a+b\,x\right)}^2}{24}+\frac{1}{24}}{b\,\left({\sin\left(a+b\,x\right)}^3-{\sin\left(a+b\,x\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(16*b) - ((5*sin(a + b*x)^2)/24 - (5*sin(a + b*x)^4
)/16 + 1/24)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(a+bx\right)\csc^3\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**3, x)
```

3.44 $\int \csc(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{384b}+\frac{35\sec(a+bx)}{128b}-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}$$

```
[Out] -35/128*arctanh(cos(b*x+a))/b+35/128*sec(b*x+a)/b+35/384*sec(b*x+a)^3/b-7/1
28*csc(b*x+a)^2*sec(b*x+a)^3/b-1/64*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{384b}+\frac{35\sec(a+bx)}{128b}-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(128*b) + (35*Sec[a + b*x])/(128*b) + (35*Sec[a
 + b*x]^3)/(384*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(128*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(64*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{64b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\right.}{1\cdots}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\,\text{Subst}\left(\int \left(1+\cdots\right.\right.}{\cdots}$$
$$= \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx}{6\cdots}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{128b} + \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a+bx}{12\cdots}$$

Mathematica [B] time = 0.51, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6(a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -1/384*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
 + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*C
os[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
 + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
 + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2
]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log
[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(
a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.49, size = 148, normalized size = 1.66

$$\frac{210\cos(bx+a)^6 - 350\cos(bx+a)^4 + 112\cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3\right)\cdots}{768\left(b\cos(bx+a)^7 - 2\,b\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*co
s(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
+ a)^3)
```

giac [B] time = 10.22, size = 5476, normalized size = 61.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/3072*(256*(36*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 6*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^36 - 1848*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 1134*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^34 - 180*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 9*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^36 + 39276*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 -
42894*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^32 + 14532*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^33 - 2052*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 144*tan(1/2*b*x + 2*a)
*tan(1/2*a)^35 - 5*tan(1/2*a)^36 - 433836*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2
9 + 709068*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 376632*tan(1/2*b*x + 2*a)^3
*tan(1/2*a)^31 + 83367*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 8364*tan(1/2*b*
x + 2*a)*tan(1/2*a)^33 + 342*tan(1/2*a)^34 + 2430348*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^27 - 5742396*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 4466808*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^29 - 1422120*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 +
193068*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 - 9639*tan(1/2*a)^32 - 5123196*tan(
1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 20329092*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2
6 - 24275368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 11529468*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^28 - 2196396*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 141660*tan(
1/2*a)^30 - 1201860*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 18742620*tan(1/2*b
*x + 2*a)^4*tan(1/2*a)^24 + 50327784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 4
0521528*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 12145596*tan(1/2*b*x + 2*a)*ta
n(1/2*a)^27 - 1116072*tan(1/2*a)^28 + 15332100*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^21 - 42937380*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^22 + 10264680*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^23 + 37504740*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 254
25036*tan(1/2*b*x + 2*a)*tan(1/2*a)^25 + 4162356*tan(1/2*a)^26 - 11041020*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 84945240*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^20 - 153596328*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 85465080*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^22 - 5642820*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 3694080
*tan(1/2*a)^24 - 11041020*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 112912560*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^19 - 170450298*tan(1/2*b*x + 2*a)^2*tan(1/2*a
)^20 + 76700868*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 8751060*tan(1/2*a)^22 +
15332100*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 84945240*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^16 + 112912560*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^17 - 55767060*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^19 + 16730118*tan(1/2*a)^20 - 1201860*tan(1/2*b
*x + 2*a)^5*tan(1/2*a)^13 + 42937380*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 1
53596328*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 170450298*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^16 - 55767060*tan(1/2*b*x + 2*a)*tan(1/2*a)^17 - 5123196*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^11 + 18742620*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12
 + 10264680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 85465080*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^14 + 76700868*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 16730118*ta
n(1/2*a)^16 + 2430348*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 - 20329092*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^10 + 50327784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 -
37504740*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 5642820*tan(1/2*b*x + 2*a)*ta
n(1/2*a)^13 + 8751060*tan(1/2*a)^14 - 433836*tan(1/2*b*x + 2*a)^5*tan(1/2*a
)^7 + 5742396*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 24275368*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^9 + 40521528*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 25425036*
tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + 3694080*tan(1/2*a)^12 + 39276*tan(1/2*b*
x + 2*a)^5*tan(1/2*a)^5 - 709068*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 446680
8*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 11529468*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^8 + 12145596*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 4162356*tan(1/2*a)^10 -
1848*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 42894*tan(1/2*b*x + 2*a)^4*tan(1/2
*a)^4 - 376632*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 + 1422120*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^6 - 2196396*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 1116072*tan(1
/2*a)^8 + 36*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 1134*tan(1/2*b*x + 2*a)^4*ta
n(1/2*a)^2 + 14532*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 83367*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 + 193068*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 141660*tan(1
/2*a)^6 + 6*tan(1/2*b*x + 2*a)^4 - 180*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 20
52*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 8364*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
```

+ 9639*tan(1/2*a)^4 - 9*tan(1/2*b*x + 2*a)^2 + 144*tan(1/2*b*x + 2*a)*tan(
1/2*a) - 342*tan(1/2*a)^2 + 5)/((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*tan
(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8
+ 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b
*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*
a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^3) + 3*(108*
tan(1/2*b*x + 2*a)^7*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^46
+ 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4
8 + 8316*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^43 - 6120*tan(1/2*b*x + 2*a)^6*tan
(1/2*a)^44 + 1400*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^45 - 24*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^46 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^47 - 253008*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^41 + 354774*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^42 - 16
5384*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^43 + 30180*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^44 - 1400*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^46 + 3918128*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^39 - 7606080*tan(1
/2*b*x + 2*a)^6*tan(1/2*a)^40 + 5365548*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^41
- 1698920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^42 + 235368*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^43 - 10008*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^44 - 108*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^45 - 32664372*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^37 + 853525
14*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^38 - 82649988*tan(1/2*b*x + 2*a)^5*tan(1
/2*a)^39 + 37937604*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^40 - 8596476*tan(1/2*b*
x + 2*a)^3*tan(1/2*a)^41 + 879006*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^42 - 2775
6*tan(1/2*b*x + 2*a)*tan(1/2*a)^43 - 162*tan(1/2*a)^44 + 150470268*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^35 - 527449208*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^36 +
684424728*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^37 - 427365432*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^38 + 137583540*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^39 - 22235328
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^40 + 1556784*tan(1/2*b*x + 2*a)*tan(1/2*a)
^41 - 28944*tan(1/2*a)^42 - 369285568*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^33 +
1800957150*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^34 - 3157829352*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^35 + 2638274028*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36 - 114501
5352*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^37 + 257818938*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^38 - 27438800*tan(1/2*b*x + 2*a)*tan(1/2*a)^39 + 1015632*tan(1/2*a
)^40 + 332678976*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^31 - 2973661056*tan(1/2*b*
x + 2*a)^6*tan(1/2*a)^32 + 7767810012*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 -
9003276360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^34 + 5266355688*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^35 - 1589641352*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^36 + 231030
852*tan(1/2*b*x + 2*a)*tan(1/2*a)^37 - 12400752*tan(1/2*a)^38 + 498198168*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^29 + 141213844*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^30 - 6978686184*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 + 14860730619*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^32 - 12919461276*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3
3 + 5392538622*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 - 1054828476*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^35 + 76246378*tan(1/2*a)^36 - 1473564360*tan(1/2*b*x + 2*
a)^7*tan(1/2*a)^27 + 7565433072*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^28 - 104925
95088*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 - 707654192*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^30 + 11624633640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31 - 8882796672*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 + 2571218880*tan(1/2*b*x + 2*a)*tan(1/2*
a)^33 - 256007232*tan(1/2*a)^34 + 883677600*tan(1/2*b*x + 2*a)^7*tan(1/2*a)
^25 - 10018470420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^26 + 30880710896*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^27 - 37780516920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28
+ 17392225680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^29 + 478350836*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^30 - 2321948736*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 + 42019
1712*tan(1/2*a)^32 + 883677600*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^23 - 1862376
5160*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 50161528368*tan(1/2*b*x + 2*a)^4*
tan(1/2*a)^26 - 51515325744*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 2260797672
0*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^28 - 3433884120*tan(1/2*b*x + 2*a)*tan(1/
2*a)^29 - 26975424*tan(1/2*a)^30 - 1473564360*tan(1/2*b*x + 2*a)^7*tan(1/2*
a)^21 + 10018470420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^22 - 18623765160*tan(1/

2*b*x + 2*a)^5*tan(1/2*a)^23 + 31169987784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
25 - 30241452708*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 10324445736*tan(1/2*b
*x + 2*a)*tan(1/2*a)^27 - 1070628372*tan(1/2*a)^28 + 498198168*tan(1/2*b*x
+ 2*a)^7*tan(1/2*a)^19 - 7565433072*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^20 + 30
880710896*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 50161528368*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^22 + 31169987784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - 62969
40000*tan(1/2*b*x + 2*a)*tan(1/2*a)^25 + 1453140000*tan(1/2*a)^26 + 3326789
76*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^17 - 141213844*tan(1/2*b*x + 2*a)^6*tan(
1/2*a)^18 - 10492595088*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 37780516920*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 51515325744*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^21 + 30241452708*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 6296940000*tan(1/
2*b*x + 2*a)*tan(1/2*a)^23 - 369285568*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^15 +
 2973661056*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^16 - 6978686184*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^17 + 707654192*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^18 + 173922
25680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^19 - 22607976720*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^20 + 10324445736*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 1453140000*
tan(1/2*a)^22 + 150470268*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^13 - 1800957150*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^14 + 7767810012*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^15 - 14860730619*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 11624633640*tan(1
/2*b*x + 2*a)^3*tan(1/2*a)^17 - 478350836*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^1
8 - 3433884120*tan(1/2*b*x + 2*a)*tan(1/2*a)^19 + 1070628372*tan(1/2*a)^20
- 32664372*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^11 + 527449208*tan(1/2*b*x + 2*a
)^6*tan(1/2*a)^12 - 3157829352*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 9003276
360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 12919461276*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^15 + 8882796672*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 2321948736*t
an(1/2*b*x + 2*a)*tan(1/2*a)^17 + 26975424*tan(1/2*a)^18 + 3918128*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^9 - 85352514*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^10 + 6
84424728*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^11 - 2638274028*tan(1/2*b*x + 2*a)
^4*tan(1/2*a)^12 + 5266355688*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 53925386
22*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 2571218880*tan(1/2*b*x + 2*a)*tan(1
/2*a)^15 - 420191712*tan(1/2*a)^16 - 253008*tan(1/2*b*x + 2*a)^7*tan(1/2*a)
^7 + 7606080*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^8 - 82649988*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^9 + 427365432*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 - 114501535
2*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 + 1589641352*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^12 - 1054828476*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 256007232*tan(1/2
*a)^14 + 8316*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^5 - 354774*tan(1/2*b*x + 2*a)
^6*tan(1/2*a)^6 + 5365548*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 - 37937604*tan(
1/2*b*x + 2*a)^4*tan(1/2*a)^8 + 137583540*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9
 - 257818938*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 + 231030852*tan(1/2*b*x + 2
*a)*tan(1/2*a)^11 - 76246378*tan(1/2*a)^12 + 108*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^3 + 6120*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^4 - 165384*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^5 + 1698920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 - 8596476*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^7 + 22235328*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8
- 27438800*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 + 12400752*tan(1/2*a)^10 + 54*ta
n(1/2*b*x + 2*a)^6*tan(1/2*a)^2 + 1400*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 -
30180*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 + 235368*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^5 - 879006*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 + 1556784*tan(1/2*b*x +
2*a)*tan(1/2*a)^7 - 1015632*tan(1/2*a)^8 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*
a) + 24*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 - 1400*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^3 + 10008*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 - 27756*tan(1/2*b*x + 2*a
)*tan(1/2*a)^5 + 28944*tan(1/2*a)^6 + tan(1/2*b*x + 2*a)^4 - 12*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a) + 54*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 108*tan(1/2*b
*x + 2*a)*tan(1/2*a)^3 + 162*tan(1/2*a)^4)/((81*tan(1/2*a)^20 - 1080*tan(1/
2*a)^18 + 5724*tan(1/2*a)^16 - 15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 -
15240*tan(1/2*a)^10 + 5724*tan(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)
^4)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6
- 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4
 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*
a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^4) +

```
840*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2
*a) + 3*tan(1/2*a)^2 - 1)) - 840*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

maple [A] time = 0.95, size = 99, normalized size = 1.11

$$-\frac{1}{64b\sin(bx+a)^4\cos(bx+a)^3}+\frac{7}{192b\sin(bx+a)^2\cos(bx+a)^3}-\frac{35}{384b\sin(bx+a)^2\cos(bx+a)}+\frac{35}{128b\cos(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^4,x)
```

```
[Out] -1/64/b/sin(b*x+a)^4/cos(b*x+a)^3+7/192/b/sin(b*x+a)^2/cos(b*x+a)^3-35/384/
b/sin(b*x+a)^2/cos(b*x+a)+35/128/b/cos(b*x+a)+35/128/b*ln(csc(b*x+a)-cot(b*
x+a))
```

maxima [B] time = 0.50, size = 3846, normalized size = 43.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/768*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*b
*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
 cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329*
cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b
*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*a
) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*
x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*x
 + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*co
s(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x + 7
*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8*
b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) - 1
5*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2
*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4
*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x +
 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3*
cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*
a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a)
 - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8
*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(1
0*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b*
x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 +
6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b*
x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)
^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*
sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*
a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) - 3
*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2
*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3*
sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10*a
) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - s
in(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x +
```

```
 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(4
*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*co
s(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x +
4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*
(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b
*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2
 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*
b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*
x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*
cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*
x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
 - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*si
n(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*s
in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*
sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3
*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)
^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b
*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(1
3*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b*
x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*s
in(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin(
8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204*
sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x
 + a))*sin(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a)
- 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(11*b*x +
11*a) + 12*(329*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) + 329*sin(5*b*x + 5
*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*sin(10*b*x + 10*a) - 1316*(3*
sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) - 70
*sin(3*b*x + 3*a) + 105*sin(b*x + a))*sin(8*b*x + 8*a) + 816*(3*sin(6*b*x +
 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) - 84*(47*si
n(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x + 6*a) +
1316*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 420*(2*sin(
3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 280*sin(3*b*x + 3*a)*sin(
2*b*x + 2*a) - 420*sin(2*b*x + 2*a)*sin(b*x + a) + 420*cos(b*x + a))/(b*cos
(14*b*x + 14*a)^2 + b*cos(12*b*x + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*b
*cos(8*b*x + 8*a)^2 + 9*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*c
os(2*b*x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*s
in(10*b*x + 10*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*b
*sin(4*b*x + 4*a)^2 + 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x +
 2*a)^2 - 2*(b*cos(12*b*x + 12*a) + 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x
+ 8*a) - 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) -
 b)*cos(14*b*x + 14*a) + 2*(3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a) -
 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(
12*b*x + 12*a) - 6*(3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) - 3*b*cos(4
*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 6*(3*b*cos(6*b*x
 + 6*a) - 3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) -
 6*(3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 6*(b*
cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(12
*b*x + 12*a) + 3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*
x + 6*a) + 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) +
```

```
2*(3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) + 3
*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 6*(3*b*sin(8
*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) + 6*(3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) -
b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 6*(3*b*sin(4*b*x + 4*a) + b*sin(2*b*
x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.12, size = 78, normalized size = 0.88

$$\frac{\frac{35\cos(a+b\,x)^6}{128}-\frac{175\cos(a+b\,x)^4}{384}+\frac{7\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{35\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^4),x)
```

```
[Out] ((7*cos(a + b*x)^2)/48 - (175*cos(a + b*x)^4)/384 + (35*cos(a + b*x)^6)/128
 + 1/48)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*ata
nh(cos(a + b*x)))/(128*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(a+bx\right)\csc^4\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**4, x)
```

3.45 $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=155

$$-\frac{128\sin^5(a+bx)\cos^9(a+bx)}{7b}-\frac{160\sin^3(a+bx)\cos^9(a+bx)}{21b}-\frac{16\sin(a+bx)\cos^9(a+bx)}{7b}+\frac{2\sin(a+bx)\cos}{7b}$$

```
[Out] 5/8*x+5/8*cos(b*x+a)*sin(b*x+a)/b+5/12*cos(b*x+a)^3*sin(b*x+a)/b+1/3*cos(b*
x+a)^5*sin(b*x+a)/b+2/7*cos(b*x+a)^7*sin(b*x+a)/b-16/7*cos(b*x+a)^9*sin(b*x
+a)/b-160/21*cos(b*x+a)^9*sin(b*x+a)^3/b-128/7*cos(b*x+a)^9*sin(b*x+a)^5/b
```

Rubi [A] time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2568, 2635, 8}

$$-\frac{128\sin^5(a+bx)\cos^9(a+bx)}{7b}-\frac{160\sin^3(a+bx)\cos^9(a+bx)}{21b}-\frac{16\sin(a+bx)\cos^9(a+bx)}{7b}+\frac{2\sin(a+bx)\cos}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (5*x)/8 + (5*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (5*Cos[a + b*x]^3*Sin[a + b
*x])/(12*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(3*b) + (2*Cos[a + b*x]^7*Sin[a
 + b*x])/(7*b) - (16*Cos[a + b*x]^9*Sin[a + b*x])/(7*b) - (160*Cos[a + b*x]
^9*Sin[a + b*x]^3)/(21*b) - (128*Cos[a + b*x]^9*Sin[a + b*x]^5)/(7*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx &= 256\int \cos^8(a+bx)\sin^6(a+bx)\,dx \\
&= -\frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{640}{7}\int \cos^8(a+bx)\sin^4(a+bx)\,dx \\
&= -\frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{160}{7}\int \mathrm{co} \\
&= -\frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos^9(a+ \\
&= \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+bx)}{21b} \\
&= \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx)\sin(}{7b} \\
&= \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)\sin(a}{7b} \\
&= \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a}{3b} \\
&= \frac{5x}{8} + \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 85, normalized size = 0.55

$$
\frac{105\sin(2(a+bx)) - 315\sin(4(a+bx)) - 63\sin(6(a+bx)) + 63\sin(8(a+bx)) + 21\sin(10(a+bx)) - 7\sin(12(a}{1344b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (840*a + 840*b*x + 105*Sin[2*(a + b*x)] - 315*Sin[4*(a + b*x)] - 63*Sin[6*(
a + b*x)] + 63*Sin[8*(a + b*x)] + 21*Sin[10*(a + b*x)] - 7*Sin[12*(a + b*x)
] - 3*Sin[14*(a + b*x)])/(1344*b)
```

fricas [A] time = 0.46, size = 87, normalized size = 0.56

$$
\frac{105\,bx - \left(3072\,\cos(bx+a)^{13} - 7424\,\cos(bx+a)^{11} + 4736\,\cos(bx+a)^{9} - 48\,\cos(bx+a)^{7} - 56\,\cos(bx+a)^{5}}{168\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 1/168*(105*b*x - (3072*cos(b*x + a)^13 - 7424*cos(b*x + a)^11 + 4736*cos(b*
x + a)^9 - 48*cos(b*x + a)^7 - 56*cos(b*x + a)^5 - 70*cos(b*x + a)^3 - 105*
cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 1.61, size = 95, normalized size = 0.61

$$
\frac{105\,bx + 105\,a + \frac{105\,\tan(bx+a)^{13} + 700\,\tan(bx+a)^{11} + 1981\,\tan(bx+a)^{9} + 3072\,\tan(bx+a)^{7} - 1981\,\tan(bx+a)^{5} - 700\,\tan(bx+a)^{3} - 105\,\tan(bx+a)}{\left(\tan(bx+a)^{2}+1\right)^{7}}}{168\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*b*x + 105*a + (105*tan(b*x + a)^13 + 700*tan(b*x + a)^11 + 1981*
tan(b*x + a)^9 + 3072*tan(b*x + a)^7 - 1981*tan(b*x + a)^5 - 700*tan(b*x +
a)^3 - 105*tan(b*x + a))/(tan(b*x + a)^2 + 1)^7)/b
```

maple [A] time = 1.41, size = 111, normalized size = 0.72

$$\frac{-\frac{128\left(\sin^{5}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{7}-\frac{160\left(\sin^{3}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{21}-\frac{16\sin(bx+a)\left(\cos^{9}(bx+a)\right)}{7}+\frac{2\left(\cos^{7}(bx+a)+\frac{7\left(\cos^{5}(bx+a)\right)}{6}+\frac{35\left(\cos^{3}(bx+a)\right)}{24}+\cdots\right.}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/14*sin(b*x+a)^5*cos(b*x+a)^9-5/168*sin(b*x+a)^3*cos(b*x+a)^9-1/11
2*sin(b*x+a)*cos(b*x+a)^9+1/896*(cos(b*x+a)^7+7/6*cos(b*x+a)^5+35/24*cos(b*
x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+5/2048*b*x+5/2048*a)
```

maxima [A] time = 0.36, size = 87, normalized size = 0.56

$$\frac{840\,bx-3\,\sin\left(14\,bx+14\,a\right)-7\,\sin\left(12\,bx+12\,a\right)+21\,\sin\left(10\,bx+10\,a\right)+63\,\sin\left(8\,bx+8\,a\right)-63\,\sin\left(6\,bx\cdots\right.}{1344\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/1344*(840*b*x - 3*sin(14*b*x + 14*a) - 7*sin(12*b*x + 12*a) + 21*sin(10*b
*x + 10*a) + 63*sin(8*b*x + 8*a) - 63*sin(6*b*x + 6*a) - 315*sin(4*b*x + 4*
a) + 105*sin(2*b*x + 2*a))/b
```

mupad [B] time = 2.27, size = 149, normalized size = 0.96

$$\frac{5\,x}{8}+\frac{\frac{5\,\tan\left(a+b\,x\right)^{13}}{8}+\frac{25\,\tan\left(a+b\,x\right)^{11}}{6}+\frac{283\,\tan\left(a+b\,x\right)^{9}}{24}+\frac{128\,\tan\left(a+b\,x\right)^{7}}{7}-\frac{283\,\tan\left(a+b\,x\right)^{5}}{24}-\frac{25\,\tan\left(a+b\,x\right)^{3}}{6}\cdots}{b\,\left(\tan\left(a+b\,x\right)^{14}+7\,\tan\left(a+b\,x\right)^{12}+21\,\tan\left(a+b\,x\right)^{10}+35\,\tan\left(a+b\,x\right)^{8}+35\,\tan\left(a+b\,x\right)^{6}+21\,\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x)^2,x)
```

```
[Out] (5*x)/8 + ((128*tan(a + b*x)^7)/7 - (25*tan(a + b*x)^3)/6 - (283*tan(a + b*
x)^5)/24 - (5*tan(a + b*x))/8 + (283*tan(a + b*x)^9)/24 + (25*tan(a + b*x)^
11)/6 + (5*tan(a + b*x)^13)/8)/(b*(7*tan(a + b*x)^2 + 21*tan(a + b*x)^4 + 3
5*tan(a + b*x)^6 + 35*tan(a + b*x)^8 + 21*tan(a + b*x)^10 + 7*tan(a + b*x)^
12 + tan(a + b*x)^14 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

3.46 $\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{32\cos^{12}(a+bx)}{3b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{16\cos^{8}(a+bx)}{b}$$

```
[Out] -16*cos(b*x+a)^8/b+128/5*cos(b*x+a)^10/b-32/3*cos(b*x+a)^12/b
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2565, 266, 43}

$$-\frac{32\cos^{12}(a+bx)}{3b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{16\cos^{8}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-16*Cos[a + b*x]^8)/b + (128*Cos[a + b*x]^10)/(5*b) - (32*Cos[a + b*x]^12)
/(3*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{128\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\,\text{Subst}\left(\int (1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{64\,\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.18, size = 48, normalized size = 1.09

$$\frac{16\left(-10\sin^{12}(a+bx)+36\sin^{10}(a+bx)-45\sin^8(a+bx)+20\sin^6(a+bx)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (16*(20*Sin[a + b*x]^6 - 45*Sin[a + b*x]^8 + 36*Sin[a + b*x]^10 - 10*Sin[a
+ b*x]^12))/(15*b)
```

fricas [A] time = 0.42, size = 36, normalized size = 0.82

$$-\frac{16\left(10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] -16/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

giac [B] time = 0.99, size = 183, normalized size = 4.16

$$-\frac{4096\left(\frac{5\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{15\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{39\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{42\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\frac{39\,(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}+\frac{15\,(\cos(bx+a)-1)^8}{(\cos(bx+a)+1)^8}+\right.}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -4096/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 15*(cos(b*x + a) -
1)^4/(cos(b*x + a) + 1)^4 + 39*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 +
42*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 39*(cos(b*x + a) - 1)^7/(cos
(b*x + a) + 1)^7 + 15*(cos(b*x + a) - 1)^8/(cos(b*x + a) + 1)^8 + 5*(cos(b*
x + a) - 1)^9/(cos(b*x + a) + 1)^9)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) +
1) - 1)^12)
```

maple [A] time = 0.73, size = 53, normalized size = 1.20

$$\frac{-\frac{32\left(\sin^4(bx+a)\right)\left(\cos^8(bx+a)\right)}{3}-\frac{64\left(\sin^2(bx+a)\right)\left(\cos^8(bx+a)\right)}{15}-\frac{16\left(\cos^8(bx+a)\right)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/12*sin(b*x+a)^4*cos(b*x+a)^8-1/30*sin(b*x+a)^2*cos(b*x+a)^8-1/120
*cos(b*x+a)^8)
```

maxima [A] time = 0.34, size = 72, normalized size = 1.64

$$-\frac{5\cos\left(12\,bx+12\,a\right)+12\,\cos\left(10\,bx+10\,a\right)-30\,\cos\left(8\,bx+8\,a\right)-100\,\cos\left(6\,bx+6\,a\right)+75\,\cos\left(4\,bx+4\,a\right)+}{960\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] -1/960*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
- 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

mupad [B] time = 0.15, size = 35, normalized size = 0.80

$$-\frac{16\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x)^2,x)
```

```
[Out] -(16*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

3.47 $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=111

$$-\frac{32\sin^3(a+bx)\cos^7(a+bx)}{5b}-\frac{12\sin(a+bx)\cos^7(a+bx)}{5b}+\frac{2\sin(a+bx)\cos^5(a+bx)}{5b}+\frac{\sin(a+bx)\cos^3(a+$$

```
[Out] 3/4*x+3/4*cos(b*x+a)*sin(b*x+a)/b+1/2*cos(b*x+a)^3*sin(b*x+a)/b+2/5*cos(b*x
+a)^5*sin(b*x+a)/b-12/5*cos(b*x+a)^7*sin(b*x+a)/b-32/5*cos(b*x+a)^7*sin(b*x
+a)^3/b
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2568, 2635, 8}

$$-\frac{32\sin^3(a+bx)\cos^7(a+bx)}{5b}-\frac{12\sin(a+bx)\cos^7(a+bx)}{5b}+\frac{2\sin(a+bx)\cos^5(a+bx)}{5b}+\frac{\sin(a+bx)\cos^3(a+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (3*x)/4 + (3*Cos[a + b*x]*Sin[a + b*x])/(4*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(2*b) + (2*Cos[a + b*x]^5*Sin[a + b*x])/(5*b) - (12*Cos[a + b*x]^7*Sin[a
 + b*x])/(5*b) - (32*Cos[a + b*x]^7*Sin[a + b*x]^3)/(5*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = 64\int \cos^6(a+bx)\sin^4(a+bx)\,dx$$
$$= -\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b} + \frac{96}{5}\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{12\cos^7(a+bx)\sin(a+bx)}{5b} - \frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b} + \frac{12}{5}\int \cos^6(a$$
$$= \frac{2\cos^5(a+bx)\sin(a+bx)}{5b} - \frac{12\cos^7(a+bx)\sin(a+bx)}{5b} - \frac{32\cos^7(a+bx)\,\mathrm{s}}{5b}$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+bx)\sin(a+bx)}{5b} - \frac{12\cos^7(a+bx)\sin(}{5b}$$
$$= \frac{3\cos(a+bx)\sin(a+bx)}{4b} + \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+bx)\sin(a}{5b}$$
$$= \frac{3x}{4} + \frac{3\cos(a+bx)\sin(a+bx)}{4b} + \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+bx)}{5b}$$

Mathematica [A] time = 0.20, size = 62, normalized size = 0.56

$$\frac{20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx)) + 120bx}{160b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(160*b)
```

fricas [A] time = 0.46, size = 66, normalized size = 0.59

$$\frac{15\,bx + \left(128\,\cos(bx+a)^9 - 176\,\cos(bx+a)^7 + 8\,\cos(bx+a)^5 + 10\,\cos(bx+a)^3 + 15\,\cos(bx+a)\right)\sin(bx+a}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 1.02, size = 75, normalized size = 0.68

$$\frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^9 + 70\,\tan(bx+a)^7 + 128\,\tan(bx+a)^5 - 70\,\tan(bx+a)^3 - 15\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^5}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/20*(15*b*x + 15*a + (15*tan(b*x + a)^9 + 70*tan(b*x + a)^7 + 128*tan(b*x
+ a)^5 - 70*tan(b*x + a)^3 - 15*tan(b*x + a))/(tan(b*x + a)^2 + 1)^5)/b
```

maple [A] time = 0.97, size = 83, normalized size = 0.75

$$\frac{-\frac{32\left(\sin^3(bx+a)\right)\left(\cos^7(bx+a)\right)}{5} - \frac{12\sin(bx+a)\left(\cos^7(bx+a)\right)}{5} + \frac{2\left(\cos^5(bx+a) + \frac{5\left(\cos^3(bx+a)\right)}{4} + \frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{5} + \frac{3bx}{4} + \frac{3a}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/10*sin(b*x+a)^3*cos(b*x+a)^7-3/80*sin(b*x+a)*cos(b*x+a)^7+1/160*(c
os(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+3/256*b*x+3/256*a)
```

maxima [A] time = 0.35, size = 65, normalized size = 0.59

$$\frac{120\,bx + 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) - 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) + 20\,\sin\left(2\,bx + 2\,a\right)}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/160*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

mupad [B] time = 1.69, size = 109, normalized size = 0.98

$$\frac{3\,x}{4}+\frac{\frac{3\,\tan\left(a+b\,x\right)^9}{4}+\frac{7\,\tan\left(a+b\,x\right)^7}{2}+\frac{32\,\tan\left(a+b\,x\right)^5}{5}-\frac{7\,\tan\left(a+b\,x\right)^3}{2}-\frac{3\,\tan\left(a+b\,x\right)}{4}}{b\,\left({\tan\left(a+b\,x\right)}^{10}+5\,{\tan\left(a+b\,x\right)}^8+10\,{\tan\left(a+b\,x\right)}^6+10\,{\tan\left(a+b\,x\right)}^4+5\,{\tan\left(a+b\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x)^2,x)
```

```
[Out] (3*x)/4 + ((32*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/2 - (3*tan(a + b*x))/
4 + (7*tan(a + b*x)^7)/2 + (3*tan(a + b*x)^9)/4)/(b*(5*tan(a + b*x)^2 + 10*
tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 + 1
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

3.48 $\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=29

$$\frac{4\cos^8(a+bx)}{b}-\frac{16\cos^6(a+bx)}{3b}$$

```
[Out] -16/3*cos(b*x+a)^6/b+4*cos(b*x+a)^8/b
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 14}

$$\frac{4\cos^8(a+bx)}{b}-\frac{16\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-16*Cos[a + b*x]^6)/(3*b) + (4*Cos[a + b*x]^8)/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{32\,\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{32\,\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^6(a+bx)}{3b}+\frac{4\cos^8(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 48, normalized size = 1.66

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(96*b)
```

fricas [A] time = 0.42, size = 26, normalized size = 0.90

$$\frac{4\left(3\cos\left(bx+a\right)^{8}-4\cos\left(bx+a\right)^{6}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 4/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

giac [B] time = 0.68, size = 139, normalized size = 4.79

$$\frac{128\left(\frac{3(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{4(\cos(bx+a)-1)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{10(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}+\frac{4(\cos(bx+a)-1)^{5}}{(\cos(bx+a)+1)^{5}}+\frac{3(\cos(bx+a)-1)^{6}}{(\cos(bx+a)+1)^{6}}\right)}{3b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 128/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 4*(cos(b*x + a) - 1)^3
/(cos(b*x + a) + 1)^3 + 10*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 4*(c
os(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 + 3*(cos(b*x + a) - 1)^6/(cos(b*x +
 a) + 1)^6)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^8)
```

maple [A] time = 0.58, size = 35, normalized size = 1.21

$$\frac{-4\left(\sin^{2}\left(bx+a\right)\right)\left(\cos^{6}\left(bx+a\right)\right)-\frac{4\left(\cos^{6}\left(bx+a\right)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] 32/b*(-1/8*sin(b*x+a)^2*cos(b*x+a)^6-1/24*cos(b*x+a)^6)
```

maxima [A] time = 0.35, size = 50, normalized size = 1.72

$$\frac{3\cos\left(8bx+8a\right)+8\cos\left(6bx+6a\right)-12\cos\left(4bx+4a\right)-72\cos\left(2bx+2a\right)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*co
s(2*b*x + 2*a))/b
```

mupad [B] time = 0.05, size = 25, normalized size = 0.86

$$\frac{4\cos\left(a+b\,x\right)^{6}\,\left(3\cos\left(a+b\,x\right)^{2}-4\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x)^2,x)
```

```
[Out] (4*cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(3*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.49 $\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=60

$$-\frac{8\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{2\sin(a+bx)\cos^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos(a+bx)}{b}+x$$

```
[Out] x+cos(b*x+a)*sin(b*x+a)/b+2/3*cos(b*x+a)^3*sin(b*x+a)/b-8/3*cos(b*x+a)^5*si
n(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2568, 2635, 8}

$$-\frac{8\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{2\sin(a+bx)\cos^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] x + (Cos[a + b*x]*Sin[a + b*x])/b + (2*Cos[a + b*x]^3*Sin[a + b*x])/(3*b) -
 (8*Cos[a + b*x]^5*Sin[a + b*x])/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^4(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{8\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{8}{3}\int \cos^4(a+bx)\,dx$$
$$= \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin(a+bx)}{3b} + 2\int \cos^2(a+bx)\,d$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{b} + \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin(a}{3b}$$
$$= x + \frac{\cos(a+bx)\sin(a+bx)}{b} + \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin}{3b}$$

Mathematica [A] time = 0.10, size = 40, normalized size = 0.67

$$-\frac{-3\sin(2(a+bx)) + 3\sin(4(a+bx)) + \sin(6(a+bx)) - 12bx}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
[Out] -1/12*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)]
)/b
```

fricas [A] time = 0.46, size = 47, normalized size = 0.78

$$\frac{3\,bx - \left(8\,\cos\left(bx+a\right)^5 - 2\,\cos\left(bx+a\right)^3 - 3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
[Out] 1/3*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*x
 + a))/b
```

giac [A] time = 0.77, size = 55, normalized size = 0.92

$$\frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^5 + 8\,\tan(bx+a)^3 - 3\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^5 + 8*tan(b*x + a)^3 - 3*tan(b*x + a))/(
tan(b*x + a)^2 + 1)^3)/b
```

maple [A] time = 0.91, size = 55, normalized size = 0.92

$$\frac{-\frac{8\sin(bx+a)\left(\cos^5(bx+a)\right)}{3} + \frac{2\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{3} + bx + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x)
[Out] 16/b*(-1/6*sin(b*x+a)*cos(b*x+a)^5+1/24*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b
*x+a)+1/16*b*x+1/16*a)
```

maxima [A] time = 0.36, size = 43, normalized size = 0.72

$$\frac{12\,bx - \sin\left(6\,bx + 6\,a\right) - 3\,\sin\left(4\,bx + 4\,a\right) + 3\,\sin\left(2\,bx + 2\,a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

mupad [B] time = 0.49, size = 65, normalized size = 1.08

$$x + \frac{\tan\left(a + b\,x\right)^5 + \frac{8\tan\left(a+b\,x\right)^3}{3} - \tan\left(a + b\,x\right)}{b\,\left(\tan\left(a + b\,x\right)^6 + 3\tan\left(a + b\,x\right)^4 + 3\tan\left(a + b\,x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x)^2,x)
```

```
[Out] x + ((8*tan(a + b*x)^3)/3 - tan(a + b*x) + tan(a + b*x)^5)/(b*(3*tan(a + b*
x)^2 + 3*tan(a + b*x)^4 + tan(a + b*x)^6 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.50 $\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{2\cos^4(a+bx)}{b}$$

```
[Out] -2*cos(b*x+a)^4/b
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4288, 2565, 30}

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-2*Cos[a + b*x]^4)/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{2\cos^4(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-2*Cos[a + b*x]^4)/b
```

fricas [A] time = 0.45, size = 13, normalized size = 1.00

$$-\frac{2\cos(bx+a)^4}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -2*cos(b*x + a)^4/b
```

giac [B] time = 0.49, size = 69, normalized size = 5.31

$$-\frac{16\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}\right)}{b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -16*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^3/(cos(b*x
+ a) + 1)^3)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^4)
```

maple [A] time = 0.57, size = 14, normalized size = 1.08

$$-\frac{2\left(\cos^4(bx+a)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x)
[Out] -2*cos(b*x+a)^4/b
```

maxima [A] time = 0.34, size = 26, normalized size = 2.00

$$-\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
[Out] -1/4*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

mupad [B] time = 0.11, size = 13, normalized size = 1.00

$$-\frac{2\cos(a+b\,x)^4}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x)^2,x)
[Out] -(2*cos(a + b*x)^4)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**3,x)
[Out] Timed out
```

3.51 $\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=21

$$\frac{2\sin(a+bx)\cos(a+bx)}{b}+2x$$

[Out] 2*x+2*cos(b*x+a)*sin(b*x+a)/b

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2635, 8}

$$\frac{2\sin(a+bx)\cos(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]

[Out] 2*x + (2*Cos[a + b*x]*Sin[a + b*x])/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\sin(a+bx)}{b}+2\int 1\,dx\\ &= 2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.95

$$\frac{2(a+bx)+\sin(2(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]

[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/b

fricas [**A**] time = 0.41, size = 22, normalized size = 1.05

$$\frac{2\,(bx+\cos{(bx+a)}\sin{(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] 2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

giac [**A**] time = 0.49, size = 29, normalized size = 1.38

$$\frac{2\left(bx+a+\frac{\tan(bx+a)}{\tan(bx+a)^2+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
[Out] 2*(b*x + a + tan(b*x + a)/(tan(b*x + a)^2 + 1))/b
```

maple [**A**] time = 0.59, size = 28, normalized size = 1.33

$$\frac{2\cos{(bx+a)}\sin{(bx+a)}+2bx+2a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x)
[Out] 4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

maxima [**A**] time = 0.34, size = 18, normalized size = 0.86

$$\frac{2\,bx+\sin{(2\,bx+2\,a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] (2*b*x + sin(2*b*x + 2*a))/b
```

mupad [**B**] time = 0.15, size = 17, normalized size = 0.81

$$2\,x+\frac{\sin{(2\,a+2\,b\,x)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x)^2,x)
[Out] 2*x + sin(2*a + 2*b*x)/b
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**2,x)
[Out] Timed out
```

3.52 $\int \csc^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=12

$$\frac{2\log(\sin(a+bx))}{b}$$

```
[Out] 2*ln(sin(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4288, 3475}

$$\frac{2\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Log[Sin[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\,dx\\ &= \frac{2\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.67

$$\frac{2(\log(\tan(a+bx))+\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b
```

fricas [A] time = 0.45, size = 14, normalized size = 1.17

$$\frac{2\,\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*log(1/2*sin(b*x + a))/b
```

giac [B] time = 0.99, size = 55, normalized size = 4.58

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] (log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.28, size = 13, normalized size = 1.08

$$\frac{2\ln\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a),x)
```

```
[Out] 2*ln(sin(b*x+a))/b
```

maxima [B] time = 0.35, size = 81, normalized size = 6.75

$$\frac{\log\left(\cos\left(bx\right)^2 + 2\,\cos\left(bx\right)\cos(a) + \cos(a)^2 + \sin\left(bx\right)^2 - 2\,\sin\left(bx\right)\sin(a) + \sin(a)^2\right) + \log\left(\cos\left(bx\right)^2 - 2\,\cos}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] (log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*si
n(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)
^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.13, size = 13, normalized size = 1.08

$$\frac{\ln\left(\sin\left(a+b\,x\right)^2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x)^2,x)
```

```
[Out] log(sin(a + b*x)^2)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

3.53 $\int \csc^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\tan(a+bx))}{2b}-\frac{\cot^2(a+bx)}{4b}$$

```
[Out] -1/4*cot(b*x+a)^2/b+1/2*ln(tan(b*x+a))/b
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2620, 14}

$$\frac{\log(\tan(a+bx))}{2b}-\frac{\cot^2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -Cot[a + b*x]^2/(4*b) + Log[Tan[a + b*x]]/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc^3(a+bx)\sec(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\tan(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan(a+bx)\right)}{2b}\\
&= -\frac{\cot^2(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{2b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/4*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/b
```

fricas [B] time = 0.43, size = 65, normalized size = 2.17

$$-\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\cos\left(bx+a\right)^2\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{4\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 1.01, size = 813, normalized size = 27.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*((3*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 180*tan(b*x + 4*a)^2*tan(1/2*a)^2
2 + 24*tan(b*x + 4*a)*tan(1/2*a)^23 + tan(1/2*a)^24 + 4230*tan(b*x + 4*a)^2
*tan(1/2*a)^20 - 1592*tan(b*x + 4*a)*tan(1/2*a)^21 + 48*tan(1/2*a)^22 - 486
12*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 31704*tan(b*x + 4*a)*tan(1/2*a)^19 - 38
46*tan(1/2*a)^20 + 277965*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 259128*tan(b*x +
 4*a)*tan(1/2*a)^17 + 51632*tan(1/2*a)^18 - 737640*tan(b*x + 4*a)^2*tan(1/2
*a)^14 + 982192*tan(b*x + 4*a)*tan(1/2*a)^15 - 274545*tan(1/2*a)^16 + 10084
68*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 1871088*tan(b*x + 4*a)*tan(1/2*a)^13 +
733728*tan(1/2*a)^14 - 737640*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 1871088*tan(
b*x + 4*a)*tan(1/2*a)^11 - 1018132*tan(1/2*a)^12 + 277965*tan(b*x + 4*a)^2*
tan(1/2*a)^8 - 982192*tan(b*x + 4*a)*tan(1/2*a)^9 + 733728*tan(1/2*a)^10 -
48612*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 259128*tan(b*x + 4*a)*tan(1/2*a)^7 -
274545*tan(1/2*a)^8 + 4230*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 31704*tan(b*x +
4*a)*tan(1/2*a)^5 + 51632*tan(1/2*a)^6 - 180*tan(b*x + 4*a)^2*tan(1/2*a)^2
+ 1592*tan(b*x + 4*a)*tan(1/2*a)^3 - 3846*tan(1/2*a)^4 + 3*tan(b*x + 4*a)^2
 - 24*tan(b*x + 4*a)*tan(1/2*a) + 48*tan(1/2*a)^2 + 1)/((tan(1/2*a)^12 - 30
*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6 + 255*tan(1/2*a)^4 - 3
0*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/
2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3
- tan(b*x + 4*a) + 6*tan(1/2*a))^2) - 2*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6
 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(
1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) + 2*log(abs(6*
tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3
 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

maple [A] time = 0.73, size = 27, normalized size = 0.90

$$-\frac{1}{4b\sin\left(bx+a\right)^2}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a),x)
```

```
[Out] -1/4/b/sin(b*x+a)^2+1/2*ln(tan(b*x+a))/b
```

maxima [B] time = 0.37, size = 656, normalized size = 21.87

$$\frac{4\cos\left(4\,bx+4\,a\right)\cos\left(2\,bx+2\,a\right)-8\cos\left(2\,bx+2\,a\right)^2+\left(2\left(2\cos\left(2\,bx+2\,a\right)-1\right)\cos\left(4\,bx+4\,a\right)-\cos\left(4\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2*
(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*s
in(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos
(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b
*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4*
a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*c
os(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

mupad [B] time = 0.14, size = 36, normalized size = 1.20

$$-\frac{\frac{\ln(\cos(a+b\,x))}{2}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{4}+\frac{1}{4\sin(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)),x)
```

```
[Out] -(log(cos(a + b*x))/2 - log(sin(a + b*x)^2)/4 + 1/(4*sin(a + b*x)^2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x), x)
```

3.54 $\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan(a+bx)}{4b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot(a+bx)}{2b}$$

```
[Out] -1/2*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b+1/4*tan(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2620, 270}

$$\frac{\tan(a+bx)}{4b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(2*b) - Cot[a + b*x]^3/(12*b) + Tan[a + b*x]/(4*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^4(a+bx)\sec^2(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(a+bx)\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan(a+bx)\right)}{4b}\\
&= -\frac{\cot(a+bx)}{2b}-\frac{\cot^3(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.14

$$\frac{\tan(a+bx)}{4b}-\frac{5\cot(a+bx)}{12b}-\frac{\cot(a+bx)\csc^2(a+bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-5*Cot[a + b*x])/(12*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(12*b) + Tan[a + b
*x]/(4*b)
```

fricas [A] time = 0.43, size = 54, normalized size = 1.29

$$-\frac{8\cos\left(bx+a\right)^{4}-12\cos\left(bx+a\right)^{2}+3}{12\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

giac [B] time = 1.81, size = 1079, normalized size = 25.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*a)
^6 + 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((6*tan(b*x + 4*a)*tan(1/2*a)^5
- tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b
*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)*(3*tan(1/2*a)^5 - 10*tan(1/2*a)
^3 + 3*tan(1/2*a))) + 2*(6*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 216*tan(b*x + 4
*a)^2*tan(1/2*a)^34 + 54*tan(b*x + 4*a)*tan(1/2*a)^35 + tan(1/2*a)^36 + 307
8*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 1638*tan(b*x + 4*a)*tan(1/2*a)^33 + 126*
tan(1/2*a)^34 - 23328*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 19872*tan(b*x + 4*a)
*tan(1/2*a)^31 - 3159*tan(1/2*a)^32 + 62856*tan(b*x + 4*a)^2*tan(1/2*a)^28
- 100224*tan(b*x + 4*a)*tan(1/2*a)^29 + 29232*tan(1/2*a)^30 + 328608*tan(b*
x + 4*a)^2*tan(1/2*a)^26 - 127176*tan(b*x + 4*a)*tan(1/2*a)^27 - 26460*tan(
1/2*a)^28 - 96504*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 808488*tan(b*x + 4*a)*ta
n(1/2*a)^25 - 228600*tan(1/2*a)^26 - 879840*tan(b*x + 4*a)^2*tan(1/2*a)^22
+ 833472*tan(b*x + 4*a)*tan(1/2*a)^23 + 237588*tan(1/2*a)^24 + 30564*tan(b*
x + 4*a)^2*tan(1/2*a)^20 - 1653792*tan(b*x + 4*a)*tan(1/2*a)^21 + 944208*ta
n(1/2*a)^22 + 1149552*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 1673388*tan(b*x + 4*
a)*tan(1/2*a)^19 - 142434*tan(1/2*a)^20 + 30564*tan(b*x + 4*a)^2*tan(1/2*a)
^16 + 1673388*tan(b*x + 4*a)*tan(1/2*a)^17 - 1358860*tan(1/2*a)^18 - 879840
*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 1653792*tan(b*x + 4*a)*tan(1/2*a)^15 - 14
2434*tan(1/2*a)^16 - 96504*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 833472*tan(b*x
+ 4*a)*tan(1/2*a)^13 + 944208*tan(1/2*a)^14 + 328608*tan(b*x + 4*a)^2*tan(1
/2*a)^10 - 808488*tan(b*x + 4*a)*tan(1/2*a)^11 + 237588*tan(1/2*a)^12 + 628
56*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 127176*tan(b*x + 4*a)*tan(1/2*a)^9 - 228
600*tan(1/2*a)^10 - 23328*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 100224*tan(b*x +
4*a)*tan(1/2*a)^7 - 26460*tan(1/2*a)^8 + 3078*tan(b*x + 4*a)^2*tan(1/2*a)^4
 - 19872*tan(b*x + 4*a)*tan(1/2*a)^5 + 29232*tan(1/2*a)^6 - 216*tan(b*x + 4
*a)^2*tan(1/2*a)^2 + 1638*tan(b*x + 4*a)*tan(1/2*a)^3 - 3159*tan(1/2*a)^4 +
 6*tan(b*x + 4*a)^2 - 54*tan(b*x + 4*a)*tan(1/2*a) + 126*tan(1/2*a)^2 + 1)/
((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*tan(1/2*a)^14 - 4728*tan(1/2*a)^12
 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8 + 4728*tan(1/2*a)^6 - 720*tan(1
/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x +
4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*ta
n(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^3))/b
```

maple [**A**] time = 1.11, size = 51, normalized size = 1.21

$$\frac{-\frac{1}{3\sin(bx+a)^3\cos(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x)
```

```
[Out] 1/4/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

maxima [**B**] time = 0.36, size = 308, normalized size = 7.33

$$\frac{4\,((2\,\mathrm{co}}{3\left(b\cos\left(8\,bx+8\,a\right)^2+4\,b\cos\left(6\,bx+6\,a\right)^2+4\,b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(8\,bx+8\,a\right)^2+4\,b\sin\left(6\,bx+6\,a\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

mupad [**B**] time = 0.14, size = 37, normalized size = 0.88

$$\frac{\tan\left(a+b\,x\right)}{4\,b}-\frac{\frac{\tan(a+b\,x)^2}{2}+\frac{1}{12}}{b\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^2),x)
```

```
[Out] tan(a + b*x)/(4*b) - (tan(a + b*x)^2/2 + 1/12)/(b*tan(a + b*x)^3)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2\left(a+bx\right)\csc^2\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**2, x)
```

3.55 $\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}-\frac{3\cot^2(a+bx)}{16b}+\frac{3\log(\tan(a+bx))}{8b}$$

```
[Out] -3/16*cot(b*x+a)^2/b-1/32*cot(b*x+a)^4/b+3/8*ln(tan(b*x+a))/b+1/16*tan(b*x+
a)^2/b
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}-\frac{3\cot^2(a+bx)}{16b}+\frac{3\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(16*b) - Cot[a + b*x]^4/(32*b) + (3*Log[Tan[a + b*x]])/
(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \tan^2(a+bx)\right)}{16b}$$
$$= -\frac{3\cot^2(a+bx)}{16b} - \frac{\cot^4(a+bx)}{32b} + \frac{3\log(\tan(a+bx))}{8b} + \frac{\tan^2(a+bx)}{16b}$$

Mathematica [A] time = 0.36, size = 54, normalized size = 0.90

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/32*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Si
n[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

fricas [B] time = 0.51, size = 138, normalized size = 2.30

$$\frac{6\cos(bx+a)^4 - 9\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 6\,(}{32\left(b\cos(bx+a)^6 - 2\,b\cos(bx+a)^4 + b\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
 a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

giac [B] time = 3.36, size = 2808, normalized size = 46.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((12*tan(b*x + 4*a)*tan(1/2*a)^23 - tan(1/2*a)^24 + 11664*tan(b*x + 4*
a)^2*tan(1/2*a)^20 - 4036*tan(b*x + 4*a)*tan(1/2*a)^21 + 384*tan(1/2*a)^22
- 155520*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 96852*tan(b*x + 4*a)*tan(1/2*a)^1
9 - 11994*tan(1/2*a)^20 + 824256*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 781884*ta
n(b*x + 4*a)*tan(1/2*a)^17 + 150592*tan(1/2*a)^18 - 2194560*tan(b*x + 4*a)^
2*tan(1/2*a)^14 + 2930936*tan(b*x + 4*a)*tan(1/2*a)^15 - 827919*tan(1/2*a)^
16 + 3065184*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 5623464*tan(b*x + 4*a)*tan(1/
2*a)^13 + 2209344*tan(1/2*a)^14 - 2194560*tan(b*x + 4*a)^2*tan(1/2*a)^10 +
5623464*tan(b*x + 4*a)*tan(1/2*a)^11 - 3036716*tan(1/2*a)^12 + 824256*tan(b
```

```
*x + 4*a)^2*tan(1/2*a)^8 - 2930936*tan(b*x + 4*a)*tan(1/2*a)^9 + 2209344*ta
n(1/2*a)^10 - 155520*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 781884*tan(b*x + 4*a)*
tan(1/2*a)^7 - 827919*tan(1/2*a)^8 + 11664*tan(b*x + 4*a)^2*tan(1/2*a)^4 -
96852*tan(b*x + 4*a)*tan(1/2*a)^5 + 150592*tan(1/2*a)^6 + 4036*tan(b*x + 4*
a)*tan(1/2*a)^3 - 11994*tan(1/2*a)^4 - 12*tan(b*x + 4*a)*tan(1/2*a) + 384*t
an(1/2*a)^2 - 1)/((9*tan(1/2*a)^10 - 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 6
0*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a
)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*t
an(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) - 2*(25*tan(b*x + 4*a)^4*tan(1/2*a)^48
- 3000*tan(b*x + 4*a)^4*tan(1/2*a)^46 + 528*tan(b*x + 4*a)^3*tan(1/2*a)^47
+ 6*tan(b*x + 4*a)^2*tan(1/2*a)^48 + 160500*tan(b*x + 4*a)^4*tan(1/2*a)^44
- 60656*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 4032*tan(b*x + 4*a)^2*tan(1/2*a)^4
6 + 48*tan(b*x + 4*a)*tan(1/2*a)^47 + tan(1/2*a)^48 - 5040200*tan(b*x + 4*a
)^4*tan(1/2*a)^42 + 2998224*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 459720*tan(b*x
 + 4*a)^2*tan(1/2*a)^44 + 13808*tan(b*x + 4*a)*tan(1/2*a)^45 + 96*tan(1/2*a
)^46 + 102947250*tan(b*x + 4*a)^4*tan(1/2*a)^40 - 84754224*tan(b*x + 4*a)^3
*tan(1/2*a)^41 + 20740800*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 1553616*tan(b*x
+ 4*a)*tan(1/2*a)^43 + 17940*tan(1/2*a)^44 - 1432641000*tan(b*x + 4*a)^4*ta
n(1/2*a)^38 + 1525423600*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 518242932*tan(b*x
 + 4*a)^2*tan(1/2*a)^40 + 62627184*tan(b*x + 4*a)*tan(1/2*a)^41 - 1976672*t
an(1/2*a)^42 + 13850865700*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 18347115792*tan
(b*x + 4*a)^3*tan(1/2*a)^37 + 8061782976*tan(b*x + 4*a)^2*tan(1/2*a)^38 - 1
352088880*tan(b*x + 4*a)*tan(1/2*a)^39 + 69043458*tan(1/2*a)^40 - 934243830
00*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 150554954928*tan(b*x + 4*a)^3*tan(1/2*a
)^35 - 82205969064*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 17773339152*tan(b*x + 4
*a)*tan(1/2*a)^37 - 1252335840*tan(1/2*a)^38 + 438276972375*tan(b*x + 4*a)^
4*tan(1/2*a)^32 - 849352828496*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 56333896416
0*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 150682961328*tan(b*x + 4*a)*tan(1/2*a)^3
5 + 13569524420*tan(1/2*a)^36 - 1429067476400*tan(b*x + 4*a)^4*tan(1/2*a)^3
0 + 3312518959776*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 2637116003430*tan(b*x +
4*a)^2*tan(1/2*a)^32 + 853300409104*tan(b*x + 4*a)*tan(1/2*a)^33 - 94325029
920*tan(1/2*a)^34 + 3274543905000*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 90295262
76960*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 8569312295808*tan(b*x + 4*a)^2*tan(1
/2*a)^30 - 3315261850656*tan(b*x + 4*a)*tan(1/2*a)^31 + 440742337263*tan(1/
2*a)^32 - 5348018130000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 17444688475680*tan
(b*x + 4*a)^3*tan(1/2*a)^27 - 19627171727760*tan(b*x + 4*a)^2*tan(1/2*a)^28
 + 9019269429600*tan(b*x + 4*a)*tan(1/2*a)^29 - 1427485388352*tan(1/2*a)^30
 + 6290345645500*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 24168979539936*tan(b*x +
4*a)^3*tan(1/2*a)^25 + 32090952253824*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 1743
5372647456*tan(b*x + 4*a)*tan(1/2*a)^27 + 3267744813864*tan(1/2*a)^28 - 534
8018130000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 24168979539936*tan(b*x + 4*a)^3
*tan(1/2*a)^23 - 37769347277400*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 2418250446
8064*tan(b*x + 4*a)*tan(1/2*a)^25 - 5348887137216*tan(1/2*a)^26 + 327454390
5000*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 17444688475680*tan(b*x + 4*a)^3*tan(1
/2*a)^21 + 32090952253824*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 24182504468064*t
an(b*x + 4*a)*tan(1/2*a)^23 + 6299635484700*tan(1/2*a)^24 - 1429067476400*t
an(b*x + 4*a)^4*tan(1/2*a)^18 + 9029526276960*tan(b*x + 4*a)^3*tan(1/2*a)^1
9 - 19627171727760*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17435372647456*tan(b*x
+ 4*a)*tan(1/2*a)^21 - 5348887137216*tan(1/2*a)^22 + 438276972375*tan(b*x +
 4*a)^4*tan(1/2*a)^16 - 3312518959776*tan(b*x + 4*a)^3*tan(1/2*a)^17 + 8569
312295808*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 9019269429600*tan(b*x + 4*a)*tan
(1/2*a)^19 + 3267744813864*tan(1/2*a)^20 - 93424383000*tan(b*x + 4*a)^4*tan
(1/2*a)^14 + 849352828496*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 2637116003430*ta
n(b*x + 4*a)^2*tan(1/2*a)^16 + 3315261850656*tan(b*x + 4*a)*tan(1/2*a)^17 -
 1427485388352*tan(1/2*a)^18 + 13850865700*tan(b*x + 4*a)^4*tan(1/2*a)^12 -
 150554954928*tan(b*x + 4*a)^3*tan(1/2*a)^13 + 563338964160*tan(b*x + 4*a)^
2*tan(1/2*a)^14 - 853300409104*tan(b*x + 4*a)*tan(1/2*a)^15 + 440742337263*
tan(1/2*a)^16 - 1432641000*tan(b*x + 4*a)^4*tan(1/2*a)^10 + 18347115792*tan
(b*x + 4*a)^3*tan(1/2*a)^11 - 82205969064*tan(b*x + 4*a)^2*tan(1/2*a)^12 +
```

```
150682961328*tan(b*x + 4*a)*tan(1/2*a)^13 - 94325029920*tan(1/2*a)^14 + 102
947250*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 1525423600*tan(b*x + 4*a)^3*tan(1/2*
a)^9 + 8061782976*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 17773339152*tan(b*x + 4*
a)*tan(1/2*a)^11 + 13569524420*tan(1/2*a)^12 - 5040200*tan(b*x + 4*a)^4*tan
(1/2*a)^6 + 84754224*tan(b*x + 4*a)^3*tan(1/2*a)^7 - 518242932*tan(b*x + 4*
a)^2*tan(1/2*a)^8 + 1352088880*tan(b*x + 4*a)*tan(1/2*a)^9 - 1252335840*tan
(1/2*a)^10 + 160500*tan(b*x + 4*a)^4*tan(1/2*a)^4 - 2998224*tan(b*x + 4*a)^
3*tan(1/2*a)^5 + 20740800*tan(b*x + 4*a)^2*tan(1/2*a)^6 - 62627184*tan(b*x
+ 4*a)*tan(1/2*a)^7 + 69043458*tan(1/2*a)^8 - 3000*tan(b*x + 4*a)^4*tan(1/2
*a)^2 + 60656*tan(b*x + 4*a)^3*tan(1/2*a)^3 - 459720*tan(b*x + 4*a)^2*tan(1
/2*a)^4 + 1553616*tan(b*x + 4*a)*tan(1/2*a)^5 - 1976672*tan(1/2*a)^6 + 25*t
an(b*x + 4*a)^4 - 528*tan(b*x + 4*a)^3*tan(1/2*a) + 4032*tan(b*x + 4*a)^2*t
an(1/2*a)^2 - 13808*tan(b*x + 4*a)*tan(1/2*a)^3 + 17940*tan(1/2*a)^4 + 6*ta
n(b*x + 4*a)^2 - 48*tan(b*x + 4*a)*tan(1/2*a) + 96*tan(1/2*a)^2 + 1)/((tan(
1/2*a)^24 - 60*tan(1/2*a)^22 + 1410*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 9
2655*tan(1/2*a)^16 - 245880*tan(1/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*t
an(1/2*a)^10 + 92655*tan(1/2*a)^8 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4
- 60*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan
(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)
^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^4) + 24*log(abs(tan(b*x + 4*a)*tan(1/2*
a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*
tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 24*log(a
bs(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2
*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1
)))/b
```

maple [A] time = 0.86, size = 69, normalized size = 1.15

$$-\frac{1}{32b\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{32b\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{16b\sin(bx+a)^2}+\frac{3\ln(\tan(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x)

[Out] -1/32/b/sin(b*x+a)^4/cos(b*x+a)^2+3/32/b/sin(b*x+a)^2/cos(b*x+a)^2-3/16/b/s
in(b*x+a)^2+3/8*ln(tan(b*x+a))/b
```

maxima [B] time = 0.45, size = 3188, normalized size = 53.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")

[Out] 1/16*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
```

```
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(10*b*x + 10*a) + b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) - b)*cos(12*b*x + 12*a) + 4*(b*cos(8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 2*(4
*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*
```

```
b*x + 8*a) - 8*(b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) + 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a)
 - 2*(2*b*sin(10*b*x + 10*a) + b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) +
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 4*(b*sin(8*
b*x + 8*a) - 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 2*(4*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 8*(b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.22, size = 82, normalized size = 1.37

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{16\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{8\,b}+\frac{\frac{3\cos(a+b\,x)^4}{16}-\frac{9\cos(a+b\,x)^2}{32}+\frac{1}{16}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(16*b) - (3*log(cos(a + b*x)))/(8*b) + ((3*cos(a +
b*x)^4)/16 - (9*cos(a + b*x)^2)/32 + 1/16)/(b*(cos(a + b*x)^2 - 2*cos(a + b
*x)^4 + cos(a + b*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\csc^2\left(a+bx\right)\csc^3\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**3, x)
```

3.56 $\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=72

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{4b}-\frac{\cot^5(a+bx)}{80b}-\frac{\cot^3(a+bx)}{12b}-\frac{3\cot(a+bx)}{8b}$$

```
[Out] -3/8*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b-1/80*cot(b*x+a)^5/b+1/4*tan(b*x+a)/b+
1/48*tan(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2620, 270}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{4b}-\frac{\cot^5(a+bx)}{80b}-\frac{\cot^3(a+bx)}{12b}-\frac{3\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/(8*b) - Cot[a + b*x]^3/(12*b) - Cot[a + b*x]^5/(80*b) + T
an[a + b*x]/(4*b) + Tan[a + b*x]^3/(48*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^6(a+bx)\sec^4(a+bx)\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^6}\,dx, x, \tan(a+bx)\right)}{16b}\\
&= \frac{\operatorname{Subst}\left(\int \left(4+\frac{1}{x^6}+\frac{4}{x^4}+\frac{6}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{16b}\\
&= -\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(a+bx)}{48b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.25

$$\frac{11\tan(a+bx)}{48b}-\frac{73\cot(a+bx)}{240b}-\frac{\cot(a+bx)\csc^4(a+bx)}{80b}-\frac{7\cot(a+bx)\csc^2(a+bx)}{120b}+\frac{\tan(a+bx)\sec^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-73*Cot[a + b*x])/(240*b) - (7*Cot[a + b*x]*Csc[a + b*x]^2)/(120*b) - (Cot
[a + b*x]*Csc[a + b*x]^4)/(80*b) + (11*Tan[a + b*x])/(48*b) + (Sec[a + b*x]
^2*Tan[a + b*x])/(48*b)
```

fricas [A] time = 0.42, size = 86, normalized size = 1.19

$$-\frac{128 \cos(bx+a)^8 - 320 \cos(bx+a)^6 + 240 \cos(bx+a)^4 - 40 \cos(bx+a)^2 - 5}{240 \left(b\cos(bx+a)^7 - 2\,b\cos(bx+a)^5 + b\cos(bx+a)^3\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(128*cos(b*x + a)^8 - 320*cos(b*x + a)^6 + 240*cos(b*x + a)^4 - 40*c
os(b*x + a)^2 - 5)/((b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x + a)
^3)*sin(b*x + a))
```

giac [B] time = 2.83, size = 3445, normalized size = 47.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/1920*(5*(108*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 18*tan(b*x + 4*a)*tan(1/2*
a)^35 + tan(1/2*a)^36 + 12240*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 3774*tan(b*x
 + 4*a)*tan(1/2*a)^33 + 342*tan(1/2*a)^34 - 85632*tan(b*x + 4*a)^2*tan(1/2*
a)^30 + 75456*tan(b*x + 4*a)*tan(1/2*a)^31 - 9783*tan(1/2*a)^32 + 58608*tan
(b*x + 4*a)^2*tan(1/2*a)^28 - 253152*tan(b*x + 4*a)*tan(1/2*a)^29 + 86064*t
an(1/2*a)^30 + 631440*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 389448*tan(b*x + 4*a
)*tan(1/2*a)^27 - 85500*tan(1/2*a)^28 - 679344*tan(b*x + 4*a)^2*tan(1/2*a)^
24 + 2047464*tan(b*x + 4*a)*tan(1/2*a)^25 - 702936*tan(1/2*a)^26 - 2420352*
tan(b*x + 4*a)^2*tan(1/2*a)^22 + 1465056*tan(b*x + 4*a)*tan(1/2*a)^23 + 675
636*tan(1/2*a)^24 + 1394928*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 6297216*tan(b*
x + 4*a)*tan(1/2*a)^21 + 2768976*tan(1/2*a)^22 + 5321736*tan(b*x + 4*a)^2*t
an(1/2*a)^18 - 5657724*tan(b*x + 4*a)*tan(1/2*a)^19 - 514818*tan(1/2*a)^20
+ 1394928*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 5657724*tan(b*x + 4*a)*tan(1/2*a
)^17 - 4173820*tan(1/2*a)^18 - 2420352*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 629
7216*tan(b*x + 4*a)*tan(1/2*a)^15 - 514818*tan(1/2*a)^16 - 679344*tan(b*x +
 4*a)^2*tan(1/2*a)^12 - 1465056*tan(b*x + 4*a)*tan(1/2*a)^13 + 2768976*tan(
1/2*a)^14 + 631440*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 2047464*tan(b*x + 4*a)*
tan(1/2*a)^11 + 675636*tan(1/2*a)^12 + 58608*tan(b*x + 4*a)^2*tan(1/2*a)^8
+ 389448*tan(b*x + 4*a)*tan(1/2*a)^9 - 702936*tan(1/2*a)^10 - 85632*tan(b*x
 + 4*a)^2*tan(1/2*a)^6 + 253152*tan(b*x + 4*a)*tan(1/2*a)^7 - 85500*tan(1/2
*a)^8 + 12240*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 75456*tan(b*x + 4*a)*tan(1/2*
a)^5 + 86064*tan(1/2*a)^6 + 108*tan(b*x + 4*a)^2*tan(1/2*a)^2 + 3774*tan(b*
x + 4*a)*tan(1/2*a)^3 - 9783*tan(1/2*a)^4 + 18*tan(b*x + 4*a)*tan(1/2*a) +
342*tan(1/2*a)^2 + 1)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*
a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/
2*a)^3)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*t
an(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)
^2 + 1)^3) + 8*(90*tan(b*x + 4*a)^4*tan(1/2*a)^60 - 8100*tan(b*x + 4*a)^4*t
an(1/2*a)^58 + 1800*tan(b*x + 4*a)^3*tan(1/2*a)^59 + 20*tan(b*x + 4*a)^2*ta
n(1/2*a)^60 + 337950*tan(b*x + 4*a)^4*tan(1/2*a)^56 - 151800*tan(b*x + 4*a)
^3*tan(1/2*a)^57 + 13200*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 150*tan(b*x + 4*a
)*tan(1/2*a)^59 + 3*tan(1/2*a)^60 - 8558760*tan(b*x + 4*a)^4*tan(1/2*a)^54
```

+ 5856480*tan(b*x + 4*a)^3*tan(1/2*a)^55 - 1068900*tan(b*x + 4*a)^2*tan(1/2
*a)^56 + 44050*tan(b*x + 4*a)*tan(1/2*a)^57 + 270*tan(1/2*a)^58 + 143179650
*tan(b*x + 4*a)^4*tan(1/2*a)^52 - 133967520*tan(b*x + 4*a)^3*tan(1/2*a)^53
+ 37650400*tan(b*x + 4*a)^2*tan(1/2*a)^54 - 3349740*tan(b*x + 4*a)*tan(1/2*
a)^55 + 56265*tan(1/2*a)^56 - 1610662860*tan(b*x + 4*a)^4*tan(1/2*a)^50 + 1
962664200*tan(b*x + 4*a)^3*tan(1/2*a)^51 - 761504220*tan(b*x + 4*a)^2*tan(1
/2*a)^52 + 105669900*tan(b*x + 4*a)*tan(1/2*a)^53 - 3921700*tan(1/2*a)^54 +
 11902135590*tan(b*x + 4*a)^4*tan(1/2*a)^48 - 18613230840*tan(b*x + 4*a)^3*
tan(1/2*a)^49 + 9508215600*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 1840189110*tan(
b*x + 4*a)*tan(1/2*a)^51 + 108499095*tan(1/2*a)^52 - 53242909200*tan(b*x +
4*a)^4*tan(1/2*a)^46 + 109725269760*tan(b*x + 4*a)^3*tan(1/2*a)^47 - 733098
82260*tan(b*x + 4*a)^2*tan(1/2*a)^48 + 18891597150*tan(b*x + 4*a)*tan(1/2*a
)^49 - 1563175302*tan(1/2*a)^50 + 107646377490*tan(b*x + 4*a)^4*tan(1/2*a)^
44 - 347626321920*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 324785269440*tan(b*x + 4
*a)^2*tan(1/2*a)^46 - 112787150040*tan(b*x + 4*a)*tan(1/2*a)^47 + 126029226
05*tan(1/2*a)^48 + 100499354940*tan(b*x + 4*a)^4*tan(1/2*a)^42 + 2439543517
20*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 614500276860*tan(b*x + 4*a)^2*tan(1/2*a
)^44 + 342870428760*tan(b*x + 4*a)*tan(1/2*a)^45 - 54544776360*tan(1/2*a)^4
6 - 720524857050*tan(b*x + 4*a)^4*tan(1/2*a)^40 + 1552622504040*tan(b*x + 4
*a)^3*tan(1/2*a)^41 - 592670717680*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 2218645
73610*tan(b*x + 4*a)*tan(1/2*a)^43 + 98588733975*tan(1/2*a)^44 + 5458434944
40*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 3402462596640*tan(b*x + 4*a)^3*tan(1/2*
a)^39 + 4208031154860*tan(b*x + 4*a)^2*tan(1/2*a)^40 - 1502042255310*tan(b*
x + 4*a)*tan(1/2*a)^41 + 99097279550*tan(1/2*a)^42 + 1394520156090*tan(b*x
+ 4*a)^4*tan(1/2*a)^36 - 1060535913120*tan(b*x + 4*a)^3*tan(1/2*a)^37 - 339
9607142880*tan(b*x + 4*a)^2*tan(1/2*a)^38 + 3367956274060*tan(b*x + 4*a)*ta
n(1/2*a)^39 - 681790695915*tan(1/2*a)^40 - 2195406541260*tan(b*x + 4*a)^4*t
an(1/2*a)^34 + 8940274641000*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 8188018122860
*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 893802691860*tan(b*x + 4*a)*tan(1/2*a)^37
 + 583218519780*tan(1/2*a)^38 - 793687329810*tan(b*x + 4*a)^4*tan(1/2*a)^32
 - 4368381430680*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 13505898965040*tan(b*x +
4*a)^2*tan(1/2*a)^34 - 8980226310150*tan(b*x + 4*a)*tan(1/2*a)^35 + 1327591
075915*tan(1/2*a)^36 + 3207851661600*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 10136
128700160*tan(b*x + 4*a)^3*tan(1/2*a)^31 + 4668559700220*tan(b*x + 4*a)^2*t
an(1/2*a)^32 + 4628264277390*tan(b*x + 4*a)*tan(1/2*a)^33 - 2309792723910*t
an(1/2*a)^34 - 793687329810*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 10136128700160
*tan(b*x + 4*a)^3*tan(1/2*a)^29 - 19695904506240*tan(b*x + 4*a)^2*tan(1/2*a
)^30 + 10330657752240*tan(b*x + 4*a)*tan(1/2*a)^31 - 756295285575*tan(1/2*a
)^32 - 2195406541260*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 4368381430680*tan(b*x
 + 4*a)^3*tan(1/2*a)^27 + 4668559700220*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 10
330657752240*tan(b*x + 4*a)*tan(1/2*a)^29 + 3368788208080*tan(1/2*a)^30 + 1
394520156090*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 8940274641000*tan(b*x + 4*a)^
3*tan(1/2*a)^25 + 13505898965040*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 462826427
7390*tan(b*x + 4*a)*tan(1/2*a)^27 - 756295285575*tan(1/2*a)^28 + 5458434944
40*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 1060535913120*tan(b*x + 4*a)^3*tan(1/2*
a)^23 - 8188018122860*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 8980226310150*tan(b*
x + 4*a)*tan(1/2*a)^25 - 2309792723910*tan(1/2*a)^26 - 720524857050*tan(b*x
 + 4*a)^4*tan(1/2*a)^20 + 3402462596640*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 33
99607142880*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 893802691860*tan(b*x + 4*a)*ta
n(1/2*a)^23 + 1327591075915*tan(1/2*a)^24 + 100499354940*tan(b*x + 4*a)^4*t
an(1/2*a)^18 - 1552622504040*tan(b*x + 4*a)^3*tan(1/2*a)^19 + 4208031154860
*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 3367956274060*tan(b*x + 4*a)*tan(1/2*a)^2
1 + 583218519780*tan(1/2*a)^22 + 107646377490*tan(b*x + 4*a)^4*tan(1/2*a)^1
6 - 243954351720*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 592670717680*tan(b*x + 4*
a)^2*tan(1/2*a)^18 + 1502042255310*tan(b*x + 4*a)*tan(1/2*a)^19 - 681790695
915*tan(1/2*a)^20 - 53242909200*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 3476263219
20*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 614500276860*tan(b*x + 4*a)^2*tan(1/2*a
)^16 + 221864573610*tan(b*x + 4*a)*tan(1/2*a)^17 + 99097279550*tan(1/2*a)^1
8 + 11902135590*tan(b*x + 4*a)^4*tan(1/2*a)^12 - 109725269760*tan(b*x + 4*a

```
)^3*tan(1/2*a)^13 + 324785269440*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 342870428
760*tan(b*x + 4*a)*tan(1/2*a)^15 + 98588733975*tan(1/2*a)^16 - 1610662860*t
an(b*x + 4*a)^4*tan(1/2*a)^10 + 18613230840*tan(b*x + 4*a)^3*tan(1/2*a)^11
- 73309882260*tan(b*x + 4*a)^2*tan(1/2*a)^12 + 112787150040*tan(b*x + 4*a)*
tan(1/2*a)^13 - 54544776360*tan(1/2*a)^14 + 143179650*tan(b*x + 4*a)^4*tan(
1/2*a)^8 - 1962664200*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 9508215600*tan(b*x +
4*a)^2*tan(1/2*a)^10 - 18891597150*tan(b*x + 4*a)*tan(1/2*a)^11 + 126029226
05*tan(1/2*a)^12 - 8558760*tan(b*x + 4*a)^4*tan(1/2*a)^6 + 133967520*tan(b*
x + 4*a)^3*tan(1/2*a)^7 - 761504220*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1840189
110*tan(b*x + 4*a)*tan(1/2*a)^9 - 1563175302*tan(1/2*a)^10 + 337950*tan(b*x
 + 4*a)^4*tan(1/2*a)^4 - 5856480*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 37650400*t
an(b*x + 4*a)^2*tan(1/2*a)^6 - 105669900*tan(b*x + 4*a)*tan(1/2*a)^7 + 1084
99095*tan(1/2*a)^8 - 8100*tan(b*x + 4*a)^4*tan(1/2*a)^2 + 151800*tan(b*x +
4*a)^3*tan(1/2*a)^3 - 1068900*tan(b*x + 4*a)^2*tan(1/2*a)^4 + 3349740*tan(b
*x + 4*a)*tan(1/2*a)^5 - 3921700*tan(1/2*a)^6 + 90*tan(b*x + 4*a)^4 - 1800*
tan(b*x + 4*a)^3*tan(1/2*a) + 13200*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 44050*t
an(b*x + 4*a)*tan(1/2*a)^3 + 56265*tan(1/2*a)^4 + 20*tan(b*x + 4*a)^2 - 150
*tan(b*x + 4*a)*tan(1/2*a) + 270*tan(1/2*a)^2 + 3)/((tan(1/2*a)^30 - 75*tan
(1/2*a)^28 + 2325*tan(1/2*a)^26 - 38255*tan(1/2*a)^24 + 356925*tan(1/2*a)^2
2 - 1880175*tan(1/2*a)^20 + 5430385*tan(1/2*a)^18 - 9069075*tan(1/2*a)^16 +
 9069075*tan(1/2*a)^14 - 5430385*tan(1/2*a)^12 + 1880175*tan(1/2*a)^10 - 35
6925*tan(1/2*a)^8 + 38255*tan(1/2*a)^6 - 2325*tan(1/2*a)^4 + 75*tan(1/2*a)^
2 - 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*ta
n(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4
*a) + 6*tan(1/2*a))^5))/b
```

maple [A] time = 1.24, size = 87, normalized size = 1.21

$$\frac{-\frac{1}{5 \sin(bx+a)^5 \cos(bx+a)^3}+\frac{8}{15 \sin(bx+a)^3 \cos(bx+a)^3}-\frac{16}{15 \sin(bx+a)^3 \cos(bx+a)}+\frac{64}{15 \sin(bx+a) \cos(bx+a)}-\frac{128 \cot(bx+a)}{15}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x)
```

```
[Out] 1/16/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^3+8/15/sin(b*x+a)^3/cos(b*x+a)^3-16/15
/sin(b*x+a)^3/cos(b*x+a)+64/15/sin(b*x+a)/cos(b*x+a)-128/15*cot(b*x+a))
```

maxima [B] time = 0.37, size = 1227, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 16/15*(2*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(16*
b*x + 16*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
cos(14*b*x + 14*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*cos(12*b*x + 12*a) + 12*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*cos(10*b*x + 10*a) - (6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a
) - 2*cos(2*b*x + 2*a) + 1)*sin(16*b*x + 16*a) + 2*(6*cos(6*b*x + 6*a) - 2*
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(14*b*x + 14*a) + 2*(6*cos(6*
b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(12*b*x + 12*a
) - 6*(6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*si
n(10*b*x + 10*a))/(b*cos(16*b*x + 16*a)^2 + 4*b*cos(14*b*x + 14*a)^2 + 4*b*
cos(12*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(16*b*x + 16*a)^2
+ 4*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x + 12*a)^2 + 36*b*sin(10*b*x + 1
0*a)^2 + 36*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 + 8*b*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(14*b*x + 14*a)
```

```
 + 2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) -
 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) + 4*(2
*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) - 2*b
*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(14*b*x + 14*a) - 4*(6*b*c
os(10*b*x + 10*a) - 6*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) + 2*b*cos(2
*b*x + 2*a) - b)*cos(12*b*x + 12*a) - 12*(6*b*cos(6*b*x + 6*a) - 2*b*cos(4*
b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) - 12*(2*b*cos(4*b
*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(2*b*cos(2*b*x +
 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(14*b*x + 14*a
) + b*sin(12*b*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) -
b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 8*(b*sin(12*b
*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) - b*sin(4*b*x +
4*a) - b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 8*(3*b*sin(10*b*x + 10*a) -
 3*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) - 24*(3*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 24*(b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6
*b*x + 6*a) + b)
```

mupad [B] time = 0.38, size = 55, normalized size = 0.76

$$-\frac{-5\tan\left(a+b\,x\right)^8-60\tan\left(a+b\,x\right)^6+90\tan\left(a+b\,x\right)^4+20\tan\left(a+b\,x\right)^2+3}{240\,b\tan\left(a+b\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^4),x)
```

```
[Out] -(20*tan(a + b*x)^2 + 90*tan(a + b*x)^4 - 60*tan(a + b*x)^6 - 5*tan(a + b*x
)^8 + 3)/(240*b*tan(a + b*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**4, x)
```

3.57 $\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=90

$$\frac{\tan^4(a+bx)}{128b}+\frac{5\tan^2(a+bx)}{64b}-\frac{\cot^6(a+bx)}{192b}-\frac{5\cot^4(a+bx)}{128b}-\frac{5\cot^2(a+bx)}{32b}+\frac{5\log(\tan(a+bx))}{16b}$$

```
[Out] -5/32*cot(b*x+a)^2/b-5/128*cot(b*x+a)^4/b-1/192*cot(b*x+a)^6/b+5/16*ln(tan(
b*x+a))/b+5/64*tan(b*x+a)^2/b+1/128*tan(b*x+a)^4/b
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^4(a+bx)}{128b}+\frac{5\tan^2(a+bx)}{64b}-\frac{\cot^6(a+bx)}{192b}-\frac{5\cot^4(a+bx)}{128b}-\frac{5\cot^2(a+bx)}{32b}+\frac{5\log(\tan(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-5*Cot[a + b*x]^2)/(32*b) - (5*Cot[a + b*x]^4)/(128*b) - Cot[a + b*x]^6/(1
92*b) + (5*Log[Tan[a + b*x]])/(16*b) + (5*Tan[a + b*x]^2)/(64*b) + Tan[a +
b*x]^4/(128*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^7(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^7}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^5}{x^4}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(5+\frac{1}{x^4}+\frac{5}{x^3}+\frac{10}{x^2}+\frac{10}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{5\cot^2(a+bx)}{32b} - \frac{5\cot^4(a+bx)}{128b} - \frac{\cot^6(a+bx)}{192b} + \frac{5\log(\tan(a+bx))}{16b} + \frac{5\,\text{ta}}{}$$

Mathematica [**A**] time = 0.42, size = 76, normalized size = 0.84

$$-\frac{2\csc^6(a+bx)+9\csc^4(a+bx)+36\csc^2(a+bx)-3\sec^4(a+bx)-24\sec^2(a+bx)-120\log(\sin(a+bx))+12}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -1/384*(36*Csc[a + b*x]^2 + 9*Csc[a + b*x]^4 + 2*Csc[a + b*x]^6 + 120*Log[C
os[a + b*x]] - 120*Log[Sin[a + b*x]] - 24*Sec[a + b*x]^2 - 3*Sec[a + b*x]^4
)/b
```

fricas [**B**] time = 0.57, size = 194, normalized size = 2.16

$$\frac{60\cos(bx+a)^8 - 150\cos(bx+a)^6 + 110\cos(bx+a)^4 - 15\cos(bx+a)^2 - 60\left(\cos(bx+a)^{10} - 3\cos(bx+a)^{8}\right.}{384\left(b\cos(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(60*cos(b*x + a)^8 - 150*cos(b*x + a)^6 + 110*cos(b*x + a)^4 - 15*cos
(b*x + a)^2 - 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 - c
os(b*x + a)^4)*log(cos(b*x + a)^2) + 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8
 + 3*cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1/4) - 3)/(
b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*x + a
)^4)
```

giac [**B**] time = 7.13, size = 6374, normalized size = 70.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/6144*((2592*tan(b*x + 4*a)^3*tan(1/2*a)^45 - 648*tan(b*x + 4*a)^2*tan(1/2
*a)^46 + 72*tan(b*x + 4*a)*tan(1/2*a)^47 - 3*tan(1/2*a)^48 + 339552*tan(b*x
 + 4*a)^3*tan(1/2*a)^43 - 131760*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 18120*tan
(b*x + 4*a)*tan(1/2*a)^45 - 720*tan(1/2*a)^46 + 419904000*tan(b*x + 4*a)^4*
tan(1/2*a)^40 - 289600704*tan(b*x + 4*a)^3*tan(1/2*a)^41 + 76838472*tan(b*x
 + 4*a)^2*tan(1/2*a)^42 - 9139608*tan(b*x + 4*a)*tan(1/2*a)^43 + 409140*tan
```

```
(1/2*a)^44 - 11197440000*tan(b*x + 4*a)^4*tan(1/2*a)^38 + 10808414400*tan(b
*x + 4*a)^3*tan(1/2*a)^39 - 3590371872*tan(b*x + 4*a)^2*tan(1/2*a)^40 + 507
604968*tan(b*x + 4*a)*tan(1/2*a)^41 - 26159856*tan(1/2*a)^42 + 133996032000
*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 169676010720*tan(b*x + 4*a)^3*tan(1/2*a)^
37 + 72375177960*tan(b*x + 4*a)^2*tan(1/2*a)^38 - 12560378760*tan(b*x + 4*a
)*tan(1/2*a)^39 + 768063258*tan(1/2*a)^40 - 949294080000*tan(b*x + 4*a)^4*t
an(1/2*a)^34 + 1509418412640*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 810503332080*
tan(b*x + 4*a)^2*tan(1/2*a)^36 + 174786910200*tan(b*x + 4*a)*tan(1/2*a)^37
- 12927376560*tan(1/2*a)^38 + 4424378112000*tan(b*x + 4*a)^4*tan(1/2*a)^32
- 8587351903488*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 5672266009560*tan(b*x + 4*
a)^2*tan(1/2*a)^34 - 1507689425640*tan(b*x + 4*a)*tan(1/2*a)^35 + 136259884
900*tan(1/2*a)^36 - 14266506240000*tan(b*x + 4*a)^4*tan(1/2*a)^30 + 3319683
7729536*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 26480349456768*tan(b*x + 4*a)^2*ta
n(1/2*a)^32 + 8552959192472*tan(b*x + 4*a)*tan(1/2*a)^33 - 941351504400*tan
(1/2*a)^34 + 32626266624000*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 90047346555840
*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 85630491456048*tan(b*x + 4*a)^2*tan(1/2*a
)^30 - 33168296242992*tan(b*x + 4*a)*tan(1/2*a)^31 + 4402067046579*tan(1/2*
a)^32 - 53491430400000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 174200110235840*tan
(b*x + 4*a)^3*tan(1/2*a)^27 - 195958049148000*tan(b*x + 4*a)^2*tan(1/2*a)^2
8 + 90139253610960*tan(b*x + 4*a)*tan(1/2*a)^29 - 14277938470432*tan(1/2*a)
^30 + 63070929280000*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 242033974687872*tan(b
*x + 4*a)^3*tan(1/2*a)^25 + 320941776763728*tan(b*x + 4*a)^2*tan(1/2*a)^26
- 174300870723312*tan(b*x + 4*a)*tan(1/2*a)^27 + 32692804970088*tan(1/2*a)^
28 - 53491430400000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 242033974687872*tan(b*
x + 4*a)^3*tan(1/2*a)^23 - 378128484292800*tan(b*x + 4*a)^2*tan(1/2*a)^24 +
 241898880641040*tan(b*x + 4*a)*tan(1/2*a)^25 - 53487220641120*tan(1/2*a)^2
6 + 32626266624000*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 174200110235840*tan(b*x
 + 4*a)^3*tan(1/2*a)^21 + 320941776763728*tan(b*x + 4*a)^2*tan(1/2*a)^22 -
241898880641040*tan(b*x + 4*a)*tan(1/2*a)^23 + 62975177889900*tan(1/2*a)^24
 - 14266506240000*tan(b*x + 4*a)^4*tan(1/2*a)^18 + 90047346555840*tan(b*x +
 4*a)^3*tan(1/2*a)^19 - 195958049148000*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17
4300870723312*tan(b*x + 4*a)*tan(1/2*a)^21 - 53487220641120*tan(1/2*a)^22 +
 4424378112000*tan(b*x + 4*a)^4*tan(1/2*a)^16 - 33196837729536*tan(b*x + 4*
a)^3*tan(1/2*a)^17 + 85630491456048*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 901392
53610960*tan(b*x + 4*a)*tan(1/2*a)^19 + 32692804970088*tan(1/2*a)^20 - 9492
94080000*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 8587351903488*tan(b*x + 4*a)^3*ta
n(1/2*a)^15 - 26480349456768*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 3316829624299
2*tan(b*x + 4*a)*tan(1/2*a)^17 - 14277938470432*tan(1/2*a)^18 + 13399603200
0*tan(b*x + 4*a)^4*tan(1/2*a)^12 - 1509418412640*tan(b*x + 4*a)^3*tan(1/2*a
)^13 + 5672266009560*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 8552959192472*tan(b*x
 + 4*a)*tan(1/2*a)^15 + 4402067046579*tan(1/2*a)^16 - 11197440000*tan(b*x +
 4*a)^4*tan(1/2*a)^10 + 169676010720*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 81050
3332080*tan(b*x + 4*a)^2*tan(1/2*a)^12 + 1507689425640*tan(b*x + 4*a)*tan(1
/2*a)^13 - 941351504400*tan(1/2*a)^14 + 419904000*tan(b*x + 4*a)^4*tan(1/2*
a)^8 - 10808414400*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 72375177960*tan(b*x + 4*
a)^2*tan(1/2*a)^10 - 174786910200*tan(b*x + 4*a)*tan(1/2*a)^11 + 1362598849
00*tan(1/2*a)^12 + 289600704*tan(b*x + 4*a)^3*tan(1/2*a)^7 - 3590371872*tan
(b*x + 4*a)^2*tan(1/2*a)^8 + 12560378760*tan(b*x + 4*a)*tan(1/2*a)^9 - 1292
7376560*tan(1/2*a)^10 - 339552*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 76838472*tan
(b*x + 4*a)^2*tan(1/2*a)^6 - 507604968*tan(b*x + 4*a)*tan(1/2*a)^7 + 768063
258*tan(1/2*a)^8 - 2592*tan(b*x + 4*a)^3*tan(1/2*a)^3 - 131760*tan(b*x + 4*
a)^2*tan(1/2*a)^4 + 9139608*tan(b*x + 4*a)*tan(1/2*a)^5 - 26159856*tan(1/2*
a)^6 - 648*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 18120*tan(b*x + 4*a)*tan(1/2*a)^
3 + 409140*tan(1/2*a)^4 - 72*tan(b*x + 4*a)*tan(1/2*a) - 720*tan(1/2*a)^2 -
 3)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*tan(1/2*a)^16 - 15240*ta
n(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^10 + 5724*tan(1/2*a)^8
 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - ta
n(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x +
 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 16*(294*tan(b*x + 4*a)^6*tan(1
```

```
/2*a)^72 - 52920*tan(b*x + 4*a)^6*tan(1/2*a)^70 + 9864*tan(b*x + 4*a)^5*tan
(1/2*a)^71 + 60*tan(b*x + 4*a)^4*tan(1/2*a)^72 + 4418820*tan(b*x + 4*a)^6*t
an(1/2*a)^68 - 1699320*tan(b*x + 4*a)^5*tan(1/2*a)^69 + 133920*tan(b*x + 4*
a)^4*tan(1/2*a)^70 + 1080*tan(b*x + 4*a)^3*tan(1/2*a)^71 + 15*tan(b*x + 4*a
)^2*tan(1/2*a)^72 - 227030328*tan(b*x + 4*a)^6*tan(1/2*a)^66 + 134207352*ta
n(b*x + 4*a)^5*tan(1/2*a)^67 - 22587480*tan(b*x + 4*a)^4*tan(1/2*a)^68 + 96
3000*tan(b*x + 4*a)^3*tan(1/2*a)^69 + 7020*tan(b*x + 4*a)^2*tan(1/2*a)^70 +
 108*tan(b*x + 4*a)*tan(1/2*a)^71 + 2*tan(1/2*a)^72 + 8027289270*tan(b*x +
4*a)^6*tan(1/2*a)^64 - 6460519176*tan(b*x + 4*a)^5*tan(1/2*a)^65 + 16863489
60*tan(b*x + 4*a)^4*tan(1/2*a)^66 - 159789240*tan(b*x + 4*a)^3*tan(1/2*a)^6
7 + 3912570*tan(b*x + 4*a)^2*tan(1/2*a)^68 + 20700*tan(b*x + 4*a)*tan(1/2*a
)^69 + 180*tan(1/2*a)^70 - 206987901120*tan(b*x + 4*a)^6*tan(1/2*a)^62 + 21
2284461120*tan(b*x + 4*a)^5*tan(1/2*a)^63 - 75682808100*tan(b*x + 4*a)^4*ta
n(1/2*a)^64 + 11222034120*tan(b*x + 4*a)^3*tan(1/2*a)^65 - 635709780*tan(b*
x + 4*a)^2*tan(1/2*a)^66 + 8559972*tan(b*x + 4*a)*tan(1/2*a)^67 + 23220*tan
(1/2*a)^68 + 4024901399904*tan(b*x + 4*a)^6*tan(1/2*a)^60 - 5048602582464*t
an(b*x + 4*a)^5*tan(1/2*a)^61 + 2296406448000*tan(b*x + 4*a)^4*tan(1/2*a)^6
2 - 466347413440*tan(b*x + 4*a)^3*tan(1/2*a)^63 + 41675854455*tan(b*x + 4*a
)^2*tan(1/2*a)^64 - 1349403948*tan(b*x + 4*a)*tan(1/2*a)^65 + 7900020*tan(1
/2*a)^66 - 60212793009600*tan(b*x + 4*a)^6*tan(1/2*a)^58 + 89880504373440*t
an(b*x + 4*a)^5*tan(1/2*a)^59 - 50018473204800*tan(b*x + 4*a)^4*tan(1/2*a)^
60 + 12967578538560*tan(b*x + 4*a)^3*tan(1/2*a)^61 - 1590087843360*tan(b*x
+ 4*a)^2*tan(1/2*a)^62 + 81753694560*tan(b*x + 4*a)*tan(1/2*a)^63 - 1193590
350*tan(1/2*a)^64 + 701137434828600*tan(b*x + 4*a)^6*tan(1/2*a)^56 - 122215
5524626752*tan(b*x + 4*a)^5*tan(1/2*a)^57 + 809309015383680*tan(b*x + 4*a)^
4*tan(1/2*a)^58 - 256626846135360*tan(b*x + 4*a)^3*tan(1/2*a)^59 + 40162301
142000*tan(b*x + 4*a)^2*tan(1/2*a)^60 - 2834816578848*tan(b*x + 4*a)*tan(1/
2*a)^61 + 66006729504*tan(1/2*a)^62 - 6393695038795680*tan(b*x + 4*a)^6*tan
(1/2*a)^54 + 12843489105103968*tan(b*x + 4*a)^5*tan(1/2*a)^55 - 99287277370
10640*tan(b*x + 4*a)^4*tan(1/2*a)^56 + 3743548943593920*tan(b*x + 4*a)^3*ta
n(1/2*a)^57 - 715723706535840*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 643798023336
00*tan(b*x + 4*a)*tan(1/2*a)^59 - 2055305635104*tan(1/2*a)^60 + 45773543210
194800*tan(b*x + 4*a)^6*tan(1/2*a)^52 - 105001341586308000*tan(b*x + 4*a)^5
*tan(1/2*a)^53 + 93508763139878400*tan(b*x + 4*a)^4*tan(1/2*a)^54 - 4111996
0482804960*tan(b*x + 4*a)^3*tan(1/2*a)^55 + 9332822656724940*tan(b*x + 4*a)
^2*tan(1/2*a)^56 - 1023101781887712*tan(b*x + 4*a)*tan(1/2*a)^57 + 41487214
960800*tan(1/2*a)^58 - 257401775291824800*tan(b*x + 4*a)^6*tan(1/2*a)^50 +
670176117430046880*tan(b*x + 4*a)^5*tan(1/2*a)^51 - 681255358380496800*tan(
b*x + 4*a)^4*tan(1/2*a)^52 + 344752124474645280*tan(b*x + 4*a)^3*tan(1/2*a)
^53 - 91096096904718960*tan(b*x + 4*a)^2*tan(1/2*a)^54 + 11822570313904656*
tan(b*x + 4*a)*tan(1/2*a)^55 - 581936260957848*tan(1/2*a)^56 + 113717503913
0480280*tan(b*x + 4*a)^6*tan(1/2*a)^48 - 3346975359162280800*tan(b*x + 4*a)
^5*tan(1/2*a)^49 + 3859521230041977600*tan(b*x + 4*a)^4*tan(1/2*a)^50 - 222
7011325606075680*tan(b*x + 4*a)^3*tan(1/2*a)^51 + 675970919512787160*tan(b*
x + 4*a)^2*tan(1/2*a)^52 - 101853310806506160*tan(b*x + 4*a)*tan(1/2*a)^53
+ 5911352313614320*tan(1/2*a)^54 - 3952573715797761600*tan(b*x + 4*a)^6*tan
(1/2*a)^46 + 13113192238126665408*tan(b*x + 4*a)^5*tan(1/2*a)^47 - 17080215
895273086480*tan(b*x + 4*a)^4*tan(1/2*a)^48 + 11166956763854634720*tan(b*x
+ 4*a)^3*tan(1/2*a)^49 - 3858112452887270640*tan(b*x + 4*a)^2*tan(1/2*a)^50
 + 666075670217316912*tan(b*x + 4*a)*tan(1/2*a)^51 - 44716273127011152*tan(
1/2*a)^52 + 10845567549201117600*tan(b*x + 4*a)^6*tan(1/2*a)^44 - 404584101
19612759872*tan(b*x + 4*a)^5*tan(1/2*a)^45 + 59335425213582061440*tan(b*x +
 4*a)^4*tan(1/2*a)^46 - 43761763235235147840*tan(b*x + 4*a)^3*tan(1/2*a)^47
 + 17102501830780473660*tan(b*x + 4*a)^2*tan(1/2*a)^48 - 335317836314927016
0*tan(b*x + 4*a)*tan(1/2*a)^49 + 257119896646635120*tan(1/2*a)^50 - 2360665
5837981910848*tan(b*x + 4*a)^6*tan(1/2*a)^42 + 98782934174733211200*tan(b*x
 + 4*a)^5*tan(1/2*a)^43 - 162685440260092255680*tan(b*x + 4*a)^4*tan(1/2*a)
^44 + 134910442121291395520*tan(b*x + 4*a)^3*tan(1/2*a)^45 - 59381984929519
133280*tan(b*x + 4*a)^2*tan(1/2*a)^46 + 13143687759024316704*tan(b*x + 4*a)
```

*tan(1/2*a)^47 - 1141633895066211960*tan(1/2*a)^48 + 40965300945895635540*t
an(b*x + 4*a)^6*tan(1/2*a)^40 - 19186209494269693228 8*tan(b*x + 4*a)^5*tan(
1/2*a)^41 + 35399380342832325696 0*tan(b*x + 4*a)^4*tan(1/2*a)^42 - 32920769
8514963712960*tan(b*x + 4*a)^3*tan(1/2*a)^43 + 162688248623415050640*tan(b*
x + 4*a)^2*tan(1/2*a)^44 - 40487960376937634400*tan(b*x + 4*a)*tan(1/2*a)^4
5 + 3961883425271202144*tan(1/2*a)^46 - 56916645144484965840*tan(b*x + 4*a)
^6*tan(1/2*a)^38 + 297814271756856799920*tan(b*x + 4*a)^5*tan(1/2*a)^39 - 6
14369141282401310520*tan(b*x + 4*a)^4*tan(1/2*a)^40 + 639362819497412266560
*tan(b*x + 4*a)^3*tan(1/2*a)^41 - 353888122537448556000*tan(b*x + 4*a)^2*ta
n(1/2*a)^42 + 98742097283990220000*tan(b*x + 4*a)*tan(1/2*a)^43 - 108461282
70028623840*tan(1/2*a)^44 + 63490416681595775256*tan(b*x + 4*a)^6*tan(1/2*a
)^36 - 370709550220416445008*tan(b*x + 4*a)^5*tan(1/2*a)^37 + 8538128885868
53302080*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 992663809176983006640*tan(b*x + 4
*a)^3*tan(1/2*a)^39 + 614257138968343735410*tan(b*x + 4*a)^2*tan(1/2*a)^40
- 191755286425161694368*tan(b*x + 4*a)*tan(1/2*a)^41 + 23585525195746656480
*tan(1/2*a)^42 - 56916645144484965840*tan(b*x + 4*a)^6*tan(1/2*a)^34 + 3707
09550220416445008*tan(b*x + 4*a)^5*tan(1/2*a)^35 - 952539854421972061200*ta
n(b*x + 4*a)^4*tan(1/2*a)^36 + 1235898837202727952720*tan(b*x + 4*a)^3*tan(
1/2*a)^37 - 853875925497859030200*tan(b*x + 4*a)^2*tan(1/2*a)^38 + 29778341
9266886959560*tan(b*x + 4*a)*tan(1/2*a)^39 - 40942896085716296868*tan(1/2*a
)^40 + 40965300945895635540*tan(b*x + 4*a)^6*tan(1/2*a)^32 - 29781427175685
6799920*tan(b*x + 4*a)^5*tan(1/2*a)^33 + 853812888586853302080*tan(b*x + 4*
a)^4*tan(1/2*a)^34 - 1235898837202727952720*tan(b*x + 4*a)^3*tan(1/2*a)^35
+ 952725450920718220380*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 370830295637238771
864*tan(b*x + 4*a)*tan(1/2*a)^37 + 56929249131035815800*tan(1/2*a)^38 - 236
06655837981910848*tan(b*x + 4*a)^6*tan(1/2*a)^30 + 191862094942696932288*ta
n(b*x + 4*a)^5*tan(1/2*a)^31 - 614369141282401310520*tan(b*x + 4*a)^4*tan(1
/2*a)^32 + 992663809176983006640*tan(b*x + 4*a)^3*tan(1/2*a)^33 - 853875925
497859030200*tan(b*x + 4*a)^2*tan(1/2*a)^34 + 370830295637238771864*tan(b*x
 + 4*a)*tan(1/2*a)^35 - 63527544212529374408*tan(1/2*a)^36 + 10845567549201
117600*tan(b*x + 4*a)^6*tan(1/2*a)^28 - 98782934174733211200*tan(b*x + 4*a)
^5*tan(1/2*a)^29 + 353993803428323256960*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 6
39362819497412266560*tan(b*x + 4*a)^3*tan(1/2*a)^31 + 614257138968343735410
*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 297783419266886959560*tan(b*x + 4*a)*tan(
1/2*a)^33 + 56929249131035815800*tan(1/2*a)^34 - 3952573715797761600*tan(b*
x + 4*a)^6*tan(1/2*a)^26 + 40458410119612759872*tan(b*x + 4*a)^5*tan(1/2*a)
^27 - 162685440260092255680*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 32920769851496
3712960*tan(b*x + 4*a)^3*tan(1/2*a)^29 - 353888122537448556000*tan(b*x + 4*
a)^2*tan(1/2*a)^30 + 191755286425161694368*tan(b*x + 4*a)*tan(1/2*a)^31 - 4
0942896085716296868*tan(1/2*a)^32 + 1137175039130480280*tan(b*x + 4*a)^6*ta
n(1/2*a)^24 - 13113192238126665408*tan(b*x + 4*a)^5*tan(1/2*a)^25 + 5933542
5213582061440*tan(b*x + 4*a)^4*tan(1/2*a)^26 - 134910442121291395520*tan(b*
x + 4*a)^3*tan(1/2*a)^27 + 162688248623415050640*tan(b*x + 4*a)^2*tan(1/2*a
)^28 - 98742097283990220000*tan(b*x + 4*a)*tan(1/2*a)^29 + 2358552519574665
6480*tan(1/2*a)^30 - 257401775291824800*tan(b*x + 4*a)^6*tan(1/2*a)^22 + 33
46975359162280800*tan(b*x + 4*a)^5*tan(1/2*a)^23 - 17080215895273086480*tan
(b*x + 4*a)^4*tan(1/2*a)^24 + 43761763235235147840*tan(b*x + 4*a)^3*tan(1/2
*a)^25 - 59381984929519133280*tan(b*x + 4*a)^2*tan(1/2*a)^26 + 404879603769
37634400*tan(b*x + 4*a)*tan(1/2*a)^27 - 10846128270028623840*tan(1/2*a)^28
+ 45773543210194800*tan(b*x + 4*a)^6*tan(1/2*a)^20 - 670176117430046880*tan
(b*x + 4*a)^5*tan(1/2*a)^21 + 3859521230041977600*tan(b*x + 4*a)^4*tan(1/2*
a)^22 - 11166956763854634720*tan(b*x + 4*a)^3*tan(1/2*a)^23 + 1710250183078
0473660*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 13143687759024316704*tan(b*x + 4*a
)*tan(1/2*a)^25 + 3961883425271202144*tan(1/2*a)^26 - 6393695038795680*tan(
b*x + 4*a)^6*tan(1/2*a)^18 + 105001341586308000*tan(b*x + 4*a)^5*tan(1/2*a)
^19 - 681255358380496800*tan(b*x + 4*a)^4*tan(1/2*a)^20 + 22270113256060756
80*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 3858112452887270640*tan(b*x + 4*a)^2*ta
n(1/2*a)^22 + 3353178363149270160*tan(b*x + 4*a)*tan(1/2*a)^23 - 1141633895
066211960*tan(1/2*a)^24 + 701137434828600*tan(b*x + 4*a)^6*tan(1/2*a)^16 -

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12843489105103968*tan(b*x + 4*a)^5*tan(1/2*a)^17 + 93508763139878400*tan(b*
x + 4*a)^4*tan(1/2*a)^18 - 344752124474645280*tan(b*x + 4*a)^3*tan(1/2*a)^1
9 + 675970919512787160*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 666075670217316912*
tan(b*x + 4*a)*tan(1/2*a)^21 + 257119896646635120*tan(1/2*a)^22 - 602127930
09600*tan(b*x + 4*a)^6*tan(1/2*a)^14 + 1222155524626752*tan(b*x + 4*a)^5*ta
n(1/2*a)^15 - 9928727737010640*tan(b*x + 4*a)^4*tan(1/2*a)^16 + 41119960482
804960*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 91096096904718960*tan(b*x + 4*a)^2*
tan(1/2*a)^18 + 101853310806506160*tan(b*x + 4*a)*tan(1/2*a)^19 - 447162731
27011152*tan(1/2*a)^20 + 4024901399904*tan(b*x + 4*a)^6*tan(1/2*a)^12 - 898
80504373440*tan(b*x + 4*a)^5*tan(1/2*a)^13 + 809309015383680*tan(b*x + 4*a)
^4*tan(1/2*a)^14 - 3743548943593920*tan(b*x + 4*a)^3*tan(1/2*a)^15 + 933282
2656724940*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 11822570313904656*tan(b*x + 4*a
)*tan(1/2*a)^17 + 5911352313614320*tan(1/2*a)^18 - 206987901120*tan(b*x + 4
*a)^6*tan(1/2*a)^10 + 5048602582464*tan(b*x + 4*a)^5*tan(1/2*a)^11 - 500184
73204800*tan(b*x + 4*a)^4*tan(1/2*a)^12 + 256626846135360*tan(b*x + 4*a)^3*
tan(1/2*a)^13 - 715723706535840*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 1023101781
887712*tan(b*x + 4*a)*tan(1/2*a)^15 - 581936260957848*tan(1/2*a)^16 + 80272
89270*tan(b*x + 4*a)^6*tan(1/2*a)^8 - 212284461120*tan(b*x + 4*a)^5*tan(1/2
*a)^9 + 2296406448000*tan(b*x + 4*a)^4*tan(1/2*a)^10 - 12967578538560*tan(b
*x + 4*a)^3*tan(1/2*a)^11 + 40162301142000*tan(b*x + 4*a)^2*tan(1/2*a)^12 -
 64379802333600*tan(b*x + 4*a)*tan(1/2*a)^13 + 41487214960800*tan(1/2*a)^14
 - 227030328*tan(b*x + 4*a)^6*tan(1/2*a)^6 + 6460519176*tan(b*x + 4*a)^5*ta
n(1/2*a)^7 - 75682808100*tan(b*x + 4*a)^4*tan(1/2*a)^8 + 466347413440*tan(b
*x + 4*a)^3*tan(1/2*a)^9 - 1590087843360*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 2
834816578848*tan(b*x + 4*a)*tan(1/2*a)^11 - 2055305635104*tan(1/2*a)^12 + 4
418820*tan(b*x + 4*a)^6*tan(1/2*a)^4 - 134207352*tan(b*x + 4*a)^5*tan(1/2*a
)^5 + 1686348960*tan(b*x + 4*a)^4*tan(1/2*a)^6 - 11222034120*tan(b*x + 4*a)
^3*tan(1/2*a)^7 + 41675854455*tan(b*x + 4*a)^2*tan(1/2*a)^8 - 81753694560*t
an(b*x + 4*a)*tan(1/2*a)^9 + 66006729504*tan(1/2*a)^10 - 52920*tan(b*x + 4*
a)^6*tan(1/2*a)^2 + 1699320*tan(b*x + 4*a)^5*tan(1/2*a)^3 - 22587480*tan(b*
x + 4*a)^4*tan(1/2*a)^4 + 159789240*tan(b*x + 4*a)^3*tan(1/2*a)^5 - 6357097
80*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 1349403948*tan(b*x + 4*a)*tan(1/2*a)^7 -
 1193590350*tan(1/2*a)^8 + 294*tan(b*x + 4*a)^6 - 9864*tan(b*x + 4*a)^5*tan
(1/2*a) + 133920*tan(b*x + 4*a)^4*tan(1/2*a)^2 - 963000*tan(b*x + 4*a)^3*ta
n(1/2*a)^3 + 3912570*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 8559972*tan(b*x + 4*a)
*tan(1/2*a)^5 + 7900020*tan(1/2*a)^6 + 60*tan(b*x + 4*a)^4 - 1080*tan(b*x +
 4*a)^3*tan(1/2*a) + 7020*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 20700*tan(b*x + 4
*a)*tan(1/2*a)^3 + 23220*tan(1/2*a)^4 + 15*tan(b*x + 4*a)^2 - 108*tan(b*x +
 4*a)*tan(1/2*a) + 180*tan(1/2*a)^2 + 2)/((tan(1/2*a)^36 - 90*tan(1/2*a)^34
 + 3465*tan(1/2*a)^32 - 74256*tan(1/2*a)^30 + 965700*tan(1/2*a)^28 - 781020
0*tan(1/2*a)^26 + 39025140*tan(1/2*a)^24 - 119084400*tan(1/2*a)^22 + 228441
150*tan(1/2*a)^20 - 282933020*tan(1/2*a)^18 + 228441150*tan(1/2*a)^16 - 119
084400*tan(1/2*a)^14 + 39025140*tan(1/2*a)^12 - 7810200*tan(1/2*a)^10 + 965
700*tan(1/2*a)^8 - 74256*tan(1/2*a)^6 + 3465*tan(1/2*a)^4 - 90*tan(1/2*a)^2
 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan
(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*
a) + 6*tan(1/2*a))^6) + 1920*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b
*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 -
20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 1920*log(abs(6*tan(b*x
+ 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*ta
n(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

maple [A] time = 0.84, size = 111, normalized size = 1.23

$$-\frac{1}{192b\sin(bx+a)^6\cos(bx+a)^4}+\frac{5}{384b\sin(bx+a)^4\cos(bx+a)^4}-\frac{5}{192b\sin(bx+a)^4\cos(bx+a)^2}+\frac{}{64b\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x)
```

```
[Out] -1/192/b/sin(b*x+a)^6/cos(b*x+a)^4+5/384/b/sin(b*x+a)^4/cos(b*x+a)^4-5/192/
b/sin(b*x+a)^4/cos(b*x+a)^2+5/64/b/sin(b*x+a)^2/cos(b*x+a)^2-5/32/b/sin(b*x
+a)^2+5/16*ln(tan(b*x+a))/b
```

maxima [**B**] time = 0.72, size = 7650, normalized size = 85.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*cos(18*b*x + 18*a) - 30*cos(16*b*x + 16*a) - 40*cos(14*b*x + 14
*a) + 110*cos(12*b*x + 12*a) + 18*cos(10*b*x + 10*a) + 110*cos(8*b*x + 8*a)
 - 40*cos(6*b*x + 6*a) - 30*cos(4*b*x + 4*a) + 15*cos(2*b*x + 2*a))*cos(20*
b*x + 20*a) + 4*(15*cos(16*b*x + 16*a) + 200*cos(14*b*x + 14*a) - 190*cos(1
2*b*x + 12*a) - 216*cos(10*b*x + 10*a) - 190*cos(8*b*x + 8*a) + 200*cos(6*b
*x + 6*a) + 15*cos(4*b*x + 4*a) - 60*cos(2*b*x + 2*a) + 15)*cos(18*b*x + 18
*a) - 120*cos(18*b*x + 18*a)^2 - 12*(40*cos(14*b*x + 14*a) + 130*cos(12*b*x
 + 12*a) - 102*cos(10*b*x + 10*a) + 130*cos(8*b*x + 8*a) + 40*cos(6*b*x + 6
*a) - 60*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 10)*cos(16*b*x + 16*a) + 3
60*cos(16*b*x + 16*a)^2 + 32*(100*cos(12*b*x + 12*a) + 78*cos(10*b*x + 10*a
) + 100*cos(8*b*x + 8*a) - 80*cos(6*b*x + 6*a) - 15*cos(4*b*x + 4*a) + 25*c
os(2*b*x + 2*a) - 5)*cos(14*b*x + 14*a) - 1280*cos(14*b*x + 14*a)^2 - 8*(64
2*cos(10*b*x + 10*a) - 220*cos(8*b*x + 8*a) - 400*cos(6*b*x + 6*a) + 195*co
s(4*b*x + 4*a) + 95*cos(2*b*x + 2*a) - 55)*cos(12*b*x + 12*a) + 880*cos(12*
b*x + 12*a)^2 - 24*(214*cos(8*b*x + 8*a) - 104*cos(6*b*x + 6*a) - 51*cos(4*
b*x + 4*a) + 36*cos(2*b*x + 2*a) - 3)*cos(10*b*x + 10*a) - 864*cos(10*b*x +
 10*a)^2 + 40*(80*cos(6*b*x + 6*a) - 39*cos(4*b*x + 4*a) - 19*cos(2*b*x + 2
*a) + 11)*cos(8*b*x + 8*a) + 880*cos(8*b*x + 8*a)^2 - 160*(3*cos(4*b*x + 4*
a) - 5*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 1280*cos(6*b*x + 6*a)^2 + 6
0*(cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a) + 360*cos(4*b*x + 4*a)^2 - 120*co
s(2*b*x + 2*a)^2 + 15*(2*(2*cos(18*b*x + 18*a) + 3*cos(16*b*x + 16*a) - 8*c
os(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(8*
b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4*(3*cos(16*b*x + 16*a) - 8
*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(
8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a)
 - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x + 18*a)^2 + 6*(8*cos(14*b*x + 14*a)
 + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*co
s(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(16*b*x +
16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*cos(12*b*x + 12*a) - 12*cos(10*b*x +
 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a) - 64*cos(14*b*x + 14*a)^2 + 4*(12*c
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - 4*cos(12*b*x + 12*a)^2
 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos
(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 144*cos(10*b*x + 10*a)^2 - 4*(8*cos
(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*
a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1
)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 6*(2*cos(2*b*x + 2*a) - 1)*cos
(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(18*b
*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 1
2*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*
sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(20*b*x + 20*a) - sin(20*b*x + 20
*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 12*
a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*si
n(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 4*sin(18*b*x + 18
*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10
*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)^2 - 16*(2*sin(12*b*
```

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x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a)
 - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 64*sin(14*
b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x
 + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 4*s
in(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 144*sin(10*b*x + 10*
a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 + 16*(3*sin(4*b*x + 4*a) + 2*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 -
12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x +
 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*
x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - 15*(2*(2*cos(18*b*x + 18*a) +
3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos
(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4
*(3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*c
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x + 18*a)^2
 + 6*(8*cos(14*b*x + 14*a) + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) +
 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x
 + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*cos(12*b*x
 + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a)
- 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a) - 64*cos(
14*b*x + 14*a)^2 + 4*(12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*
b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a
) - 4*cos(12*b*x + 12*a)^2 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) -
3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 144*cos(1
0*b*x + 10*a)^2 - 4*(8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x
+ 2*a) + 1)*cos(8*b*x + 8*a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4*b*x + 4*a
) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 6*(2
*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 + 2*(2*sin(18*b*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*sin(14*b*x
+ 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a)
 - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(20*b*x
 + 20*a) - sin(20*b*x + 20*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin(14*b*x +
14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) -
 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(18*b*x +
 18*a) - 4*sin(18*b*x + 18*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin(12*b*x +
12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3
*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x +
 16*a)^2 - 16*(2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x +
 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(1
4*b*x + 14*a) - 64*sin(14*b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a) - 2*sin(
8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a)
)*sin(12*b*x + 12*a) - 4*sin(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x + 8*a) + 8*
sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10
*a) - 144*sin(10*b*x + 10*a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a)
 - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 + 16*(3*sin(
4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*x + 6*a)^2
 - 9*sin(4*b*x + 4*a)^2 - 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*
x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(2*cos(18*b*x
+ 18*a) + 3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a
) + 12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos
(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(20*b*x + 20*a) - cos(20*b*x + 2
0*a)^2 - 4*(3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12
*a) + 12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x
 + 18*a)^2 + 6*(8*cos(14*b*x + 14*a) + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x
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 + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2
*cos(2*b*x + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*
cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b
*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a)
 - 64*cos(14*b*x + 14*a)^2 + 4*(12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a)
- 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*
b*x + 12*a) - 4*cos(12*b*x + 12*a)^2 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x
 + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) -
 144*cos(10*b*x + 10*a)^2 - 4*(8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*
cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4
*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a
)^2 - 6*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 -
4*cos(2*b*x + 2*a)^2 + 2*(2*sin(18*b*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*s
in(14*b*x + 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*
b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*
sin(20*b*x + 20*a) - sin(20*b*x + 20*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin
(14*b*x + 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*
x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*si
n(18*b*x + 18*a) - 4*sin(18*b*x + 18*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin
(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x
+ 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*si
n(16*b*x + 16*a)^2 - 16*(2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*s
in(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2
*a))*sin(14*b*x + 14*a) - 64*sin(14*b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a
) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*
b*x + 2*a))*sin(12*b*x + 12*a) - 4*sin(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x +
 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(1
0*b*x + 10*a) - 144*sin(10*b*x + 10*a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*
b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 +
16*(3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*
x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(15*sin(
18*b*x + 18*a) - 30*sin(16*b*x + 16*a) - 40*sin(14*b*x + 14*a) + 110*sin(12
*b*x + 12*a) + 18*sin(10*b*x + 10*a) + 110*sin(8*b*x + 8*a) - 40*sin(6*b*x
+ 6*a) - 30*sin(4*b*x + 4*a) + 15*sin(2*b*x + 2*a))*sin(20*b*x + 20*a) + 4*
(15*sin(16*b*x + 16*a) + 200*sin(14*b*x + 14*a) - 190*sin(12*b*x + 12*a) -
216*sin(10*b*x + 10*a) - 190*sin(8*b*x + 8*a) + 200*sin(6*b*x + 6*a) + 15*s
in(4*b*x + 4*a) - 60*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 120*sin(18*b*x
+ 18*a)^2 - 12*(40*sin(14*b*x + 14*a) + 130*sin(12*b*x + 12*a) - 102*sin(10
*b*x + 10*a) + 130*sin(8*b*x + 8*a) + 40*sin(6*b*x + 6*a) - 60*sin(4*b*x +
4*a) - 5*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 360*sin(16*b*x + 16*a)^2 +
32*(100*sin(12*b*x + 12*a) + 78*sin(10*b*x + 10*a) + 100*sin(8*b*x + 8*a) -
 80*sin(6*b*x + 6*a) - 15*sin(4*b*x + 4*a) + 25*sin(2*b*x + 2*a))*sin(14*b*
x + 14*a) - 1280*sin(14*b*x + 14*a)^2 - 8*(642*sin(10*b*x + 10*a) - 220*sin
(8*b*x + 8*a) - 400*sin(6*b*x + 6*a) + 195*sin(4*b*x + 4*a) + 95*sin(2*b*x
+ 2*a))*sin(12*b*x + 12*a) + 880*sin(12*b*x + 12*a)^2 - 24*(214*sin(8*b*x +
 8*a) - 104*sin(6*b*x + 6*a) - 51*sin(4*b*x + 4*a) + 36*sin(2*b*x + 2*a))*s
in(10*b*x + 10*a) - 864*sin(10*b*x + 10*a)^2 + 40*(80*sin(6*b*x + 6*a) - 39
*sin(4*b*x + 4*a) - 19*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 880*sin(8*b*x +
 8*a)^2 - 160*(3*sin(4*b*x + 4*a) - 5*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
1280*sin(6*b*x + 6*a)^2 + 360*sin(4*b*x + 4*a)^2 + 60*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 120*sin(2*b*x + 2*a)^2 + 60*cos(2*b*x + 2*a))/(b*cos(20*b*x
+ 20*a)^2 + 4*b*cos(18*b*x + 18*a)^2 + 9*b*cos(16*b*x + 16*a)^2 + 64*b*cos(
14*b*x + 14*a)^2 + 4*b*cos(12*b*x + 12*a)^2 + 144*b*cos(10*b*x + 10*a)^2 +
4*b*cos(8*b*x + 8*a)^2 + 64*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 +
 4*b*cos(2*b*x + 2*a)^2 + b*sin(20*b*x + 20*a)^2 + 4*b*sin(18*b*x + 18*a)^2
 + 9*b*sin(16*b*x + 16*a)^2 + 64*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x +
12*a)^2 + 144*b*sin(10*b*x + 10*a)^2 + 4*b*sin(8*b*x + 8*a)^2 + 64*b*sin(6*
```

```
b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 + 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2
*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(18*b*x + 18*a) + 3*b*cos(16*b*x +
 16*a) - 8*b*cos(14*b*x + 14*a) - 2*b*cos(12*b*x + 12*a) + 12*b*cos(10*b*x
+ 10*a) - 2*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a
) + 2*b*cos(2*b*x + 2*a) - b)*cos(20*b*x + 20*a) + 4*(3*b*cos(16*b*x + 16*a
) - 8*b*cos(14*b*x + 14*a) - 2*b*cos(12*b*x + 12*a) + 12*b*cos(10*b*x + 10*
a) - 2*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + 2
*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 6*(8*b*cos(14*b*x + 14*a) + 2
*b*cos(12*b*x + 12*a) - 12*b*cos(10*b*x + 10*a) + 2*b*cos(8*b*x + 8*a) + 8*
b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(1
6*b*x + 16*a) + 16*(2*b*cos(12*b*x + 12*a) - 12*b*cos(10*b*x + 10*a) + 2*b*
cos(8*b*x + 8*a) + 8*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*
b*x + 2*a) + b)*cos(14*b*x + 14*a) - 4*(12*b*cos(10*b*x + 10*a) - 2*b*cos(8
*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
 2*a) - b)*cos(12*b*x + 12*a) - 24*(2*b*cos(8*b*x + 8*a) + 8*b*cos(6*b*x +
6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a)
+ 4*(8*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b
)*cos(8*b*x + 8*a) - 16*(3*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*c
os(6*b*x + 6*a) + 6*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2
*b*x + 2*a) - 2*(2*b*sin(18*b*x + 18*a) + 3*b*sin(16*b*x + 16*a) - 8*b*sin(
14*b*x + 14*a) - 2*b*sin(12*b*x + 12*a) + 12*b*sin(10*b*x + 10*a) - 2*b*sin
(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(20*b*x + 20*a) + 4*(3*b*sin(16*b*x + 16*a) - 8*b*sin(14*b*x +
14*a) - 2*b*sin(12*b*x + 12*a) + 12*b*sin(10*b*x + 10*a) - 2*b*sin(8*b*x +
8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*
sin(18*b*x + 18*a) - 6*(8*b*sin(14*b*x + 14*a) + 2*b*sin(12*b*x + 12*a) - 1
2*b*sin(10*b*x + 10*a) + 2*b*sin(8*b*x + 8*a) + 8*b*sin(6*b*x + 6*a) - 3*b*
sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 16*(2*b*sin(1
2*b*x + 12*a) - 12*b*sin(10*b*x + 10*a) + 2*b*sin(8*b*x + 8*a) + 8*b*sin(6*
b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a
) - 4*(12*b*sin(10*b*x + 10*a) - 2*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a
) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 24*(2
*b*sin(8*b*x + 8*a) + 8*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - 2*b*sin
(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*(8*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x
 + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 16*(3*b*sin(4*b*x + 4*a)
 + 2*b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.29, size = 114, normalized size = 1.27

$$\frac{5\ln\left(\sin(a+b\,x)^2\right)}{32\,b}-\frac{5\ln\left(\cos(a+b\,x)\right)}{16\,b}+\frac{-\frac{5\cos(a+b\,x)^8}{32}+\frac{25\cos(a+b\,x)^6}{64}-\frac{55\cos(a+b\,x)^4}{192}+\frac{5\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(-\cos(a+b\,x)^{10}+3\cos(a+b\,x)^8-3\cos(a+b\,x)^6+\cos(a+b\,x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^5),x)
```

```
[Out] (5*log(sin(a + b*x)^2))/(32*b) - (5*log(cos(a + b*x)))/(16*b) + ((5*cos(a +
 b*x)^2)/128 - (55*cos(a + b*x)^4)/192 + (25*cos(a + b*x)^6)/64 - (5*cos(a
+ b*x)^8)/32 + 1/128)/(b*(cos(a + b*x)^4 - 3*cos(a + b*x)^6 + 3*cos(a + b*x
)^8 - cos(a + b*x)^10))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**5, x)
```

3.58 $\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$

Optimal. Leaf size=102

$$\frac{\tan^5(a+bx)}{320b}+\frac{\tan^3(a+bx)}{32b}+\frac{15\tan(a+bx)}{64b}-\frac{\cot^7(a+bx)}{448b}-\frac{3\cot^5(a+bx)}{160b}-\frac{5\cot^3(a+bx)}{64b}-\frac{5\cot(a+bx)}{16b}$$

```
[Out] -5/16*cot(b*x+a)/b-5/64*cot(b*x+a)^3/b-3/160*cot(b*x+a)^5/b-1/448*cot(b*x+a
)^7/b+15/64*tan(b*x+a)/b+1/32*tan(b*x+a)^3/b+1/320*tan(b*x+a)^5/b
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2620, 270}

$$\frac{\tan^5(a+bx)}{320b}+\frac{\tan^3(a+bx)}{32b}+\frac{15\tan(a+bx)}{64b}-\frac{\cot^7(a+bx)}{448b}-\frac{3\cot^5(a+bx)}{160b}-\frac{5\cot^3(a+bx)}{64b}-\frac{5\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cot[a + b*x])/(16*b) - (5*Cot[a + b*x]^3)/(64*b) - (3*Cot[a + b*x]^5)/(
160*b) - Cot[a + b*x]^7/(448*b) + (15*Tan[a + b*x])/(64*b) + Tan[a + b*x]^3
/(32*b) + Tan[a + b*x]^5/(320*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx &= \frac{1}{64}\int \csc^8(a+bx)\sec^6(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^6}{x^8}\,dx,x,\tan(a+bx)\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(15+\frac{1}{x^8}+\frac{6}{x^6}+\frac{15}{x^4}+\frac{20}{x^2}+6x^2+x^4\right)dx,x,\tan(a+bx)\right)}{64b}\\
&= -\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\tan}{6}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 132, normalized size = 1.29

$$\frac{33\tan(a+bx)}{160b}-\frac{281\cot(a+bx)}{1120b}-\frac{\cot(a+bx)\csc^6(a+bx)}{448b}-\frac{27\cot(a+bx)\csc^4(a+bx)}{2240b}-\frac{53\cot(a+bx)\csc^2(a+bx)}{1120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-281*Cot[a + b*x])/(1120*b) - (53*Cot[a + b*x]*Csc[a + b*x]^2)/(1120*b) -
(27*Cot[a + b*x]*Csc[a + b*x]^4)/(2240*b) - (Cot[a + b*x]*Csc[a + b*x]^6)/(
448*b) + (33*Tan[a + b*x])/(160*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(40*b) +
 (Sec[a + b*x]^4*Tan[a + b*x])/(320*b)
```

fricas [A] time = 0.51, size = 118, normalized size = 1.16

$$-\frac{1024\cos(bx+a)^{12}-3584\cos(bx+a)^{10}+4480\cos(bx+a)^{8}-2240\cos(bx+a)^{6}+280\cos(bx+a)^{4}+28}{2240\left(b\cos(bx+a)^{11}-3\,b\cos(bx+a)^{9}+3\,b\cos(bx+a)^{7}-b\cos(bx+a)^{5}\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -1/2240*(1024*cos(b*x + a)^12 - 3584*cos(b*x + a)^10 + 4480*cos(b*x + a)^8
- 2240*cos(b*x + a)^6 + 280*cos(b*x + a)^4 + 28*cos(b*x + a)^2 + 7)/((b*cos
(b*x + a)^11 - 3*b*cos(b*x + a)^9 + 3*b*cos(b*x + a)^7 - b*cos(b*x + a)^5)*
sin(b*x + a))
```

giac [B] time = 12.22, size = 7477, normalized size = 73.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1/71680*(7*(6480*tan(b*x + 4*a)^4*tan(1/2*a)^56 - 2160*tan(b*x + 4*a)^3*ta
n(1/2*a)^57 + 360*tan(b*x + 4*a)^2*tan(1/2*a)^58 - 30*tan(b*x + 4*a)*tan(1/
2*a)^59 + tan(1/2*a)^60 + 963360*tan(b*x + 4*a)^4*tan(1/2*a)^54 - 451440*ta
n(b*x + 4*a)^3*tan(1/2*a)^55 + 84000*tan(b*x + 4*a)^2*tan(1/2*a)^56 - 6650*
tan(b*x + 4*a)*tan(1/2*a)^57 + 210*tan(1/2*a)^58 + 76410000*tan(b*x + 4*a)^
4*tan(1/2*a)^52 - 46604880*tan(b*x + 4*a)^3*tan(1/2*a)^53 + 11606800*tan(b*
x + 4*a)^2*tan(1/2*a)^54 - 1351380*tan(b*x + 4*a)*tan(1/2*a)^55 + 61875*tan
(1/2*a)^56 - 1737755520*tan(b*x + 4*a)^4*tan(1/2*a)^50 + 1779239600*tan(b*x
 + 4*a)^3*tan(1/2*a)^51 - 594557280*tan(b*x + 4*a)^2*tan(1/2*a)^52 + 841797
00*tan(b*x + 4*a)*tan(1/2*a)^53 - 4366140*tan(1/2*a)^54 + 14743616480*tan(b
*x + 4*a)^4*tan(1/2*a)^48 - 21984264480*tan(b*x + 4*a)^3*tan(1/2*a)^49 + 10
388004600*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 1902997490*tan(b*x + 4*a)*tan(1/
2*a)^51 + 120469005*tan(1/2*a)^52 - 61028652480*tan(b*x + 4*a)^4*tan(1/2*a)
^46 + 130174560480*tan(b*x + 4*a)^3*tan(1/2*a)^47 - 86273730880*tan(b*x + 4
*a)^2*tan(1/2*a)^48 + 21462983130*tan(b*x + 4*a)*tan(1/2*a)^49 - 1737306714
*tan(1/2*a)^50 + 103180381920*tan(b*x + 4*a)^4*tan(1/2*a)^44 - 371517010400
*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 365221080480*tan(b*x + 4*a)^2*tan(1/2*a)^
46 - 127981989480*tan(b*x + 4*a)*tan(1/2*a)^47 + 14001839215*tan(1/2*a)^48
+ 111160976000*tan(b*x + 4*a)^4*tan(1/2*a)^42 + 211586603040*tan(b*x + 4*a)
^3*tan(1/2*a)^43 - 637111944000*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 3760488381
20*tan(b*x + 4*a)*tan(1/2*a)^45 - 60608749080*tan(1/2*a)^46 - 724773023760*
tan(b*x + 4*a)^4*tan(1/2*a)^40 + 1597018455600*tan(b*x + 4*a)^3*tan(1/2*a)^
41 - 663483964760*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 227449079550*tan(b*x + 4
*a)*tan(1/2*a)^43 + 109533462525*tan(1/2*a)^44 + 663892404960*tan(b*x + 4*a
)^4*tan(1/2*a)^38 - 3633046154960*tan(b*x + 4*a)^3*tan(1/2*a)^39 + 44420531
```

39040*tan(b*x + 4*a)^2*tan(1/2*a)^40 - 1628950998330*tan(b*x + 4*a)*tan(1/2
*a)^41 + 110076988450*tan(1/2*a)^42 + 1382243479600*tan(b*x + 4*a)^4*tan(1/
2*a)^36 - 662161010160*tan(b*x + 4*a)^3*tan(1/2*a)^37 - 3947253799440*tan(b
*x + 4*a)^2*tan(1/2*a)^38 + 3687432890100*tan(b*x + 4*a)*tan(1/2*a)^39 - 75
7631165865*tan(1/2*a)^40 - 2679939290880*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 9
953265569040*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 8576804580640*tan(b*x + 4*a)^
2*tan(1/2*a)^36 + 813320615340*tan(b*x + 4*a)*tan(1/2*a)^37 + 647844828540*
tan(1/2*a)^38 - 755338211520*tan(b*x + 4*a)^4*tan(1/2*a)^32 - 5830705497280
*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 15767445241080*tan(b*x + 4*a)^2*tan(1/2*a
)^34 - 9992067869250*tan(b*x + 4*a)*tan(1/2*a)^35 + 1474839074345*tan(1/2*a
)^36 + 3975568027520*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 11953155299520*tan(b*
x + 4*a)^3*tan(1/2*a)^31 + 4861415944320*tan(b*x + 4*a)^2*tan(1/2*a)^32 + 5
463157048330*tan(b*x + 4*a)*tan(1/2*a)^33 - 2566729120410*tan(1/2*a)^34 - 7
55338211520*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 11953155299520*tan(b*x + 4*a)^
3*tan(1/2*a)^29 - 23059287629120*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 117070930
56720*tan(b*x + 4*a)*tan(1/2*a)^31 - 840595305645*tan(1/2*a)^32 - 267993929
0880*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 5830705497280*tan(b*x + 4*a)^3*tan(1/
2*a)^27 + 4861415944320*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 11707093056720*tan
(b*x + 4*a)*tan(1/2*a)^29 + 3742852321200*tan(1/2*a)^30 + 1382243479600*tan
(b*x + 4*a)^4*tan(1/2*a)^24 - 9953265569040*tan(b*x + 4*a)^3*tan(1/2*a)^25
+ 15767445241080*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 5463157048330*tan(b*x + 4
*a)*tan(1/2*a)^27 - 840595305645*tan(1/2*a)^28 + 663892404960*tan(b*x + 4*a
)^4*tan(1/2*a)^22 + 662161010160*tan(b*x + 4*a)^3*tan(1/2*a)^23 - 857680458
0640*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 9992067869250*tan(b*x + 4*a)*tan(1/2*
a)^25 - 2566729120410*tan(1/2*a)^26 - 724773023760*tan(b*x + 4*a)^4*tan(1/2
*a)^20 + 3633046154960*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 3947253799440*tan(b
*x + 4*a)^2*tan(1/2*a)^22 - 813320615340*tan(b*x + 4*a)*tan(1/2*a)^23 + 147
4839074345*tan(1/2*a)^24 + 111160976000*tan(b*x + 4*a)^4*tan(1/2*a)^18 - 15
97018455600*tan(b*x + 4*a)^3*tan(1/2*a)^19 + 4442053139040*tan(b*x + 4*a)^2
*tan(1/2*a)^20 - 3687432890100*tan(b*x + 4*a)*tan(1/2*a)^21 + 647844828540*
tan(1/2*a)^22 + 103180381920*tan(b*x + 4*a)^4*tan(1/2*a)^16 - 211586603040*
tan(b*x + 4*a)^3*tan(1/2*a)^17 - 663483964760*tan(b*x + 4*a)^2*tan(1/2*a)^1
8 + 1628950998330*tan(b*x + 4*a)*tan(1/2*a)^19 - 757631165865*tan(1/2*a)^20
 - 61028652480*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 371517010400*tan(b*x + 4*a)
^3*tan(1/2*a)^15 - 637111944000*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 2274490795
50*tan(b*x + 4*a)*tan(1/2*a)^17 + 110076988450*tan(1/2*a)^18 + 14743616480*
tan(b*x + 4*a)^4*tan(1/2*a)^12 - 130174560480*tan(b*x + 4*a)^3*tan(1/2*a)^1
3 + 365221080480*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 376048838120*tan(b*x + 4*
a)*tan(1/2*a)^15 + 109533462525*tan(1/2*a)^16 - 1737755520*tan(b*x + 4*a)^4
*tan(1/2*a)^10 + 21984264480*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 86273730880*t
an(b*x + 4*a)^2*tan(1/2*a)^12 + 127981989480*tan(b*x + 4*a)*tan(1/2*a)^13 -
 60608749080*tan(1/2*a)^14 + 76410000*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 17792
39600*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 10388004600*tan(b*x + 4*a)^2*tan(1/2*
a)^10 - 21462983130*tan(b*x + 4*a)*tan(1/2*a)^11 + 14001839215*tan(1/2*a)^1
2 + 963360*tan(b*x + 4*a)^4*tan(1/2*a)^6 + 46604880*tan(b*x + 4*a)^3*tan(1/
2*a)^7 - 594557280*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1902997490*tan(b*x + 4*a
)*tan(1/2*a)^9 - 1737306714*tan(1/2*a)^10 + 6480*tan(b*x + 4*a)^4*tan(1/2*a
)^4 + 451440*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 11606800*tan(b*x + 4*a)^2*tan(
1/2*a)^6 - 84179700*tan(b*x + 4*a)*tan(1/2*a)^7 + 120469005*tan(1/2*a)^8 +
2160*tan(b*x + 4*a)^3*tan(1/2*a)^3 + 84000*tan(b*x + 4*a)^2*tan(1/2*a)^4 +
1351380*tan(b*x + 4*a)*tan(1/2*a)^5 - 4366140*tan(1/2*a)^6 + 360*tan(b*x +
4*a)^2*tan(1/2*a)^2 + 6650*tan(b*x + 4*a)*tan(1/2*a)^3 + 61875*tan(1/2*a)^4
 + 30*tan(b*x + 4*a)*tan(1/2*a) + 210*tan(1/2*a)^2 + 1)/((243*tan(1/2*a)^25
 - 4050*tan(1/2*a)^23 + 28215*tan(1/2*a)^21 - 106200*tan(1/2*a)^19 + 233430
*tan(1/2*a)^17 - 304300*tan(1/2*a)^15 + 233430*tan(1/2*a)^13 - 106200*tan(1
/2*a)^11 + 28215*tan(1/2*a)^9 - 4050*tan(1/2*a)^7 + 243*tan(1/2*a)^5)*(6*ta
n(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 +
 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^5) +
32*(700*tan(b*x + 4*a)^6*tan(1/2*a)^84 - 102900*tan(b*x + 4*a)^6*tan(1/2*a)

^82 + 22050*tan(b*x + 4*a)^5*tan(1/2*a)^83 + 175*tan(b*x + 4*a)^4*tan(1/2*a
)^84 + 7200060*tan(b*x + 4*a)^6*tan(1/2*a)^80 - 3103170*tan(b*x + 4*a)^5*ta
n(1/2*a)^81 + 280770*tan(b*x + 4*a)^4*tan(1/2*a)^82 + 2940*tan(b*x + 4*a)^3
*tan(1/2*a)^83 + 42*tan(b*x + 4*a)^2*tan(1/2*a)^84 - 316457680*tan(b*x + 4*
a)^6*tan(1/2*a)^78 + 206229240*tan(b*x + 4*a)^5*tan(1/2*a)^79 - 38821965*ta
n(b*x + 4*a)^4*tan(1/2*a)^80 + 1904140*tan(b*x + 4*a)^3*tan(1/2*a)^81 + 176
40*tan(b*x + 4*a)^2*tan(1/2*a)^82 + 294*tan(b*x + 4*a)*tan(1/2*a)^83 + 5*ta
n(1/2*a)^84 + 9709765800*tan(b*x + 4*a)^6*tan(1/2*a)^76 - 8530152120*tan(b*
x + 4*a)^5*tan(1/2*a)^77 + 2449598200*tan(b*x + 4*a)^4*tan(1/2*a)^78 - 2592
55080*tan(b*x + 4*a)^3*tan(1/2*a)^79 + 7360290*tan(b*x + 4*a)^2*tan(1/2*a)^
80 + 47530*tan(b*x + 4*a)*tan(1/2*a)^81 + 462*tan(1/2*a)^82 - 218895118680*
tan(b*x + 4*a)^6*tan(1/2*a)^74 + 243907782300*tan(b*x + 4*a)^5*tan(1/2*a)^7
5 - 94820555130*tan(b*x + 4*a)^4*tan(1/2*a)^76 + 15442018200*tan(b*x + 4*a)
^3*tan(1/2*a)^77 - 976159968*tan(b*x + 4*a)^2*tan(1/2*a)^78 + 15463224*tan(
b*x + 4*a)*tan(1/2*a)^79 + 47985*tan(1/2*a)^80 + 3722263347880*tan(b*x + 4*
a)^6*tan(1/2*a)^72 - 5072116579740*tan(b*x + 4*a)^5*tan(1/2*a)^73 + 2508017
550780*tan(b*x + 4*a)^4*tan(1/2*a)^74 - 555145935120*tan(b*x + 4*a)^3*tan(1
/2*a)^75 + 54416410020*tan(b*x + 4*a)^2*tan(1/2*a)^76 - 1964255832*tan(b*x
+ 4*a)*tan(1/2*a)^77 + 13827464*tan(1/2*a)^78 - 48306853319760*tan(b*x + 4*
a)^6*tan(1/2*a)^70 + 78691575627000*tan(b*x + 4*a)^5*tan(1/2*a)^71 - 476992
27086490*tan(b*x + 4*a)^4*tan(1/2*a)^72 + 13463793136080*tan(b*x + 4*a)^3*t
an(1/2*a)^73 - 1800097458768*tan(b*x + 4*a)^2*tan(1/2*a)^74 + 101416919940*
tan(b*x + 4*a)*tan(1/2*a)^75 - 1647303294*tan(1/2*a)^76 + 478852179070860*t
an(b*x + 4*a)^6*tan(1/2*a)^68 - 920259549793080*tan(b*x + 4*a)^5*tan(1/2*a)
^69 + 668627834124600*tan(b*x + 4*a)^4*tan(1/2*a)^70 - 231870649855320*tan(
b*x + 4*a)^3*tan(1/2*a)^71 + 39617163658212*tan(b*x + 4*a)^2*tan(1/2*a)^72
- 3053123538948*tan(b*x + 4*a)*tan(1/2*a)^73 + 77871263652*tan(1/2*a)^74 -
3590491658734820*tan(b*x + 4*a)^6*tan(1/2*a)^66 + 8099233169503770*tan(b*x
+ 4*a)^5*tan(1/2*a)^67 - 6967640085374745*tan(b*x + 4*a)^4*tan(1/2*a)^68 +
2903918016468840*tan(b*x + 4*a)^3*tan(1/2*a)^69 - 610729793019360*tan(b*x +
4*a)^2*tan(1/2*a)^70 + 60236954245272*tan(b*x + 4*a)*tan(1/2*a)^71 - 21053
89907166*tan(1/2*a)^72 + 19896259338686220*tan(b*x + 4*a)^6*tan(1/2*a)^64 -
52908136354185210*tan(b*x + 4*a)^5*tan(1/2*a)^65 + 53736728263219610*tan(b
*x + 4*a)^4*tan(1/2*a)^66 - 26624174150101500*tan(b*x + 4*a)^3*tan(1/2*a)^6
7 + 6746600775679002*tan(b*x + 4*a)^2*tan(1/2*a)^68 - 820187477390456*tan(b
*x + 4*a)*tan(1/2*a)^69 + 36705931718472*tan(1/2*a)^70 - 77771578076512320*
tan(b*x + 4*a)^6*tan(1/2*a)^62 + 248694609613265760*tan(b*x + 4*a)^5*tan(1/
2*a)^63 - 300960195956707245*tan(b*x + 4*a)^4*tan(1/2*a)^64 + 1773024391563
31860*tan(b*x + 4*a)^3*tan(1/2*a)^65 - 53651600975241528*tan(b*x + 4*a)^2*t
an(1/2*a)^66 + 7877252164835550*tan(b*x + 4*a)*tan(1/2*a)^67 - 434705306175
315*tan(1/2*a)^68 + 192844994494822880*tan(b*x + 4*a)^6*tan(1/2*a)^60 - 787
509809051626080*tan(b*x + 4*a)^5*tan(1/2*a)^61 + 1173510851306099040*tan(b*
x + 4*a)^4*tan(1/2*a)^62 - 835950959826956960*tan(b*x + 4*a)^3*tan(1/2*a)^6
3 + 303563975744062530*tan(b*x + 4*a)^2*tan(1/2*a)^64 - 53507434964136174*t
an(b*x + 4*a)*tan(1/2*a)^65 + 3573865030470070*tan(1/2*a)^66 - 200273988698
142240*tan(b*x + 4*a)^6*tan(1/2*a)^58 + 1398151947986755920*tan(b*x + 4*a)^
5*tan(1/2*a)^59 - 2880661846092155160*tan(b*x + 4*a)^4*tan(1/2*a)^60 + 2628
180105420371040*tan(b*x + 4*a)^3*tan(1/2*a)^61 - 1179839882273931648*tan(b*
x + 4*a)^2*tan(1/2*a)^62 + 252811149558104928*tan(b*x + 4*a)*tan(1/2*a)^63
- 20413710218743023*tan(1/2*a)^64 - 371753255147504160*tan(b*x + 4*a)^6*tan
(1/2*a)^56 - 1005666833635111120*tan(b*x + 4*a)^5*tan(1/2*a)^57 + 2954417021
031525840*tan(b*x + 4*a)^4*tan(1/2*a)^58 - 4615789529427552000*tan(b*x + 4*
a)^3*tan(1/2*a)^59 + 2869139270141795952*tan(b*x + 4*a)^2*tan(1/2*a)^60 - 7
89182123619651552*tan(b*x + 4*a)*tan(1/2*a)^61 + 79047902485397664*tan(1/2*
a)^62 + 1588175918421104320*tan(b*x + 4*a)^6*tan(1/2*a)^54 - 57052883200744
46880*tan(b*x + 4*a)^5*tan(1/2*a)^55 + 5543116031154667560*tan(b*x + 4*a)^4
*tan(1/2*a)^56 + 291827962982908800*tan(b*x + 4*a)^3*tan(1/2*a)^57 - 290703
5119602048960*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 1372067944595797680*tan(b*x
+ 4*a)*tan(1/2*a)^59 - 190524603741625608*tan(1/2*a)^60 - 17642684176828174

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80*tan(b*x + 4*a)^6*tan(1/2*a)^52 + 118515074133824827 20*tan(b*x + 4*a)^5*t
an(1/2*a)^53 - 23636877153497649440*tan(b*x + 4*a)^4*tan(1/2*a)^54 + 188525
55003710349600*tan(b*x + 4*a)^3*tan(1/2*a)^55 - 5514689294400576528*tan(b*x
 + 4*a)^2*tan(1/2*a)^56 - 74535612123099568*tan(b*x + 4*a)*tan(1/2*a)^57 +
190797055644582576*tan(1/2*a)^58 - 1248453345588419880*tan(b*x + 4*a)^6*tan
(1/2*a)^50 - 4107878664125019420*tan(b*x + 4*a)^5*tan(1/2*a)^51 + 265217801
08972032990*tan(b*x + 4*a)^4*tan(1/2*a)^52 - 39323126797282860000*tan(b*x +
 4*a)^3*tan(1/2*a)^53 + 23453137263033690240*tan(b*x + 4*a)^2*tan(1/2*a)^54
 - 5608964036362402464*tan(b*x + 4*a)*tan(1/2*a)^55 + 366017355229752504*ta
n(1/2*a)^56 + 528627047989511660 0*tan(b*x + 4*a)^6*tan(1/2*a)^48 - 21060700
379637099300*tan(b*x + 4*a)^5*tan(1/2*a)^49 + 18283250157335063460*tan(b*x
+ 4*a)^4*tan(1/2*a)^50 + 14094690574782962200*tan(b*x + 4*a)^3*tan(1/2*a)^5
1 - 26565501133755260460*tan(b*x + 4*a)^2*tan(1/2*a)^52 + 11739546935802688
800*tan(b*x + 4*a)*tan(1/2*a)^53 - 1551520053876868320*tan(1/2*a)^54 - 4019
457841895096160*tan(b*x + 4*a)^6*tan(1/2*a)^46 + 33873365085581604240*tan(b
*x + 4*a)^5*tan(1/2*a)^47 - 79355355678133689210*tan(b*x + 4*a)^4*tan(1/2*a
)^48 + 69758331625489475640*tan(b*x + 4*a)^3*tan(1/2*a)^49 - 17832092647389
129648*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 4344999459760312724*tan(b*x + 4*a)*
tan(1/2*a)^51 + 1772974039101333642*tan(1/2*a)^52 - 3363472645057689360*tan
(b*x + 4*a)^6*tan(1/2*a)^44 - 41963462437389040 80*tan(b*x + 4*a)^5*tan(1/2*
a)^45 + 61023004440064909200*tan(b*x + 4*a)^4*tan(1/2*a)^46 - 1135940055490
24014000*tan(b*x + 4*a)^3*tan(1/2*a)^47 + 79397061913802193540*tan(b*x + 4*
a)^2*tan(1/2*a)^48 - 20785250550683362380*tan(b*x + 4*a)*tan(1/2*a)^49 + 11
58344161574237820*tan(1/2*a)^50 + 7922839707121600240*tan(b*x + 4*a)^6*tan(
1/2*a)^42 - 40062672006803795160*tan(b*x + 4*a)^5*tan(1/2*a)^43 + 504790970
14227689220*tan(b*x + 4*a)^4*tan(1/2*a)^44 + 14711499739562948560*tan(b*x +
 4*a)^3*tan(1/2*a)^45 - 61779677247183389760*tan(b*x + 4*a)^2*tan(1/2*a)^46
 + 34283272085462710800*tan(b*x + 4*a)*tan(1/2*a)^47 - 5294375878659948990*
tan(1/2*a)^48 - 3363472645057689360*tan(b*x + 4*a)^6*tan(1/2*a)^40 + 400626
72006803795160*tan(b*x + 4*a)^5*tan(1/2*a)^41 - 119703426365609444120*tan(b
*x + 4*a)^4*tan(1/2*a)^42 + 134372289532573058400*tan(b*x + 4*a)^3*tan(1/2*
a)^43 - 50496361003895322600*tan(b*x + 4*a)^2*tan(1/2*a)^44 - 4643613116469
393360*tan(b*x + 4*a)*tan(1/2*a)^45 + 4170913759258711920*tan(1/2*a)^46 - 4
019457841895096160*tan(b*x + 4*a)^6*tan(1/2*a)^38 + 4196346243738904080*tan
(b*x + 4*a)^5*tan(1/2*a)^39 + 50479097014227689220*tan(b*x + 4*a)^4*tan(1/2
*a)^40 - 134372289532573058400*tan(b*x + 4*a)^3*tan(1/2*a)^41 + 12059954353
0513343520*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 40569490418667454440*tan(b*x +
4*a)*tan(1/2*a)^43 + 3366765108190708140*tan(1/2*a)^44 + 528627047989511660
0*tan(b*x + 4*a)^6*tan(1/2*a)^36 - 33873365085581604240*tan(b*x + 4*a)^5*ta
n(1/2*a)^37 + 61023004440064909200*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 1471149
9739562948560*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 50496361003895322600*tan(b*x
 + 4*a)^2*tan(1/2*a)^40 + 40569490418667454440*tan(b*x + 4*a)*tan(1/2*a)^41
 - 8102375952750405800*tan(1/2*a)^42 - 1248453345588419880*tan(b*x + 4*a)^6
*tan(1/2*a)^34 + 21060700379637099300*tan(b*x + 4*a)^5*tan(1/2*a)^35 - 7935
5355678133689210*tan(b*x + 4*a)^4*tan(1/2*a)^36 + 113594005549024014000*tan
(b*x + 4*a)^3*tan(1/2*a)^37 - 61779677247183389760*tan(b*x + 4*a)^2*tan(1/2
*a)^38 + 4643613116469393360*tan(b*x + 4*a)*tan(1/2*a)^39 + 336676510819070
8140*tan(1/2*a)^40 - 1764268417682817480*tan(b*x + 4*a)^6*tan(1/2*a)^32 + 4
107878664125019420*tan(b*x + 4*a)^5*tan(1/2*a)^33 + 18283250157335063460*ta
n(b*x + 4*a)^4*tan(1/2*a)^34 - 69758331625489475640*tan(b*x + 4*a)^3*tan(1/
2*a)^35 + 79397061913802193540*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 34283272085
462710800*tan(b*x + 4*a)*tan(1/2*a)^37 + 4170913759258711920*tan(1/2*a)^38
+ 1588175918421104320*tan(b*x + 4*a)^6*tan(1/2*a)^30 - 11851507413382482720
*tan(b*x + 4*a)^5*tan(1/2*a)^31 + 26521780108972032990*tan(b*x + 4*a)^4*tan
(1/2*a)^32 - 14094690574782962200*tan(b*x + 4*a)^3*tan(1/2*a)^33 - 17832092
647389129648*tan(b*x + 4*a)^2*tan(1/2*a)^34 + 20785250550683362380*tan(b*x
+ 4*a)*tan(1/2*a)^35 - 5294375878659948990*tan(1/2*a)^36 - 3717532551475041
60*tan(b*x + 4*a)^6*tan(1/2*a)^28 + 5705288320074446880*tan(b*x + 4*a)^5*ta
n(1/2*a)^29 - 23636877153497649440*tan(b*x + 4*a)^4*tan(1/2*a)^30 + 3932312
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6797282860000*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 2656550113375526046O*tan(b*x
+ 4*a)^2*tan(1/2*a)^32 + 4344999459760312724*tan(b*x + 4*a)*tan(1/2*a)^33
+ 1158344161574237820*tan(1/2*a)^34 - 200273988698142240*tan(b*x + 4*a)^6*t
an(1/2*a)^26 + 1005666833635111 20*tan(b*x + 4*a)^5*tan(1/2*a)^27 + 55431160
31154667560*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 18852555003710349600*tan(b*x +
4*a)^3*tan(1/2*a)^29 + 23453137263033690240*tan(b*x + 4*a)^2*tan(1/2*a)^30
- 11739546935802688800*tan(b*x + 4*a)*tan(1/2*a)^31 + 1772974039101333642*
tan(1/2*a)^32 + 19284499449482288 0*tan(b*x + 4*a)^6*tan(1/2*a)^24 - 1398151
947986755920*tan(b*x + 4*a)^5*tan(1/2*a)^25 + 2954417021031525840*tan(b*x +
4*a)^4*tan(1/2*a)^26 - 291827962982908800*tan(b*x + 4*a)^3*tan(1/2*a)^27 -
5514689294400576528*tan(b*x + 4*a)^2*tan(1/2*a)^28 + 5608964036362402464*t
an(b*x + 4*a)*tan(1/2*a)^29 - 1551520053876868320*tan(1/2*a)^30 - 777715780
76512320*tan(b*x + 4*a)^6*tan(1/2*a)^22 + 787509809051626080*tan(b*x + 4*a)
^5*tan(1/2*a)^23 - 2880661846092155160*tan(b*x + 4*a)^4*tan(1/2*a)^24 + 461
5789529427552000*tan(b*x + 4*a)^3*tan(1/2*a)^25 - 2907035119602048960*tan(b
*x + 4*a)^2*tan(1/2*a)^26 + 74535612123099568*tan(b*x + 4*a)*tan(1/2*a)^27
+ 366017355229752504*tan(1/2*a)^28 + 19896259338686220*tan(b*x + 4*a)^6*tan
(1/2*a)^20 - 248694609613265760*tan(b*x + 4*a)^5*tan(1/2*a)^21 + 1173510851
306099040*tan(b*x + 4*a)^4*tan(1/2*a)^22 - 2628180105420371040*tan(b*x + 4*
a)^3*tan(1/2*a)^23 + 2869139270141795952*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 1
372067944595797680*tan(b*x + 4*a)*tan(1/2*a)^25 + 190797055644582576*tan(1/
2*a)^26 - 3590491658734820*tan(b*x + 4*a)^6*tan(1/2*a)^18 + 529081363541852
10*tan(b*x + 4*a)^5*tan(1/2*a)^19 - 300960195956707245*tan(b*x + 4*a)^4*tan
(1/2*a)^20 + 835950959826956960*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 1179839882
273931648*tan(b*x + 4*a)^2*tan(1/2*a)^22 + 789182123619651552*tan(b*x + 4*a
)*tan(1/2*a)^23 - 190524603741625608*tan(1/2*a)^24 + 478852179070860*tan(b*
x + 4*a)^6*tan(1/2*a)^16 - 8099233169503770*tan(b*x + 4*a)^5*tan(1/2*a)^17
+ 53736728263219610*tan(b*x + 4*a)^4*tan(1/2*a)^18 - 177302439156331860*tan
(b*x + 4*a)^3*tan(1/2*a)^19 + 303563975744062530*tan(b*x + 4*a)^2*tan(1/2*a
)^20 - 252811149558104928*tan(b*x + 4*a)*tan(1/2*a)^21 + 79047902485397664*
tan(1/2*a)^22 - 48306853319760*tan(b*x + 4*a)^6*tan(1/2*a)^14 + 92025954979
3080*tan(b*x + 4*a)^5*tan(1/2*a)^15 - 6967640085374745*tan(b*x + 4*a)^4*tan
(1/2*a)^16 + 26624174150101500*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 53651600975
241528*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 53507434964136174*tan(b*x + 4*a)*ta
n(1/2*a)^19 - 20413710218743023*tan(1/2*a)^20 + 3722263347880*tan(b*x + 4*a
)^6*tan(1/2*a)^12 - 78691575627000*tan(b*x + 4*a)^5*tan(1/2*a)^13 + 6686278
34124600*tan(b*x + 4*a)^4*tan(1/2*a)^14 - 2903918016468840*tan(b*x + 4*a)^3
*tan(1/2*a)^15 + 6746600775679002*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 78772521
64835550*tan(b*x + 4*a)*tan(1/2*a)^17 + 3573865030470070*tan(1/2*a)^18 - 21
8895118680*tan(b*x + 4*a)^6*tan(1/2*a)^10 + 5072116579740*tan(b*x + 4*a)^5*
tan(1/2*a)^11 - 47699227086490*tan(b*x + 4*a)^4*tan(1/2*a)^12 + 23187064985
5320*tan(b*x + 4*a)^3*tan(1/2*a)^13 - 610729793019360*tan(b*x + 4*a)^2*tan(
1/2*a)^14 + 820187477390456*tan(b*x + 4*a)*tan(1/2*a)^15 - 434705306175315*
tan(1/2*a)^16 + 9709765800*tan(b*x + 4*a)^6*tan(1/2*a)^8 - 243907782300*tan
(b*x + 4*a)^5*tan(1/2*a)^9 + 2508017550780*tan(b*x + 4*a)^4*tan(1/2*a)^10 -
13463793136080*tan(b*x + 4*a)^3*tan(1/2*a)^11 + 39617163658212*tan(b*x + 4
*a)^2*tan(1/2*a)^12 - 60236954245272*tan(b*x + 4*a)*tan(1/2*a)^13 + 3670593
1718472*tan(1/2*a)^14 - 316457680*tan(b*x + 4*a)^6*tan(1/2*a)^6 + 853015212
0*tan(b*x + 4*a)^5*tan(1/2*a)^7 - 94820555130*tan(b*x + 4*a)^4*tan(1/2*a)^8
+ 555145935120*tan(b*x + 4*a)^3*tan(1/2*a)^9 - 1800097458768*tan(b*x + 4*a
)^2*tan(1/2*a)^10 + 3053123538948*tan(b*x + 4*a)*tan(1/2*a)^11 - 2105389907
166*tan(1/2*a)^12 + 7200060*tan(b*x + 4*a)^6*tan(1/2*a)^4 - 206229240*tan(b
*x + 4*a)^5*tan(1/2*a)^5 + 2449598200*tan(b*x + 4*a)^4*tan(1/2*a)^6 - 15442
018200*tan(b*x + 4*a)^3*tan(1/2*a)^7 + 54416410020*tan(b*x + 4*a)^2*tan(1/2
*a)^8 - 101416919940*tan(b*x + 4*a)*tan(1/2*a)^9 + 77871263652*tan(1/2*a)^1
0 - 102900*tan(b*x + 4*a)^6*tan(1/2*a)^2 + 3103170*tan(b*x + 4*a)^5*tan(1/2
*a)^3 - 38821965*tan(b*x + 4*a)^4*tan(1/2*a)^4 + 259255080*tan(b*x + 4*a)^3
*tan(1/2*a)^5 - 976159968*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 1964255832*tan(b*
x + 4*a)*tan(1/2*a)^7 - 1647303294*tan(1/2*a)^8 + 700*tan(b*x + 4*a)^6 - 22

```
050*tan(b*x + 4*a)^5*tan(1/2*a) + 280770*tan(b*x + 4*a)^4*tan(1/2*a)^2 - 19
04140*tan(b*x + 4*a)^3*tan(1/2*a)^3 + 7360290*tan(b*x + 4*a)^2*tan(1/2*a)^4
 - 15463224*tan(b*x + 4*a)*tan(1/2*a)^5 + 13827464*tan(1/2*a)^6 + 175*tan(b
*x + 4*a)^4 - 2940*tan(b*x + 4*a)^3*tan(1/2*a) + 17640*tan(b*x + 4*a)^2*tan
(1/2*a)^2 - 47530*tan(b*x + 4*a)*tan(1/2*a)^3 + 47985*tan(1/2*a)^4 + 42*tan
(b*x + 4*a)^2 - 294*tan(b*x + 4*a)*tan(1/2*a) + 462*tan(1/2*a)^2 + 5)/((tan
(1/2*a)^42 - 105*tan(1/2*a)^40 + 4830*tan(1/2*a)^38 - 127582*tan(1/2*a)^36
+ 2131605*tan(1/2*a)^34 - 23413005*tan(1/2*a)^32 + 170737896*tan(1/2*a)^30
- 822580200*tan(1/2*a)^28 + 2607894450*tan(1/2*a)^26 - 5534841410*tan(1/2*a
)^24 + 8018138100*tan(1/2*a)^22 - 8018138100*tan(1/2*a)^20 + 5534841410*tan
(1/2*a)^18 - 2607894450*tan(1/2*a)^16 + 822580200*tan(1/2*a)^14 - 170737896
*tan(1/2*a)^12 + 23413005*tan(1/2*a)^10 - 2131605*tan(1/2*a)^8 + 127582*tan
(1/2*a)^6 - 4830*tan(1/2*a)^4 + 105*tan(1/2*a)^2 - 1)*(tan(b*x + 4*a)*tan(1
/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*
a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^7))/b
```

maple [A] time = 1.26, size = 123, normalized size = 1.21

$$\frac{-\frac{1}{7\sin(bx+a)^7\cos(bx+a)^5}+\frac{12}{35\sin(bx+a)^5\cos(bx+a)^5}-\frac{24}{35\sin(bx+a)^5\cos(bx+a)^3}+\frac{64}{35\sin(bx+a)^3\cos(bx+a)^3}-\frac{128}{35\sin(bx+a)^3\cos(bx+a)}\cdots}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x)
```

```
[Out] 1/64/b*(-1/7/sin(b*x+a)^7/cos(b*x+a)^5+12/35/sin(b*x+a)^5/cos(b*x+a)^5-24/3
5/sin(b*x+a)^5/cos(b*x+a)^3+64/35/sin(b*x+a)^3/cos(b*x+a)^3-128/35/sin(b*x+
a)^3/cos(b*x+a)+512/35/sin(b*x+a)/cos(b*x+a)-1024/35*cot(b*x+a))
```

maxima [B] time = 0.48, size = 2710, normalized size = 26.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -32/35*((20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) +
 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(24*b*x + 24*a) - 2*(20*sin(10
*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a
) + 2*sin(2*b*x + 2*a))*cos(22*b*x + 22*a) - 4*(20*sin(10*b*x + 10*a) - 5*s
in(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x +
2*a))*cos(20*b*x + 20*a) + 10*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) -
 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(18*b*x
+ 18*a) + 5*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*
a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(16*b*x + 16*a) - 20*(20*s
in(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x
+ 4*a) + 2*sin(2*b*x + 2*a))*cos(14*b*x + 14*a) - (20*cos(10*b*x + 10*a) -
5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*sin(24*b*x + 24*a) + 2*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x +
8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*s
in(22*b*x + 22*a) + 4*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(
6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(20*b*x + 20
*a) - 10*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a)
+ 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(18*b*x + 18*a) - 5*(20*c
os(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x
+ 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(16*b*x + 16*a) + 20*(20*cos(10*b*x + 1
0*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*co
s(2*b*x + 2*a) - 1)*sin(14*b*x + 14*a))/(b*cos(24*b*x + 24*a)^2 + 4*b*cos(2
2*b*x + 22*a)^2 + 16*b*cos(20*b*x + 20*a)^2 + 100*b*cos(18*b*x + 18*a)^2 +
25*b*cos(16*b*x + 16*a)^2 + 400*b*cos(14*b*x + 14*a)^2 + 400*b*cos(10*b*x +
```

```
10*a)^2 + 25*b*cos(8*b*x + 8*a)^2 + 100*b*cos(6*b*x + 6*a)^2 + 16*b*cos(4*
b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(24*b*x + 24*a)^2 + 4*b*sin(22
*b*x + 22*a)^2 + 16*b*sin(20*b*x + 20*a)^2 + 100*b*sin(18*b*x + 18*a)^2 + 2
5*b*sin(16*b*x + 16*a)^2 + 400*b*sin(14*b*x + 14*a)^2 + 400*b*sin(10*b*x +
10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 100*b*sin(6*b*x + 6*a)^2 + 16*b*sin(4*b
*x + 4*a)^2 + 16*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)
^2 - 2*(2*b*cos(22*b*x + 22*a) + 4*b*cos(20*b*x + 20*a) - 10*b*cos(18*b*x +
 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x + 14*a) - 20*b*cos(10*b*x
 + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4
*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(24*b*x + 24*a) + 4*(4*b*cos(20*b*x + 20
*a) - 10*b*cos(18*b*x + 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x +
14*a) - 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6
*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(22*b*x + 22*a) -
 8*(10*b*cos(18*b*x + 18*a) + 5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14
*a) + 20*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a
) + 4*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(20*b*x + 20*a) + 2
0*(5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14*a) + 20*b*cos(10*b*x + 10*
a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b*cos(4*b*x + 4*a) +
2*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 10*(20*b*cos(14*b*x + 14*a)
- 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) -
4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) - 40*(2
0*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b
*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(14*b*x + 14*a) - 40*(5*b*
cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2
*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 10*(10*b*cos(6*b*x + 6*a) - 4*b*cos(4
*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 20*(4*b*cos(4*b*
x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 8*(2*b*cos(2*b*x +
2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 2*(2*b*sin(22*b*x + 22*
a) + 4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) - 5*b*sin(16*b*x + 16
*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x + 8
*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*
sin(24*b*x + 24*a) + 4*(4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) -
5*b*sin(16*b*x + 16*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a)
+ 5*b*sin(8*b*x + 8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(22*b*x + 22*a) - 8*(10*b*sin(18*b*x + 18*a) + 5*b*si
n(16*b*x + 16*a) - 20*b*sin(14*b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*
sin(8*b*x + 8*a) - 10*b*sin(6*b*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2
*b*x + 2*a))*sin(20*b*x + 20*a) + 20*(5*b*sin(16*b*x + 16*a) - 20*b*sin(14*
b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) - 10*b*sin(6*b
*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(18*b*x + 18*a)
 - 10*(20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x +
8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))
*sin(16*b*x + 16*a) - 40*(20*b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) -
10*b*sin(6*b*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(14
*b*x + 14*a) - 40*(5*b*sin(8*b*x + 8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4
*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 20*(5*b*sin(6*b*x
+ 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 40*(
2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.28, size = 83, normalized size = 0.81

$$\frac{15\tan(a+bx)}{64b}+\frac{\tan(a+bx)^3}{32b}+\frac{\tan(a+bx)^5}{320b}-\frac{\cot(a+bx)^7\left(\frac{5\tan(a+bx)^6}{16}+\frac{5\tan(a+bx)^4}{64}+\frac{3\tan(a+bx)^2}{160}+\frac{1}{448}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^6),x)
```

```
[Out] (15*tan(a + b*x))/(64*b) + tan(a + b*x)^3/(32*b) + tan(a + b*x)^5/(320*b) -
 (cot(a + b*x)^7*((3*tan(a + b*x)^2)/160 + (5*tan(a + b*x)^4)/64 + (5*tan(a
```

```
 + b*x)^6)/16 + 1/448))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

3.59 $\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{1024\cos^{17}(a+bx)}{17b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{11}(a+bx)}{11b}$$

```
[Out] -1024/11*cos(b*x+a)^11/b+3072/13*cos(b*x+a)^13/b-1024/5*cos(b*x+a)^15/b+102
4/17*cos(b*x+a)^17/b
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 270}

$$\frac{1024\cos^{17}(a+bx)}{17b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{11}(a+bx)}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-1024*Cos[a + b*x]^11)/(11*b) + (3072*Cos[a + b*x]^13)/(13*b) - (1024*Cos[
a + b*x]^15)/(5*b) + (1024*Cos[a + b*x]^17)/(17*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx &= 1024\int \cos^{10}(a+bx)\sin^7(a+bx)\,dx\\ &= -\frac{1024\,\text{Subst}\left(\int x^{10}\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{1024\,\text{Subst}\left(\int \left(x^{10}-3x^{12}+3x^{14}-x^{16}\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{1024\cos^{11}(a+bx)}{11b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{1024\,\text{co}}{}\end{aligned}$$

Mathematica [A] time = 0.15, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{32b}-\frac{7\cos(3(a+bx))}{16b}+\frac{7\cos(5(a+bx))}{80b}+\frac{\cos(7(a+bx))}{8b}-\frac{5\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}+\text{cos}($$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-35*Cos[a + b*x])/(32*b) - (7*Cos[3*(a + b*x)])/(16*b) + (7*Cos[5*(a + b*x
)])/(80*b) + Cos[7*(a + b*x)]/(8*b) - (5*Cos[11*(a + b*x)])/(176*b) - Cos[1
3*(a + b*x)]/(208*b) + Cos[15*(a + b*x)]/(320*b) + Cos[17*(a + b*x)]/(1088*
b)
```

fricas [A] time = 0.65, size = 46, normalized size = 0.75

$$\frac{1024 \left(715 \cos\left(bx+a\right)^{17} - 2431 \cos\left(bx+a\right)^{15} + 2805 \cos\left(bx+a\right)^{13} - 1105 \cos\left(bx+a\right)^{11}\right)}{12155\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="fricas")
```

```
[Out] 1024/12155*(715*cos(b*x + a)^17 - 2431*cos(b*x + a)^15 + 2805*cos(b*x + a)^
13 - 1105*cos(b*x + a)^11)/b
```

giac [B] time = 3.30, size = 314, normalized size = 5.15

$$-\frac{32768 \left(\frac{17 (\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{136 (\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{680 (\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{9775 (\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{71825 (\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} + \frac{221000 (\cos(}{(\cos(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="giac")
```

```
[Out] -32768/12155*(17*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 136*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 680*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
 + 9775*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 71825*(cos(b*x + a) - 1
)^5/(cos(b*x + a) + 1)^5 + 221000*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6
 + 486200*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 668525*(cos(b*x + a)
- 1)^8/(cos(b*x + a) + 1)^8 + 692835*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1
)^9 + 466752*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 233376*(cos(b*x
+ a) - 1)^11/(cos(b*x + a) + 1)^11 + 65637*(cos(b*x + a) - 1)^12/(cos(b*x +
 a) + 1)^12 + 12155*(cos(b*x + a) - 1)^13/(cos(b*x + a) + 1)^13 - 1)/(b*((c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^17)
```

maple [A] time = 0.95, size = 71, normalized size = 1.16

$$\frac{-\frac{1024\left(\sin^{6}(bx+a)\right)\left(\cos^{11}(bx+a)\right)}{17} - \frac{2048\left(\sin^{4}(bx+a)\right)\left(\cos^{11}(bx+a)\right)}{85} - \frac{8192\left(\sin^{2}(bx+a)\right)\left(\cos^{11}(bx+a)\right)}{1105} - \frac{16384\left(\cos^{11}(bx+a)\right)}{12155}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x)
```

```
[Out] 1024/b*(-1/17*sin(b*x+a)^6*cos(b*x+a)^11-2/85*sin(b*x+a)^4*cos(b*x+a)^11-8/
1105*sin(b*x+a)^2*cos(b*x+a)^11-16/12155*cos(b*x+a)^11)
```

maxima [A] time = 0.35, size = 91, normalized size = 1.49

$$\frac{715 \cos\left(17\, bx + 17\, a\right) + 2431 \cos\left(15\, bx + 15\, a\right) - 3740 \cos\left(13\, bx + 13\, a\right) - 22100 \cos\left(11\, bx + 11\, a\right) + 97240}{777920\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="maxima")
```

```
[Out] 1/777920*(715*cos(17*b*x + 17*a) + 2431*cos(15*b*x + 15*a) - 3740*cos(13*b*
x + 13*a) - 22100*cos(11*b*x + 11*a) + 97240*cos(7*b*x + 7*a) + 68068*cos(5
*b*x + 5*a) - 340340*cos(3*b*x + 3*a) - 850850*cos(b*x + a))/b
```

mupad [B] time = 0.13, size = 46, normalized size = 0.75

$$-\frac{-\frac{1024\cos(a+b\,x)^{17}}{17}+\frac{1024\cos(a+b\,x)^{15}}{5}-\frac{3072\cos(a+b\,x)^{13}}{13}+\frac{1024\cos(a+b\,x)^{11}}{11}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^10/sin(a + b*x)^3,x)
```

```
[Out] -((1024*cos(a + b*x)^11)/11 - (3072*cos(a + b*x)^13)/13 + (1024*cos(a + b*x
)^15)/5 - (1024*cos(a + b*x)^17)/17)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**10,x)
```

```
[Out] Timed out
```

3.60 $\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{512\sin^{15}(a+bx)}{15b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{9}(a+bx)}{9b}+\frac{512\sin^{7}(a+bx)}{7b}$$

```
[Out] 512/7*sin(b*x+a)^7/b-2048/9*sin(b*x+a)^9/b+3072/11*sin(b*x+a)^11/b-2048/13*
sin(b*x+a)^13/b+512/15*sin(b*x+a)^15/b
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 270}

$$\frac{512\sin^{15}(a+bx)}{15b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{9}(a+bx)}{9b}+\frac{512\sin^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

```
[Out] (512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b*
x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15*b
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx &= 512\int \cos^9(a+bx)\sin^6(a+bx)\,dx\\
&= \frac{512\,\text{Subst}\left(\int x^6\left(1-x^2\right)^4\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{512\,\text{Subst}\left(\int \left(x^6-4x^8+6x^{10}-4x^{12}+x^{14}\right)dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{512\sin^7(a+bx)}{7b}-\frac{2048\sin^9(a+bx)}{9b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{13}(
\end{aligned}$$
13b

Mathematica [A] time = 0.33, size = 58, normalized size = 0.76

$$\frac{512\left(3003\sin^{15}(a+bx)-13860\sin^{13}(a+bx)+24570\sin^{11}(a+bx)-20020\sin^{9}(a+bx)+6435\sin^{7}(a+bx)\right)}{45045b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

```
[Out] (512*(6435*Sin[a + b*x]^7 - 20020*Sin[a + b*x]^9 + 24570*Sin[a + b*x]^11 -
13860*Sin[a + b*x]^13 + 3003*Sin[a + b*x]^15))/(45045*b)
```

fricas [A] time = 0.48, size = 83, normalized size = 1.09

$$-\frac{512\left(3003\cos(bx+a)^{14}-7161\cos(bx+a)^{12}+4473\cos(bx+a)^{10}-35\cos(bx+a)^{8}-40\cos(bx+a)^{6}-4\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="fricas")
```

```
[Out] -512/45045*(3003*cos(b*x + a)^14 - 7161*cos(b*x + a)^12 + 4473*cos(b*x + a)
^10 - 35*cos(b*x + a)^8 - 40*cos(b*x + a)^6 - 48*cos(b*x + a)^4 - 64*cos(b*
x + a)^2 - 128)*sin(b*x + a)/b
```

giac [A] time = 1.93, size = 56, normalized size = 0.74

$$\frac{512\left(3003\sin(bx+a)^{15}-13860\sin(bx+a)^{13}+24570\sin(bx+a)^{11}-20020\sin(bx+a)^{9}+6435\sin(bx+a\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="giac")
```

```
[Out] 512/45045*(3003*sin(b*x + a)^15 - 13860*sin(b*x + a)^13 + 24570*sin(b*x + a
)^11 - 20020*sin(b*x + a)^9 + 6435*sin(b*x + a)^7)/b
```

maple [A] time = 1.33, size = 107, normalized size = 1.41

$$\frac{-\frac{512(\sin^5(bx+a))(\cos^{10}(bx+a))}{15}-\frac{512(\sin^3(bx+a))(\cos^{10}(bx+a))}{39}-\frac{512\sin(bx+a)(\cos^{10}(bx+a))}{143}+\frac{512\left(\frac{128}{35}+\cos^8(bx+a)+\frac{8(\cos^6(bx+a))}{7}+\frac{48(\cos^4(\right.}{1287}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x)
```

```
[Out] 512/b*(-1/15*sin(b*x+a)^5*cos(b*x+a)^10-1/39*sin(b*x+a)^3*cos(b*x+a)^10-1/1
43*sin(b*x+a)*cos(b*x+a)^10+1/1287*(128/35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48
/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.83, size = 91, normalized size = 1.20

$$-\frac{3003\sin(15\,bx+15\,a)+10395\sin(13\,bx+13\,a)-12285\sin(11\,bx+11\,a)-85085\sin(9\,bx+9\,a)-19305\,\mathrm{s}}{1441440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="maxima")
```

```
[Out] -1/1441440*(3003*sin(15*b*x + 15*a) + 10395*sin(13*b*x + 13*a) - 12285*sin(
11*b*x + 11*a) - 85085*sin(9*b*x + 9*a) - 19305*sin(7*b*x + 7*a) + 351351*s
in(5*b*x + 5*a) + 375375*sin(3*b*x + 3*a) - 2027025*sin(b*x + a))/b
```

mupad [B] time = 0.12, size = 55, normalized size = 0.72

$$\frac{\frac{512\sin(a+b\,x)^{15}}{15}-\frac{2048\sin(a+b\,x)^{13}}{13}+\frac{3072\sin(a+b\,x)^{11}}{11}-\frac{2048\sin(a+b\,x)^{9}}{9}+\frac{512\sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^9/sin(a + b*x)^3,x)
```

```
[Out] ((512*sin(a + b*x)^7)/7 - (2048*sin(a + b*x)^9)/9 + (3072*sin(a + b*x)^11)/
11 - (2048*sin(a + b*x)^13)/13 + (512*sin(a + b*x)^15)/15)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**9,x)
```

```
[Out] Timed out
```

3.61 $\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{256\cos^{13}(a+bx)}{13b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{9}(a+bx)}{9b}$$

```
[Out] -256/9*cos(b*x+a)^9/b+512/11*cos(b*x+a)^11/b-256/13*cos(b*x+a)^13/b
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 270}

$$-\frac{256\cos^{13}(a+bx)}{13b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{9}(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (512*Cos[a + b*x]^11)/(11*b) - (256*Cos[a + b
*x]^13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx &= 256\int \cos^8(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{256\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}
\end{aligned}$$

Mathematica [B] time = 0.10, size = 104, normalized size = 2.26

$$-\frac{5\cos(a+bx)}{4b}-\frac{25\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{16b}+\frac{\cos(7(a+bx))}{8b}+\frac{\cos(9(a+bx))}{72b}-\frac{3\cos(11(a+bx))}{176b}-\frac{\cos(13(}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-5*Cos[a + b*x])/(4*b) - (25*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(
16*b) + Cos[7*(a + b*x)]/(8*b) + Cos[9*(a + b*x)]/(72*b) - (3*Cos[11*(a + b
*x)])/(176*b) - Cos[13*(a + b*x)]/(208*b)
```

fricas [A] time = 0.45, size = 36, normalized size = 0.78

$$-\frac{256\left(99\cos\left(bx+a\right)^{13}-234\cos\left(bx+a\right)^{11}+143\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] -256/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

giac [B] time = 1.34, size = 248, normalized size = 5.39

$$-\frac{4096\left(\frac{13\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-\frac{78\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}-\frac{572\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}}-\frac{3718\left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}}-\frac{7722\left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}}-\frac{13728\left(\cos(bx+a)\right.}{\left(\cos(bx+a)+1\right.}}{1287\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -4096/1287*(13*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 78*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 572*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
3718*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 7722*(cos(b*x + a) - 1)^5/
(cos(b*x + a) + 1)^5 - 13728*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 12
012*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 9009*(cos(b*x + a) - 1)^8/(
cos(b*x + a) + 1)^8 - 3003*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9 - 858*
(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 - 1)/(b*((cos(b*x + a) - 1)/(co
s(b*x + a) + 1) - 1)^13)
```

maple [A] time = 0.77, size = 53, normalized size = 1.15

$$\frac{-\frac{256\left(\sin^{4}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{13}-\frac{1024\left(\sin^{2}(bx+a)\right)\left(\cos^{9}(bx+a)\right)}{143}-\frac{2048\left(\cos^{9}(bx+a)\right)}{1287}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/13*sin(b*x+a)^4*cos(b*x+a)^9-4/143*sin(b*x+a)^2*cos(b*x+a)^9-8/12
87*cos(b*x+a)^9)
```

maxima [A] time = 1.02, size = 80, normalized size = 1.74

$$-\frac{99\cos\left(13\,bx+13\,a\right)+351\cos\left(11\,bx+11\,a\right)-286\cos\left(9\,bx+9\,a\right)-2574\cos\left(7\,bx+7\,a\right)-1287\cos\left(5\,b\right.}{20592\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] -1/20592*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3*
a) + 25740*cos(b*x + a))/b
```

mupad [B] time = 0.14, size = 36, normalized size = 0.78

$$-\frac{256\,\left(99\cos\left(a+b\,x\right)^{13}-234\cos\left(a+b\,x\right)^{11}+143\cos\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x)^3,x)
```

```
[Out] -(256*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(128
7*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

3.62 $\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{3b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^5(a+bx)}{5b}$$

```
[Out] 128/5*sin(b*x+a)^5/b-384/7*sin(b*x+a)^7/b+128/3*sin(b*x+a)^9/b-128/11*sin(b
*x+a)^11/b
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 270}

$$-\frac{128\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{3b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x]
^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{128\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{128\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.17, size = 48, normalized size = 0.79

$$\frac{128\left(-105\sin^{11}(a+bx)+385\sin^9(a+bx)-495\sin^7(a+bx)+231\sin^5(a+bx)\right)}{1155b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(231*Sin[a + b*x]^5 - 495*Sin[a + b*x]^7 + 385*Sin[a + b*x]^9 - 105*Si
n[a + b*x]^11))/(1155*b)
```

fricas [A] time = 0.43, size = 63, normalized size = 1.03

$$\frac{128 (105 \cos(bx+a)^{10} - 140 \cos(bx+a)^{8} + 5 \cos(bx+a)^{6} + 6 \cos(bx+a)^{4} + 8 \cos(bx+a)^{2} + 16) \sin(bx +}{1155\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*c
os(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

giac [A] time = 1.34, size = 46, normalized size = 0.75

$$-\frac{128 \left(105 \sin(bx+a)^{11} - 385 \sin(bx+a)^{9} + 495 \sin(bx+a)^{7} - 231 \sin(bx+a)^{5}\right)}{1155\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 -
231*sin(b*x + a)^5)/b
```

maple [A] time = 0.99, size = 79, normalized size = 1.30

$$\frac{-\frac{128\left(\sin^{3}(bx+a)\right)\left(\cos^{8}(bx+a)\right)}{11} - \frac{128 \sin(bx+a)\left(\cos^{8}(bx+a)\right)}{33} + \frac{128\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{231}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/11*sin(b*x+a)^3*cos(b*x+a)^8-1/33*sin(b*x+a)*cos(b*x+a)^8+1/231*(
16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.75, size = 69, normalized size = 1.13

$$\frac{105 \sin(11\,bx + 11\,a) + 385 \sin(9\,bx + 9\,a) - 165 \sin(7\,bx + 7\,a) - 2541 \sin(5\,bx + 5\,a) - 2310 \sin(3\,bx + 3\,a)}{9240\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*a
) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

mupad [B] time = 0.12, size = 45, normalized size = 0.74

$$\frac{-\frac{128 \sin(a+b\,x)^{11}}{11} + \frac{128 \sin(a+b\,x)^{9}}{3} - \frac{384 \sin(a+b\,x)^{7}}{7} + \frac{128 \sin(a+b\,x)^{5}}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x)^3,x)
```

```
[Out] ((128*sin(a + b*x)^5)/5 - (384*sin(a + b*x)^7)/7 + (128*sin(a + b*x)^9)/3 -
 (128*sin(a + b*x)^11)/11)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

3.63 $\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{64\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2565, 14}

$$\frac{64\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^3(a+bx)\,dx\\ &= -\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.15, size = 27, normalized size = 0.87

$$\frac{32\cos^7(a+bx)(7\cos(2(a+bx))-11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (32*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

fricas [A] time = 0.44, size = 26, normalized size = 0.84

$$\frac{64 \left(7 \cos\left(bx + a\right)^{9} - 9 \cos\left(bx + a\right)^{7}\right)}{63 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

giac [B] time = 1.44, size = 182, normalized size = 5.87

$$-\frac{256 \left(\frac{9 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1} + \frac{27 \left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}} + \frac{189 \left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}} + \frac{189 \left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}} + \frac{315 \left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}} + \frac{105 \left(\cos(bx+a)-1\right)^{6}}{\left(\cos(bx+a)+1\right)^{6}} -}{63 \, b \left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -256/63*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 27*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 189*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 189*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 315*(cos(b*x + a) - 1)^5/(cos(b
*x + a) + 1)^5 + 105*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 63*(cos(b*
x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 1)^9)
```

maple [A] time = 0.67, size = 35, normalized size = 1.13

$$\frac{-\frac{64\left(\sin^{2}(bx+a)\right)\left(\cos^{7}(bx+a)\right)}{9} - \frac{128\left(\cos^{7}(bx+a)\right)}{63}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/9*sin(b*x+a)^2*cos(b*x+a)^7-2/63*cos(b*x+a)^7)
```

maxima [A] time = 1.05, size = 47, normalized size = 1.52

$$\frac{7 \cos\left(9 \, bx + 9 \, a\right) + 27 \cos\left(7 \, bx + 7 \, a\right) - 168 \cos\left(3 \, bx + 3 \, a\right) - 378 \cos\left(bx + a\right)}{252 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/252*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 37
8*cos(b*x + a))/b
```

mupad [B] time = 0.05, size = 26, normalized size = 0.84

$$-\frac{64 \left(9 \cos\left(a + b\,x\right)^{7} - 7 \cos\left(a + b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x)^3,x)
[Out] -(64*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**6,x)
[Out] Timed out
```

3.64 $\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^3(a+bx)}{3b}$$

```
[Out] 32/3*sin(b*x+a)^3/b-64/5*sin(b*x+a)^5/b+32/7*sin(b*x+a)^7/b
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4288, 2564, 270}

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^3)/(3*b) - (64*Sin[a + b*x]^5)/(5*b) + (32*Sin[a + b*x]^7)
/(7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx&=32\int \cos^5(a+bx)\sin^2(a+bx)\,dx\\&=\frac{32\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\&=\frac{32\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\&=\frac{32\sin^3(a+bx)}{3b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 37, normalized size = 0.80

$$\frac{4\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(105*
b)
```

fricas [A] time = 0.69, size = 43, normalized size = 0.93

$$-\frac{32\left(15\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

giac [A] time = 0.78, size = 36, normalized size = 0.78

$$\frac{32\left(15\sin\left(bx+a\right)^{7}-42\sin\left(bx+a\right)^{5}+35\sin\left(bx+a\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

maple [A] time = 0.96, size = 51, normalized size = 1.11

$$\frac{-\frac{32\sin(bx+a)\left(\cos^{6}(bx+a)\right)}{7}+\frac{32\left(\frac{8}{3}+\cos^{4}(bx+a)+\frac{4\left(\cos^{2}(bx+a)\right)}{3}\right)\sin(bx+a)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] 32/b*(-1/7*sin(b*x+a)*cos(b*x+a)^6+1/35*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)
*sin(b*x+a))
```

maxima [A] time = 0.45, size = 47, normalized size = 1.02

$$-\frac{15\sin\left(7\,bx+7\,a\right)+63\sin\left(5\,bx+5\,a\right)+35\sin\left(3\,bx+3\,a\right)-525\sin\left(bx+a\right)}{210\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/210*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) - 5
25*sin(b*x + a))/b
```

mupad [B] time = 0.12, size = 36, normalized size = 0.78

$$\frac{32\,\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x)^3,x)
```

```
[Out] (32*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**5,x)
[Out] Timed out
```

3.65 $\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] -16/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4288, 2565, 30}

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin(a+bx)\,dx\\ &= -\frac{16\,\text{Subst}\left(\int x^4\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{16\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

fricas [A] time = 0.51, size = 13, normalized size = 0.87

$$-\frac{16 \cos\left(bx+a\right)^5}{5\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/5*cos(b*x + a)^5/b
```

giac [B] time = 0.98, size = 74, normalized size = 4.93

$$\frac{32\left(\frac{10\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1\right)}{5\, b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 32/5*(10*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5*(cos(b*x + a) - 1)^4
/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^5
)
```

maple [A] time = 0.58, size = 14, normalized size = 0.93

$$-\frac{16\left(\cos^5\left(bx+a\right)\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] -16/5*cos(b*x+a)^5/b
```

maxima [B] time = 0.82, size = 34, normalized size = 2.27

$$-\frac{\cos\left(5\, bx+5\, a\right)+5\, \cos\left(3\, bx+3\, a\right)+10\, \cos\left(bx+a\right)}{5\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] -1/5*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$-\frac{16\cos\left(a+b\,x\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x)^3,x)
```

```
[Out] -(16*cos(a + b*x)^5)/(5*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.66 $\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=27

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

```
[Out] 8*sin(b*x+a)/b-8/3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4288, 2633}

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
[Out] (8*Sin[a + b*x])/b - (8*Sin[a + b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int \left(1-x^2\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.04

$$8\left(\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
[Out] 8*(Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b))
```

fricas [A] time = 0.64, size = 21, normalized size = 0.78

$$\frac{8\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

giac [A] time = 0.51, size = 22, normalized size = 0.81

$$-\frac{8\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

maple [A] time = 0.82, size = 22, normalized size = 0.81

$$\frac{8\left(2+\cos^{2}\left(bx+a\right)\right)\sin\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] 8/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

maxima [A] time = 1.87, size = 23, normalized size = 0.85

$$\frac{2\left(\sin\left(3\,bx+3\,a\right)+9\,\sin\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(3*b*x + 3*a) + 9*sin(b*x + a))/b
```

mupad [B] time = 0.10, size = 24, normalized size = 0.89

$$\frac{8\,\left(3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x)^3,x)
```

```
[Out] (8*(3*sin(a + b*x) - sin(a + b*x)^3))/(3*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

3.67 $\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=24

$$\frac{4\cos(a+bx)}{b}-\frac{4\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -4*arctanh(cos(b*x+a))/b+4*cos(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4288, 2592, 321, 206}

$$\frac{4\cos(a+bx)}{b}-\frac{4\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*ArcTanh[Cos[a + b*x]])/b + (4*Cos[a + b*x])/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx = 4\int \cos(a+bx)\cot(a+bx)\,dx$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= \frac{4\cos(a+bx)}{b} - \frac{4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\tanh^{-1}(\cos(a+bx))}{b} + \frac{4\cos(a+bx)}{b}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.83

$$4\left(\frac{\cos(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] 4*(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)
```

fricas [A] time = 0.62, size = 38, normalized size = 1.58

$$\frac{2\left(2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) + 1
/2))/b
```

giac [B] time = 0.57, size = 57, normalized size = 2.38

$$-\frac{2\left(\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a) +
 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.57, size = 34, normalized size = 1.42

$$\frac{4\cos(bx+a)}{b} + \frac{4\ln(\csc(bx+a) - \cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] 4*cos(b*x+a)/b+4/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.99, size = 92, normalized size = 3.83

$$\frac{2\left(2\,\cos\left(bx+a\right)-\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^{2}\right)+\log\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 2*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.11, size = 22, normalized size = 0.92

$$\frac{4\,\cos\left(a+b\,x\right)-4\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x)^3,x)
```

```
[Out] (4*cos(a + b*x) - 4*atanh(cos(a + b*x)))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

3.68 $\int \csc^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$-\frac{2\csc(a+bx)}{b}$$

```
[Out] -2*csc(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4288, 2606, 8}

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Csc[a + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\csc(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int 1\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{2\csc(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Csc[a + b*x])/b
```

fricas [**A**] time = 0.52, size = 13, normalized size = 1.18

$$-\frac{2}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] -2/(b*sin(b*x + a))
```

giac [**A**] time = 0.35, size = 13, normalized size = 1.18

$$-\frac{2}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
[Out] -2/(b*sin(b*x + a))
```

maple [**A**] time = 0.29, size = 14, normalized size = 1.27

$$-\frac{2}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a),x)
[Out] -2/b/sin(b*x+a)
```

maxima [**B**] time = 0.35, size = 84, normalized size = 7.64

$$-\frac{4\left(\cos(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\sin(bx+a)+\sin(bx+a)\right)}{b\cos(2\,bx+2\,a)^{2}+b\sin(2\,bx+2\,a)^{2}-2\,b\cos(2\,bx+2\,a)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
[Out] -4*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x
 + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a)
+ b)
```

mupad [**B**] time = 0.04, size = 13, normalized size = 1.18

$$-\frac{2}{b\,\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x)^3,x)
[Out] -2/(b*sin(a + b*x))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a),x)
[Out] Timed out
```

3.69 $\int \csc^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\csc^3(a+bx)}{6b}-\frac{\csc(a+bx)}{2b}+\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b-1/6*csc(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{4288, 2621, 302, 207\}$

$$-\frac{\csc^3(a+bx)}{6b}-\frac{\csc(a+bx)}{2b}+\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b) - Csc[a + b*x]^3/(6*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{\csc(a+bx)}{2b} - \frac{\csc^3(a+bx)}{6b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{2b} - \frac{\csc^3(a+bx)}{6b}$$

Mathematica [C] time = 0.02, size = 31, normalized size = 0.72

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/6*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.43, size = 94, normalized size = 2.19

$$\frac{3\left(\cos(bx+a)^2-1\right)\log(\sin(bx+a)+1)\sin(bx+a)-3\left(\cos(bx+a)^2-1\right)\log(-\sin(bx+a)+1)\sin(bx+}{12\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [B] time = 1.07, size = 2096, normalized size = 48.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/48*((27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan(
1/2*a)^35 + tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 630*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^32 - 267*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 18*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^34 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 25458*tan(1/2*b
*x + 2*a)^5*tan(1/2*a)^30 + 28818*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 1005
3*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 1077*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^33 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 202086*tan(1/2*b*x + 2*a)^5*tan
(1/2*a)^28 - 396738*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 244788*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^30 - 58788*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 4518*t
an(1/2*b*x + 2*a)*tan(1/2*a)^32 + 54*tan(1/2*a)^33 - 644166*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^26 + 2016678*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 - 1980792*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 + 780948*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
```

```
^29 - 123954*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 5778*tan(1/2*a)^31 + 616590
*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 4024566*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^25 + 6570108*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 4036848*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^27 + 993942*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 - 80658*tan(
1/2*a)^29 + 1290870*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 545670*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^23 - 6240000*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 8134
776*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^25 - 3273030*tan(1/2*b*x + 2*a)*tan(1/2
*a)^26 + 402730*tan(1/2*a)^27 - 3012066*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20
+ 12765366*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 13579740*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^22 + 1241820*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 3103230*tan
(1/2*b*x + 2*a)*tan(1/2*a)^24 - 796770*tan(1/2*a)^25 - 10040220*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^19 + 29284146*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 255
75756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 + 6696630*tan(1/2*b*x + 2*a)*tan(1
/2*a)^22 - 99990*tan(1/2*a)^23 + 3012066*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16
 - 10040220*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 19709370*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^19 - 14753394*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 2525334*tan
(1/2*a)^21 - 1290870*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 + 12765366*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^15 - 29284146*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 +
19709370*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 2087550*tan(1/2*a)^19 - 61659
0*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^12 - 545670*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^13 + 13579740*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 - 25575756*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^15 + 14753394*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 208755
0*tan(1/2*a)^17 + 644166*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 4024566*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^11 + 6240000*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12
+ 1241820*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 - 6696630*tan(1/2*b*x + 2*a)*t
an(1/2*a)^14 + 2525334*tan(1/2*a)^15 - 202086*tan(1/2*b*x + 2*a)^5*tan(1/2*
a)^8 + 2016678*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 6570108*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^10 + 8134776*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 3103230*t
an(1/2*b*x + 2*a)*tan(1/2*a)^12 - 99990*tan(1/2*a)^13 + 25458*tan(1/2*b*x +
 2*a)^5*tan(1/2*a)^6 - 396738*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 + 1980792*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 4036848*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
9 + 3273030*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 796770*tan(1/2*a)^11 - 630*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 28818*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5
- 244788*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 780948*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^7 - 993942*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 402730*tan(1/2*a)^9 -
 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 267*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^3 + 10053*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 58788*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^5 + 123954*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 80658*tan(1/2*a)^7 -
 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 18*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 +
 1077*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 4518*tan(1/2*b*x + 2*a)*tan(1/2*a
)^4 + 5778*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^2*tan
(1/2*a) - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*tan(1/
2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*t
an(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2
*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*
a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^3) - 24*log(abs(tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*
a) - 1)) + 24*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2
*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

maple [A] time = 0.89, size = 47, normalized size = 1.09

$$-\frac{1}{6b\sin\left(bx+a\right)^{3}}-\frac{1}{2b\sin\left(bx+a\right)}+\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a),x)
```

```
[Out] -1/6/b/sin(b*x+a)^3-1/2/b/sin(b*x+a)+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 1.14, size = 834, normalized size = 19.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

mupad [B] time = 0.13, size = 38, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)}{2\,b}-\frac{\frac{\sin(a+b\,x)^2}{2}+\frac{1}{6}}{b\,\sin\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)),x)
```

```
[Out] atanh(sin(a + b*x))/(2*b) - (sin(a + b*x)^2/2 + 1/6)/(b*sin(a + b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x), x)
```

3.70 $\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{32b}-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}$$

```
[Out] -15/32*arctanh(cos(b*x+a))/b+15/32*sec(b*x+a)/b-5/32*csc(b*x+a)^2*sec(b*x+a
)/b-1/16*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4288, 2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{32b}-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(32*b) + (15*Sec[a + b*x])/(32*b) - (5*Csc[a +
b*x]^2*Sec[a + b*x])/(32*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(16*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{15\,\text{Subst}\left(\int \frac{x}{-1+}\right.}{}$$
$$= \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b} +$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{32b} + \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{32b} -$$

Mathematica [A] time = 4.47, size = 129, normalized size = 1.84

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right) + 14\csc^2\left(\frac{1}{2}(a+bx)\right) + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(-14\tan^2\left(\frac{1}{2}(a+bx)\right)+\cos(a+bx)\left(\sec^4\left(\frac{1}{2}(a+bx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right.\right.\right.\right.}{\tan^2\left(\frac{1}{2}(a+bx)\right)-1}}{256b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/256*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(7
8 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2
]]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^
2))/b
```

fricas [B] time = 0.44, size = 132, normalized size = 1.89

$$\frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \right.}{64\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*
x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 -
 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos
(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

giac [B] time = 4.39, size = 3775, normalized size = 53.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

[Out] -1/256*(128*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2
*b*x + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2
*a)^7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*
x + 2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) -
 12*tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 +
 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
+ 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)
 - 15*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 -
 1)) + (108*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^6*ta
n(1/2*a)^46 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^4*
tan(1/2*a)^48 + 4428*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^43 - 2880*tan(1/2*b*x
+ 2*a)^6*tan(1/2*a)^44 + 536*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^45 + 48*tan(1/
2*b*x + 2*a)^4*tan(1/2*a)^46 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^47 - 1532
16*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^41 + 210486*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^42 - 95832*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^43 + 16740*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^44 - 536*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^45 - 54*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^46 + 2486912*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^39 - 4777
344*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^40 + 3337020*tan(1/2*b*x + 2*a)^5*tan(1
/2*a)^41 - 1046736*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^42 + 142488*tan(1/2*b*x
+ 2*a)^3*tan(1/2*a)^43 - 5472*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^44 - 108*tan(
1/2*b*x + 2*a)*tan(1/2*a)^45 - 20891364*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^37
+ 54465762*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^38 - 52518388*tan(1/2*b*x + 2*a)
^5*tan(1/2*a)^39 + 23995332*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^40 - 5409324*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^41 + 546366*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4
2 - 16092*tan(1/2*b*x + 2*a)*tan(1/2*a)^43 - 162*tan(1/2*a)^44 + 95841468*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^35 - 336486592*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^36 + 436639944*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^37 - 272279280*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^38 + 87480580*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^39 - 14
094720*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^40 + 977472*tan(1/2*b*x + 2*a)*tan(1
/2*a)^41 - 17280*tan(1/2*a)^42 - 234315648*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^
33 + 1144864350*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^34 - 2010477960*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^35 + 1680996460*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36 - 7
29548040*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^37 + 164195226*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^38 - 17453120*tan(1/2*b*x + 2*a)*tan(1/2*a)^39 + 642384*tan(1/
2*a)^40 + 211641984*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^31 - 1888388352*tan(1/2
*b*x + 2*a)^6*tan(1/2*a)^32 + 4935828252*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33
 - 5726288880*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^34 + 3352125960*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^35 - 1012326432*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^36 + 147
189588*tan(1/2*b*x + 2*a)*tan(1/2*a)^37 - 7902336*tan(1/2*a)^38 + 314808792
*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^29 + 96589076*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^30 - 4440981672*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 + 9447759099*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^32 - 8215340892*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33
 + 3430559358*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 - 671389308*tan(1/2*b*x +
2*a)*tan(1/2*a)^35 + 48570730*tan(1/2*a)^36 - 939066728*tan(1/2*b*x + 2*a)^
7*tan(1/2*a)^27 + 4808309376*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^28 - 665521204
8*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 - 466539808*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^30 + 7397865576*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31 - 5649253632*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^32 + 1635149568*tan(1/2*b*x + 2*a)*tan(1/2*a)^33
 - 162842112*tan(1/2*a)^34 + 564924672*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^25 -
 6391344852*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^26 + 19663795984*tan(1/2*b*x +
2*a)^5*tan(1/2*a)^27 - 24029264184*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 110
50166160*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^29 + 310355316*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^30 - 1477416960*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 + 267141600*t
an(1/2*a)^32 + 564924672*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^23 - 11879791560*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 31957055712*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^26 - 32793412368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 14381865792*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^28 - 2181545880*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
9 - 17553920*tan(1/2*a)^30 - 939066728*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^21 +

6391344852*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^22 - 11879791560*tan(1/2*b*x +
2*a)^5*tan(1/2*a)^23 + 19856454696*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 192
58200036*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 6571946504*tan(1/2*b*x + 2*a)
*tan(1/2*a)^27 - 680794644*tan(1/2*a)^28 + 314808792*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^19 - 4808309376*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^20 + 19663795984*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 31957055712*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^22 + 19856454696*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - 4012160256*tan(
1/2*b*x + 2*a)*tan(1/2*a)^25 + 925883136*tan(1/2*a)^26 + 211641984*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^17 - 96589076*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^18 -
6655212048*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 24029264184*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^20 - 32793412368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 1925
8200036*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 4012160256*tan(1/2*b*x + 2*a)*
tan(1/2*a)^23 - 234315648*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^15 + 1888388352*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^16 - 4440981672*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^17 + 466539808*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^18 + 11050166160*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^19 - 14381865792*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2
0 + 6571946504*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 925883136*tan(1/2*a)^22 +
95841468*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^13 - 1144864350*tan(1/2*b*x + 2*a
)^6*tan(1/2*a)^14 + 4935828252*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 9447759
099*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 7397865576*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^17 - 310355316*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2181545880*tan
(1/2*b*x + 2*a)*tan(1/2*a)^19 + 680794644*tan(1/2*a)^20 - 20891364*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^11 + 336486592*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^12 -
2010477960*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 5726288880*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^14 - 8215340892*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 56492
53632*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 1477416960*tan(1/2*b*x + 2*a)*ta
n(1/2*a)^17 + 17553920*tan(1/2*a)^18 + 2486912*tan(1/2*b*x + 2*a)^7*tan(1/2
*a)^9 - 54465762*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^10 + 436639944*tan(1/2*b*x
+ 2*a)^5*tan(1/2*a)^11 - 1680996460*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 3
352125960*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 3430559358*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^14 + 1635149568*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 267141600
*tan(1/2*a)^16 - 153216*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^7 + 4777344*tan(1/2
*b*x + 2*a)^6*tan(1/2*a)^8 - 52518388*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 + 2
72279280*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 - 729548040*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^11 + 1012326432*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 671389308
*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 162842112*tan(1/2*a)^14 + 4428*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^5 - 210486*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^6 + 3337
020*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 - 23995332*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^8 + 87480580*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 - 164195226*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^10 + 147189588*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - 485
70730*tan(1/2*a)^12 + 108*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^3 + 2880*tan(1/2*
b*x + 2*a)^6*tan(1/2*a)^4 - 95832*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^5 + 10467
36*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 - 5409324*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^7 + 14094720*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 - 17453120*tan(1/2*b*x +
2*a)*tan(1/2*a)^9 + 7902336*tan(1/2*a)^10 + 54*tan(1/2*b*x + 2*a)^6*tan(1/
2*a)^2 + 536*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 - 16740*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^4 + 142488*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 546366*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^6 + 977472*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 - 642384*
tan(1/2*a)^8 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 48*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^2 - 536*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 5472*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 - 16092*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 17280*tan(1/2
*a)^6 + tan(1/2*b*x + 2*a)^4 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 54*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 108*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 162*t
an(1/2*a)^4)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*tan(1/2*a)^16 -
15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^10 + 5724*tan
(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(3*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*
b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a

```
)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^4) + 120*log(abs(tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - 1)) - 1
20*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - tan(1/2*b*x +
 2*a) + 3*tan(1/2*a))))/b
```

maple [A] time = 0.74, size = 78, normalized size = 1.11

$$-\frac{1}{16b\sin(bx+a)^4\cos(bx+a)}-\frac{5}{32b\sin(bx+a)^2\cos(bx+a)}+\frac{15}{32b\cos(bx+a)}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x)
```

```
[Out] -1/16/b/sin(b*x+a)^4/cos(b*x+a)-5/32/b/sin(b*x+a)^2/cos(b*x+a)+15/32/b/cos(
b*x+a)+15/32/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.59, size = 2237, normalized size = 31.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*x
 + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)
*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos(
3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
+ 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*cos
(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) + 7
2*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*co
s(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a)
 - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(8
*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
- 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*
cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x +
 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
- 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*co
s(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - s
in(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a)
 - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*
x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*cos
(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 1
0*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(
4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a
)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2
 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b
*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*
a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*sin
(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
- 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x
)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*si
```

```
n(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
- 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(
2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x + 5
*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*
(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3*
a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/
(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 +
 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 +
 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 -
 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*c
os(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*b
*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x
 + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4
*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
- b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2*
b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b*
x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.15, size = 66, normalized size = 0.94

$$\frac{\frac{15\cos(a+b\,x)^4}{32}-\frac{25\cos(a+b\,x)^2}{32}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^2),x)
```

```
[Out] ((15*cos(a + b*x)^4)/32 - (25*cos(a + b*x)^2)/32 + 1/4)/(b*(cos(a + b*x) -
2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(32*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**2, x)
```

3.71 $\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=81

$$-\frac{7\csc^5(a+bx)}{80b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc(a+bx)}{16b}+\frac{7\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 7/16*arctanh(sin(b*x+a))/b-7/16*csc(b*x+a)/b-7/48*csc(b*x+a)^3/b-7/80*csc(b
*x+a)^5/b+1/16*csc(b*x+a)^5*sec(b*x+a)^2/b
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{7\csc^5(a+bx)}{80b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc(a+bx)}{16b}+\frac{7\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (7*ArcTanh[Sin[a + b*x]])/(16*b) - (7*Csc[a + b*x])/(16*b) - (7*Csc[a + b*x
]^3)/(48*b) - (7*Csc[a + b*x]^5)/(80*b) + (Csc[a + b*x]^5*Sec[a + b*x]^2)/(
16*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^6(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}$$
$$= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\,\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \csc(a$$
$$= -\frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b} + \frac{\csc^5(a+bx)\sec^2(a+$$
$$= \frac{7\tanh^{-1}(\sin(a+bx))}{16b} - \frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b}$$

Mathematica [C] time = 0.04, size = 31, normalized size = 0.38

$$-\frac{\csc^5(a+bx)\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\sin^2(a+bx)\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
[Out] -1/40*(Csc[a + b*x]^5*Hypergeometric2F1[-5/2, 2, -3/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.56, size = 166, normalized size = 2.05

$$-\frac{210\cos(bx+a)^6 - 490\cos(bx+a)^4 - 105\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)\log(\sin(bx+a}{480\left(b\cos(bx+a)^6 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -1/480*(210*cos(b*x + a)^6 - 490*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - 2*c
os(b*x + a)^4 + cos(b*x + a)^2)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(c
os(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-sin(b*x + a) + 1)*s
in(b*x + a) + 322*cos(b*x + a)^2 - 30)/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

giac [B] time = 9.37, size = 6318, normalized size = 78.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -1/3840*(480*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*t
an(1/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^2
3 + 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27
```

*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
17 - 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 -
 2331*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668
*tan(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2
*a)*tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^6 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan
(1/2*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 457
8*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6
 - 2178*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1
/2*a)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*t
an(1/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(
1/2*a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*t
an(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*t
an(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) + (1215*tan(1/2*b*x
 + 2*a)^9*tan(1/2*a)^56 - 810*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^57 + 270*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^58 - 45*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^59 + 3*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^60 + 71280*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^5
4 - 59940*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^55 + 17640*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^56 - 795*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^57 - 360*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^58 + 45*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^59 + 2407725*tan(1
/2*b*x + 2*a)^9*tan(1/2*a)^52 - 2568780*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^53
 + 1013880*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^54 - 144900*tan(1/2*b*x + 2*a)^6*
tan(1/2*a)^55 - 1635*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^56 + 795*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^57 + 270*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^58 - 164020320*
tan(1/2*b*x + 2*a)^9*tan(1/2*a)^50 + 294234150*tan(1/2*b*x + 2*a)^8*tan(1/2
*a)^51 - 204257160*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^52 + 68740740*tan(1/2*b*
x + 2*a)^6*tan(1/2*a)^53 - 10958240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^54 + 56
7720*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^55 + 17640*tan(1/2*b*x + 2*a)^3*tan(1/
2*a)^56 + 810*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^57 + 2847265170*tan(1/2*b*x +
 2*a)^9*tan(1/2*a)^48 - 7061366340*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^49 + 684
8508750*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^50 - 3360350485*tan(1/2*b*x + 2*a)^
6*tan(1/2*a)^51 + 882126585*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^52 - 115984800*
tan(1/2*b*x + 2*a)^4*tan(1/2*a)^53 + 5446200*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^54 + 86670*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^55 + 1215*tan(1/2*b*x + 2*a)*t
an(1/2*a)^56 - 25466109120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^46 + 82199042640
*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^47 - 104745094160*tan(1/2*b*x + 2*a)^7*tan
(1/2*a)^48 + 68829859005*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^49 - 25315144296*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^50 + 5205627685*tan(1/2*b*x + 2*a)^4*tan(1/2
*a)^51 - 546245640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^52 + 20895030*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^53 + 187920*tan(1/2*b*x + 2*a)*tan(1/2*a)^54 + 1458*t
an(1/2*a)^55 + 137905936830*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^44 - 5629893348
00*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^45 + 911808828960*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^46 - 768698936880*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^47 + 3700499622
95*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^48 - 104054449965*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^49 + 16540981710*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^50 - 1323596190*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^51 + 37166445*tan(1/2*b*x + 2*a)*tan(1/2*a)
^52 + 172530*tan(1/2*a)^53 - 472782926880*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^4
2 + 2429658984060*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^43 - 4936681833840*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^44 + 5227899617200*tan(1/2*b*x + 2*a)^6*tan(1/2*a
)^45 - 3184356059520*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^46 + 1153540125240*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^47 - 245839387280*tan(1/2*b*x + 2*a)^3*tan(1/2

*a)^48 + 28942646220*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^49 - 1600947360*tan(1/2*b*x + 2*a)*tan(1/2*a)^50 + 25793640*tan(1/2*a)^51 + 985566407445*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^40 - 6594701846310*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^41 + 17020352439990*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^42 - 22628389243545*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^43 + 17235607351635*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^44 - 7834375816360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^45 + 2124636283680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^46 - 329145687420*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^47 + 25919270610*tan(1/2*b*x + 2*a)*tan(1/2*a)^48 - 753655320*tan(1/2*a)^49 - 941937351120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^38 + 10024213003740*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^39 - 35619165658440*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^40 + 61660501611345*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^41 - 59571399047240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 33928676906985*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^43 - 11501138427120*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^44 + 2245614722020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^45 - 228535536960*tan(1/2*b*x + 2*a)*tan(1/2*a)^46 + 9122039460*tan(1/2*a)^47 - 756859069305*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^36 - 2938166946540*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^37 + 33963536732760*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^38 - 93824947269780*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^39 + 124880209066749*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^40 - 92522313305985*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41 + 39714192559670*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^42 - 9707168584980*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^43 + 1237500036990*tan(1/2*b*x + 2*a)*tan(1/2*a)^44 - 62831824476*tan(1/2*a)^45 + 3403775131200*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^34 - 19290846651030*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^35 + 27663473408040*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^36 + 27109136872980*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^37 - 118939479927840*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 + 140811947469040*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^39 - 83200102704840*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^40 + 26404824927630*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^41 - 4255065187040*tan(1/2*b*x + 2*a)*tan(1/2*a)^42 + 270445183560*tan(1/2*a)^43 - 3650513198580*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^32 + 36323787336840*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^33 - 122659045084170*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^34 + 180766125659295*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^35 - 97447951446495*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^36 - 40571312613240*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^37 + 79276457792280*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 - 40160124218470*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^39 + 8888251621845*tan(1/2*b*x + 2*a)*tan(1/2*a)^40 - 731644643640*tan(1/2*a)^41 - 19469403725760*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^31 + 130628272644000*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^32 - 338314460497335*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^33 + 429435328442040*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 - 271342502839935*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 + 64809044724520*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 11678391455970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^37 - 8484705136560*tan(1/2*b*x + 2*a)*tan(1/2*a)^38 + 1111051397230*tan(1/2*a)^39 + 3650513198580*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^28 - 19469403725760*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^29 + 180715347146880*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^31 - 456090384021345*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^32 + 507272799301335*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 - 286258153276170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 + 77334987640230*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 6866537353465*tan(1/2*b*x + 2*a)*tan(1/2*a)^36 - 329444622930*tan(1/2*a)^37 - 3403775131200*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^26 + 36323787336840*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^27 - 130628272644000*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^28 + 180715347146880*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^29 - 270808903495440*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 + 304337156718240*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 - 145126844230520*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^33 + 30650585964480*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 - 2135547006768*tan(1/2*a)^35 + 756859069305*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^24 - 19290846651030*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^25 + 122659045084170*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^26 - 338314460497335*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27 + 456090384021345*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 - 270808903495440*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 77638534829400*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 - 32750159843700*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 + 4042939650000*tan(1/2*a)^33 + 941937351120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^22 - 2938166946

540*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^23 - 27663473408040*tan(1/2*b*x + 2*a)^
7*tan(1/2*a)^24 + 180766125659295*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^25 - 4294
35328442040*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 + 507272799301335*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^27 - 304337156718240*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
28 + 77638534829400*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 2175308641800*tan(
1/2*a)^31 - 985566407445*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^20 + 1002421300374
0*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^21 - 33963536732760*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^22 + 27109136872980*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^23 + 9744795
1446495*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 271342502839935*tan(1/2*b*x +
2*a)^4*tan(1/2*a)^25 + 286258153276170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 -
145126844230520*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 + 32750159843700*tan(1/
2*b*x + 2*a)*tan(1/2*a)^28 - 2175308641800*tan(1/2*a)^29 + 472782926880*tan
(1/2*b*x + 2*a)^9*tan(1/2*a)^18 - 6594701846310*tan(1/2*b*x + 2*a)^8*tan(1/
2*a)^19 + 35619165658440*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^20 - 9382494726978
0*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^21 + 118939479927840*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^22 - 40571312613240*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 648090
44724520*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 77334987640230*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^25 - 30650585964480*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 + 40
42939650000*tan(1/2*a)^27 - 137905936830*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^16
+ 2429658984060*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^17 - 17020352439990*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^18 + 61660501611345*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^19 - 124880209066749*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20 + 14081194746904
0*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 79276457792280*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^22 + 11678391455970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 6866537
353465*tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 2135547006768*tan(1/2*a)^25 + 254
66109120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^14 - 562989334800*tan(1/2*b*x + 2*
a)^8*tan(1/2*a)^15 + 4936681833840*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^16 - 226
28389243545*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^17 + 59571399047240*tan(1/2*b*x
+ 2*a)^5*tan(1/2*a)^18 - 92522313305985*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^19
+ 83200102704840*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 40160124218470*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^21 + 8484705136560*tan(1/2*b*x + 2*a)*tan(1/2*a)
^22 - 329444622930*tan(1/2*a)^23 - 2847265170*tan(1/2*b*x + 2*a)^9*tan(1/2*
a)^12 + 82199042640*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^13 - 911808828960*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^14 + 5227899617200*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^15 - 17235607351635*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 + 33928676906985*
tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 - 39714192559670*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^18 + 26404824927630*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 - 888825162
1845*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 1111051397230*tan(1/2*a)^21 + 16402
0320*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^10 - 7061366340*tan(1/2*b*x + 2*a)^8*t
an(1/2*a)^11 + 104745094160*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^12 - 7686989368
80*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 3184356059520*tan(1/2*b*x + 2*a)^5*
tan(1/2*a)^14 - 7834375816360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 + 11501138
427120*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 9707168584980*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^17 + 4255065187040*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 - 731644
643640*tan(1/2*a)^19 - 2407725*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^8 + 29423415
0*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^9 - 6848508750*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^10 + 68829859005*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 370049962295*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^12 + 1153540125240*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^13 - 2124636283680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 2245614722020
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 1237500036990*tan(1/2*b*x + 2*a)*tan(
1/2*a)^16 + 270445183560*tan(1/2*a)^17 - 71280*tan(1/2*b*x + 2*a)^9*tan(1/2
*a)^6 - 2568780*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^7 + 204257160*tan(1/2*b*x +
2*a)^7*tan(1/2*a)^8 - 3360350485*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^9 + 25315
144296*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 104054449965*tan(1/2*b*x + 2*a)
^4*tan(1/2*a)^11 + 245839387280*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 - 329145
687420*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 + 228535536960*tan(1/2*b*x + 2*a)
*tan(1/2*a)^14 - 62831824476*tan(1/2*a)^15 - 1215*tan(1/2*b*x + 2*a)^9*tan(
1/2*a)^4 - 59940*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^5 - 1013880*tan(1/2*b*x +
2*a)^7*tan(1/2*a)^6 + 68740740*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^7 - 88212658

```
5*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 5205627685*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^9 - 16540981710*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 + 28942646220*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 25919270610*tan(1/2*b*x + 2*a)*tan(1/2*a)^
12 + 9122039460*tan(1/2*a)^13 - 810*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^3 - 176
40*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^4 - 144900*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^5 + 10958240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 115984800*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^7 + 546245640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 132359
6190*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 + 1600947360*tan(1/2*b*x + 2*a)*tan(
1/2*a)^10 - 753655320*tan(1/2*a)^11 - 270*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2
 - 795*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^3 + 1635*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^4 + 567720*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 5446200*tan(1/2*b*x + 2
*a)^3*tan(1/2*a)^6 + 20895030*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 37166445*
tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 25793640*tan(1/2*a)^9 - 45*tan(1/2*b*x +
2*a)^6*tan(1/2*a) + 360*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 + 795*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^3 - 17640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 + 86670*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - 187920*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 +
172530*tan(1/2*a)^7 - 3*tan(1/2*b*x + 2*a)^5 + 45*tan(1/2*b*x + 2*a)^4*tan(
1/2*a) - 270*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 810*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^3 - 1215*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1458*tan(1/2*a)^5)/((2
43*tan(1/2*a)^25 - 4050*tan(1/2*a)^23 + 28215*tan(1/2*a)^21 - 106200*tan(1/
2*a)^19 + 233430*tan(1/2*a)^17 - 304300*tan(1/2*a)^15 + 233430*tan(1/2*a)^1
3 - 106200*tan(1/2*a)^11 + 28215*tan(1/2*a)^9 - 4050*tan(1/2*a)^7 + 243*tan
(1/2*a)^5)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/
2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1
/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b
*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a
))^5) - 1680*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)
*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*
a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 1680*log(abs(tan(1/2*b*x +
 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*t
an(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(
1/2*a) - 1)))/b
```

maple [A] time = 0.71, size = 97, normalized size = 1.20

$$-\frac{1}{40b\sin(bx+a)^5\cos(bx+a)^2}-\frac{7}{120b\sin(bx+a)^3\cos(bx+a)^2}+\frac{7}{48b\sin(bx+a)\cos(bx+a)^2}-\frac{7}{16b\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x)
```

```
[Out] -1/40/b/sin(b*x+a)^5/cos(b*x+a)^2-7/120/b/sin(b*x+a)^3/cos(b*x+a)^2+7/48/b/
sin(b*x+a)/cos(b*x+a)^2-7/16/b/sin(b*x+a)+7/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 1.05, size = 3095, normalized size = 38.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/480*(4*(105*sin(13*b*x + 13*a) - 350*sin(11*b*x + 11*a) + 231*sin(9*b*x +
 9*a) + 412*sin(7*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a)
+ 105*sin(b*x + a))*cos(14*b*x + 14*a) + 420*(3*sin(12*b*x + 12*a) - sin(10
*b*x + 10*a) - 5*sin(8*b*x + 8*a) + 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) -
 3*sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 12*(350*sin(11*b*x + 11*a) - 231*
sin(9*b*x + 9*a) - 412*sin(7*b*x + 7*a) - 231*sin(5*b*x + 5*a) + 350*sin(3*
b*x + 3*a) - 105*sin(b*x + a))*cos(12*b*x + 12*a) + 1400*(sin(10*b*x + 10*a
) + 5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*
```

```
x + 2*a))*cos(11*b*x + 11*a) + 4*(231*sin(9*b*x + 9*a) + 412*sin(7*b*x + 7*
a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(10
*b*x + 10*a) - 924*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4
*a) + 3*sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 20*(412*sin(7*b*x + 7*a) + 231
*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(8*b*x + 8*
a) + 1648*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(
7*b*x + 7*a) - 140*(33*sin(5*b*x + 5*a) - 50*sin(3*b*x + 3*a) + 15*sin(b*x
+ a))*cos(6*b*x + 6*a) + 924*(sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(5*
b*x + 5*a) + 140*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) +
105*(2*(3*cos(12*b*x + 12*a) - cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*
cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(14*b*x +
14*a) - cos(14*b*x + 14*a)^2 + 6*(cos(10*b*x + 10*a) + 5*cos(8*b*x + 8*a) -
 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(12*b*x
 + 12*a) - 9*cos(12*b*x + 12*a)^2 - 2*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6
*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(1
0*b*x + 10*a)^2 + 10*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x +
 2*a) + 1)*cos(8*b*x + 8*a) - 25*cos(8*b*x + 8*a)^2 - 10*(cos(4*b*x + 4*a)
- 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 25*cos(6*b*x + 6*a)^2 + 2*(3*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 9*cos(2*b*x +
2*a)^2 + 2*(3*sin(12*b*x + 12*a) - sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a)
+ 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(14*b*x +
14*a) - sin(14*b*x + 14*a)^2 + 6*(sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) -
 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 1
2*a) - 9*sin(12*b*x + 12*a)^2 - 2*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a)
- sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x +
10*a)^2 + 10*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*s
in(8*b*x + 8*a) - 25*sin(8*b*x + 8*a)^2 - 10*(sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*sin(6*b*x + 6*a) - 25*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 + 6
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2
*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a
)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) - 4*(105*cos(13*b*x + 13*a) - 350*cos(11*b*x + 11*a) + 231*cos
(9*b*x + 9*a) + 412*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x
 + 3*a) + 105*cos(b*x + a))*sin(14*b*x + 14*a) - 420*(3*cos(12*b*x + 12*a)
- cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*cos(6*b*x + 6*a) + cos(4*b*x
+ 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(13*b*x + 13*a) - 12*(350*cos(11*b*x +
11*a) - 231*cos(9*b*x + 9*a) - 412*cos(7*b*x + 7*a) - 231*cos(5*b*x + 5*a)
+ 350*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(12*b*x + 12*a) - 1400*(cos(1
0*b*x + 10*a) + 5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a)
+ 3*cos(2*b*x + 2*a) - 1)*sin(11*b*x + 11*a) - 4*(231*cos(9*b*x + 9*a) + 41
2*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(
b*x + a))*sin(10*b*x + 10*a) + 924*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a)
 - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*sin(9*b*x + 9*a) - 20*(412*co
s(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(b*x
+ a))*sin(8*b*x + 8*a) - 1648*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*co
s(2*b*x + 2*a) + 1)*sin(7*b*x + 7*a) + 140*(33*cos(5*b*x + 5*a) - 50*cos(3*
b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a) - 924*(cos(4*b*x + 4*a) - 3*
cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 140*(10*cos(3*b*x + 3*a) - 3*cos(b
*x + a))*sin(4*b*x + 4*a) - 1400*(3*cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)
+ 4200*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 1260*cos(b*x + a)*sin(2*b*x + 2*
a) + 1260*cos(2*b*x + 2*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x +
 14*a)^2 + 9*b*cos(12*b*x + 12*a)^2 + b*cos(10*b*x + 10*a)^2 + 25*b*cos(8*b
*x + 8*a)^2 + 25*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*
x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + 9*b*sin(12*b*x + 12*a)^2 + b*sin(10*b
*x + 10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 25*b*sin(6*b*x + 6*a)^2 + b*sin(4*
b*x + 4*a)^2 - 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)
^2 - 2*(3*b*cos(12*b*x + 12*a) - b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a
) + 5*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*c
```

```
os(14*b*x + 14*a) - 6*(b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) - 5*b*co
s(6*b*x + 6*a) - b*cos(4*b*x + 4*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(12*b*x
+ 12*a) + 2*(5*b*cos(8*b*x + 8*a) - 5*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*
a) + 3*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 10*(5*b*cos(6*b*x + 6*a
) + b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 10*(b
*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 2*(3*b*cos
(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(12*
b*x + 12*a) - b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) + 5*b*sin(6*b*x +
 6*a) + b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 6*(
b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x + 8*a) - 5*b*sin(6*b*x + 6*a) - b*sin(
4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 2*(5*b*sin(8*b*x
+ 8*a) - 5*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*
sin(10*b*x + 10*a) - 10*(5*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 3*b*si
n(2*b*x + 2*a))*sin(8*b*x + 8*a) + 10*(b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x +
 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.18, size = 71, normalized size = 0.88

$$\frac{7\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b}-\frac{-\frac{7\,{\sin\left(a+b\,x\right)}^6}{16}+\frac{7\,{\sin\left(a+b\,x\right)}^4}{24}+\frac{7\,{\sin\left(a+b\,x\right)}^2}{120}+\frac{1}{40}}{b\,\left({\sin\left(a+b\,x\right)}^5-{\sin\left(a+b\,x\right)}^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (7*atanh(sin(a + b*x)))/(16*b) - ((7*sin(a + b*x)^2)/120 + (7*sin(a + b*x)^
4)/24 - (7*sin(a + b*x)^6)/16 + 1/40)/(b*(sin(a + b*x)^5 - sin(a + b*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**3, x)
```

3.72 $\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=112

$$\frac{35\sec^3(a+bx)}{256b}+\frac{105\sec(a+bx)}{256b}-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}-\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b}$$

```
[Out] -105/256*arctanh(cos(b*x+a))/b+105/256*sec(b*x+a)/b+35/256*sec(b*x+a)^3/b-2
1/256*csc(b*x+a)^2*sec(b*x+a)^3/b-3/128*csc(b*x+a)^4*sec(b*x+a)^3/b-1/96*cs
c(b*x+a)^6*sec(b*x+a)^3/b
```

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{256b}+\frac{105\sec(a+bx)}{256b}-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}-\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-105*ArcTanh[Cos[a + b*x]])/(256*b) + (105*Sec[a + b*x])/(256*b) + (35*Sec
[a + b*x]^3)/(256*b) - (21*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (3*Csc[
a + b*x]^4*Sec[a + b*x]^3)/(128*b) - (Csc[a + b*x]^6*Sec[a + b*x]^3)/(96*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^7(a+bx)\sec^4(a+bx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^4}\,dx, x, \sec(a+bx)\right)}{16b}$$

$$= -\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{3\,\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$

$$= -\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{21\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+b}{}$$

$$= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+b}{}$$

$$= \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc}{}$$

$$= -\frac{105\tanh^{-1}(\cos(a+bx))}{256b} + \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\csc^2}{}$$

Mathematica [B] time = 0.85, size = 278, normalized size = 2.48

$$\frac{\csc^{12}(a+bx)\left(-4752\cos(2(a+bx)) + 1600\cos(3(a+bx)) + 504\cos(4(a+bx)) + 1680\cos(6(a+bx)) - 600\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (Csc[a + b*x]^12*(1150 - 4752*Cos[2*(a + b*x)] + 1600*Cos[3*(a + b*x)] + 50
4*Cos[4*(a + b*x)] + 1680*Cos[6*(a + b*x)] - 600*Cos[7*(a + b*x)] - 630*Cos
[8*(a + b*x)] + 200*Cos[9*(a + b*x)] + 2520*Cos[3*(a + b*x)]*Log[Cos[(a + b
*x)/2]] - 945*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 315*Cos[9*(a + b*x)]
*Log[Cos[(a + b*x)/2]] - 30*Cos[a + b*x]*(40 + 63*Log[Cos[(a + b*x)/2]] - 6
3*Log[Sin[(a + b*x)/2]]) - 2520*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 94
5*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]] - 315*Cos[9*(a + b*x)]*Log[Sin[(a
+ b*x)/2]]))/(3072*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.55, size = 194, normalized size = 1.73

$$\frac{630\cos(bx+a)^8 - 1680\cos(bx+a)^6 + 1386\cos(bx+a)^4 - 288\cos(bx+a)^2 - 315\left(\cos(bx+a)^9 - 3\cos\right.}{1536\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/1536*(630*cos(b*x + a)^8 - 1680*cos(b*x + a)^6 + 1386*cos(b*x + a)^4 - 28
8*cos(b*x + a)^2 - 315*(cos(b*x + a)^9 - 3*cos(b*x + a)^7 + 3*cos(b*x + a)^
```

```
5 - cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 315*(cos(b*x + a)^9 - 3*c
os(b*x + a)^7 + 3*cos(b*x + a)^5 - cos(b*x + a)^3)*log(-1/2*cos(b*x + a) +
1/2) - 32)/(b*cos(b*x + a)^9 - 3*b*cos(b*x + a)^7 + 3*b*cos(b*x + a)^5 - b*
cos(b*x + a)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.85, size = 120, normalized size = 1.07

$$-\frac{1}{96b\sin(bx+a)^6\cos(bx+a)^3}-\frac{3}{128b\sin(bx+a)^4\cos(bx+a)^3}+\frac{7}{128b\sin(bx+a)^2\cos(bx+a)^3}-\frac{}{256b\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x)
```

```
[Out] -1/96/b/sin(b*x+a)^6/cos(b*x+a)^3-3/128/b/sin(b*x+a)^4/cos(b*x+a)^3+7/128/b
/sin(b*x+a)^2/cos(b*x+a)^3-35/256/b/sin(b*x+a)^2/cos(b*x+a)+105/256/b/cos(b
*x+a)+105/256/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 1.57, size = 4268, normalized size = 38.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(315*cos(17*b*x + 17*a) - 840*cos(15*b*x + 15*a) - 252*cos(13*b*x
 + 13*a) + 2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x
 + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*c
os(18*b*x + 18*a) - 1260*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
 2*a) - 1)*cos(17*b*x + 17*a) + 12*(840*cos(15*b*x + 15*a) + 252*cos(13*b*x
 + 13*a) - 2376*cos(11*b*x + 11*a) + 1150*cos(9*b*x + 9*a) - 2376*cos(7*b*x
 + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) - 315*cos(b*x + a))*c
os(16*b*x + 16*a) - 3360*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*c
os(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(15*b*x +
 15*a) - 1008*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x +
8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(13*b*x + 13*a) + 32
*(2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x + 7*a) -
 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*cos(12*b*x
 + 12*a) - 9504*(6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x +
6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 24*(1150*cos(9*b*x + 9*
a) - 2376*cos(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) -
315*cos(b*x + a))*cos(10*b*x + 10*a) + 4600*(6*cos(8*b*x + 8*a) - 8*cos(6*b
*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(9*b*x + 9*a) - 72*(792*cos(7*b*x +
7*a) - 84*cos(5*b*x + 5*a) - 280*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 9504*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x
+ 7*a) - 672*(12*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) - 15*cos(b*x + a))*
cos(6*b*x + 6*a) + 1008*(3*cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 3360*(3
*cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 3780*cos(2*b*x + 2*a)*cos(b*x + a
) + 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*cos(10*b*x + 10
*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos
```

```
(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x + 12*a) - 6*cos(10
*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a)
 + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*cos(10*b*x + 10*a
) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(1
2*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x + 8*a) - 8*cos(6*
b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 36*cos(10*b*x + 1
0*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a)
 - 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64
*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*x + 16*a) - 8*si
n(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x
+ 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*x + 18*a)^2 + 6*
(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a) + 8*sin(6
*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)
^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3
*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*a)^2 - 12*(6*sin
(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a)
 - 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*s
in(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*a)^2 + 48*sin(6*
b*x + 6*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1
)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*s
in(a) + sin(a)^2) - 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6
*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x
 + 2*a) - 1)*cos(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x +
12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*
cos(2*b*x + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
 2*a) - 1)*cos(12*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x +
 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 3
6*cos(10*b*x + 10*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*c
os(8*b*x + 8*a) - 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6
*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*
x + 16*a) - 8*sin(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a
) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*
x + 18*a)^2 + 6*(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x
+ 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*si
n(16*b*x + 16*a)^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(
6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*
a)^2 - 12*(6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) - 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin
(2*b*x + 2*a))*sin(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*
a)^2 + 48*sin(6*b*x + 6*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(
2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^
2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(315*sin(17*b*x + 17*a) - 840*sin(15*
b*x + 15*a) - 252*sin(13*b*x + 13*a) + 2376*sin(11*b*x + 11*a) - 1150*sin(9
*b*x + 9*a) + 2376*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) - 840*sin(3*b*x
+ 3*a) + 315*sin(b*x + a))*sin(18*b*x + 18*a) - 1260*(3*sin(16*b*x + 16*a)
- 8*sin(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(
6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(17*b*x + 17*a) + 12*(840*sin(15*b*x
+ 15*a) + 252*sin(13*b*x + 13*a) - 2376*sin(11*b*x + 11*a) + 1150*sin(9*b*x
 + 9*a) - 2376*sin(7*b*x + 7*a) + 252*sin(5*b*x + 5*a) + 840*sin(3*b*x + 3*
a) - 315*sin(b*x + a))*sin(16*b*x + 16*a) - 3360*(8*sin(12*b*x + 12*a) - 6*
sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x
+ 2*a))*sin(15*b*x + 15*a) - 1008*(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10
*a) - 6*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(13*
b*x + 13*a) + 32*(2376*sin(11*b*x + 11*a) - 1150*sin(9*b*x + 9*a) + 2376*si
n(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) - 840*sin(3*b*x + 3*a) + 315*sin(b*x
+ a))*sin(12*b*x + 12*a) - 9504*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a)
- 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(11*b*x + 11*a) + 24*(1150*si
n(9*b*x + 9*a) - 2376*sin(7*b*x + 7*a) + 252*sin(5*b*x + 5*a) + 840*sin(3*b
```

```
*x + 3*a) - 315*sin(b*x + a))*sin(10*b*x + 10*a) + 4600*(6*sin(8*b*x + 8*a)
 - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(9*b*x + 9*a) - 72*(792*sin(
7*b*x + 7*a) - 84*sin(5*b*x + 5*a) - 280*sin(3*b*x + 3*a) + 105*sin(b*x + a
))*sin(8*b*x + 8*a) + 9504*(8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(7*
b*x + 7*a) - 672*(12*sin(5*b*x + 5*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x +
a))*sin(6*b*x + 6*a) + 3024*sin(5*b*x + 5*a)*sin(2*b*x + 2*a) + 10080*sin(3
*b*x + 3*a)*sin(2*b*x + 2*a) - 3780*sin(2*b*x + 2*a)*sin(b*x + a) + 1260*co
s(b*x + a))/(b*cos(18*b*x + 18*a)^2 + 9*b*cos(16*b*x + 16*a)^2 + 64*b*cos(1
2*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(8*b*x + 8*a)^2 + 64*
b*cos(6*b*x + 6*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(18*b*x + 18*a)^2 + 9*
b*sin(16*b*x + 16*a)^2 + 64*b*sin(12*b*x + 12*a)^2 + 36*b*sin(10*b*x + 10*a
)^2 + 36*b*sin(8*b*x + 8*a)^2 + 64*b*sin(6*b*x + 6*a)^2 - 48*b*sin(6*b*x +
6*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(16*b*x + 16*a)
- 8*b*cos(12*b*x + 12*a) + 6*b*cos(10*b*x + 10*a) + 6*b*cos(8*b*x + 8*a) -
8*b*cos(6*b*x + 6*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 6*(8*
b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) - 6*b*cos(8*b*x + 8*a) + 8*b*
cos(6*b*x + 6*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) - 16*(6*b*c
os(10*b*x + 10*a) + 6*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(2
*b*x + 2*a) - b)*cos(12*b*x + 12*a) + 12*(6*b*cos(8*b*x + 8*a) - 8*b*cos(6*
b*x + 6*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 12*(8*b*cos(6*b
*x + 6*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 16*(3*b*cos(2*b*x
+ 2*a) - b)*cos(6*b*x + 6*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(16*b*x + 1
6*a) - 8*b*sin(12*b*x + 12*a) + 6*b*sin(10*b*x + 10*a) + 6*b*sin(8*b*x + 8*
a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 6*(8
*b*sin(12*b*x + 12*a) - 6*b*sin(10*b*x + 10*a) - 6*b*sin(8*b*x + 8*a) + 8*b
*sin(6*b*x + 6*a) - 3*b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 16*(6*b*sin(
10*b*x + 10*a) + 6*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(2*b*
x + 2*a))*sin(12*b*x + 12*a) + 12*(6*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6
*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 12*(8*b*sin(6*b*x + 6*a) -
 3*b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

mupad [B] time = 0.23, size = 100, normalized size = 0.89

$$\frac{-\frac{105\cos(a+bx)^8}{256}+\frac{35\cos(a+bx)^6}{32}-\frac{231\cos(a+bx)^4}{256}+\frac{3\cos(a+bx)^2}{16}+\frac{1}{48}}{b\left(-\cos\left(a+b\,x\right)^9+3\cos\left(a+b\,x\right)^7-3\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{105\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^4),x)
```

```
[Out] ((3*cos(a + b*x)^2)/16 - (231*cos(a + b*x)^4)/256 + (35*cos(a + b*x)^6)/32
- (105*cos(a + b*x)^8)/256 + 1/48)/(b*(cos(a + b*x)^3 - 3*cos(a + b*x)^5 +
3*cos(a + b*x)^7 - cos(a + b*x)^9)) - (105*atanh(cos(a + b*x)))/(256*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**4, x)
```

3.73 $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}-\frac{5\sqrt{\sin(2a+2}}{1}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+5/24*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/6*cos(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b-5/16*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4302, 4301, 4305}

$$\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (5*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) - (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} - \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{8}\int \sin(a+bx$$
$$= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} - \cos(a+bx)$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{s}\right.}{32b}$$

Mathematica [A] time = 0.30, size = 98, normalized size = 0.72

$$\frac{15\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}\,(1}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (15*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(14*Cos[a + b*x] + 3*Cos[3*(a + b*x)] - 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/(96*b)
```

fricas [B] time = 0.62, size = 291, normalized size = 2.14

$$8\sqrt{2}\left(32\cos(bx+a)^5-52\cos(bx+a)^3+5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(8*sqrt(2)*(32*cos(b*x + a)^5 - 52*cos(b*x + a)^3 + 5*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) + 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 30*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 89.38, size = 183661406, normalized size = 1350451.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx + 2\,a\right)^{\frac{5}{2}} \sin\left(bx + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a + b\,x\right)\,\sin\left(2\,a + 2\,b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
[Out] Timed out
```

3.74 $\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}-\frac{3\log\left(\sin(a+bx\right.}{}$$

```
[Out] -3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-1/4*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+3/8*sin(b*x+a)*sin(2
*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4302, 4301, 4306}

$$\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\sin(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (3*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{8}\int \frac{\sin(a+}{\sqrt{\sin(2a}}\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\mathrm{si}}\right.}{16b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 86, normalized size = 0.78

$$\frac{2\sqrt{\sin(2(a+bx))}\,(2\sin(a+bx)-\sin(3(a+bx)))-3\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\ }\right.\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(2*Sin[a + b*x] - Sin
[3*(a + b*x)]))/(16*b)
```

fricas [B] time = 0.48, size = 280, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-}{\cos(bx+a)^2+2\cos(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-
32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*si
n(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x +
 a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(2\,bx+2\,a)^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

maple [B] time = 25.67, size = 65166862, normalized size = 592426.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(2\,bx+2\,a)^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.75 $\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a\right.}{4b}$$

```
[Out] -1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-1/2*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4302, 4305}

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{s}\right.}{4b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.86

$$\frac{-\sin^{-1}(\cos(a+bx)-\sin(a+bx))-2\sqrt{\sin(2(a+bx))}\cos(a+bx)+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\text{cc}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Cos[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

fricas [B] time = 0.78, size = 266, normalized size = 3.17

$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) - 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) - 2*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(2bx + 2a)}\sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

maple [B] time = 2.85, size = 6214674, normalized size = 73984.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(2bx + 2a)}\sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a + b\,x)\,\sqrt{\sin(2\,a + 2\,b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.76 $\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4306}

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2b\right.}}{2b}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.86

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]])/b
```

fricas [B] time = 0.49, size = 240, normalized size = 4.14

$$2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx}{\cos(bx+a)-\sin(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 3.21, size = 15972263, normalized size = 275383.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+b\,x)}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.77 $\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=23

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4292}

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx=\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.96

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]])
```

fricas [A] time = 0.47, size = 39, normalized size = 1.70

$$\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\cos(bx+a)}{2\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + cos(b*x + a))/(b*cos(b*x + a
))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [**B**] time = 11.40, size = 59119746, normalized size = 2570423.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [**B**] time = 0.28, size = 34, normalized size = 1.48

$$\frac{\cos(a+b\,x)\,\sqrt{\sin(2\,a+2\,b\,x)}}{b\,(\cos(2\,a+2\,b\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
[Out] (cos(a + b*x)*sin(2*a + 2*b*x)^(1/2))/(b*(cos(2*a + 2*b*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.78 $\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4304, 4291}

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{12}\tan(a+bx)\sec(a+bx)-\frac{1}{4}\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sin[2*(a + b*x)]]*(-1/4*Csc[a + b*x] + (Sec[a + b*x]*Tan[a + b*x])/12
))/b
```

fricas [A] time = 0.45, size = 69, normalized size = 1.30

$$-\frac{4\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\left(4\cos(bx+a)^2-1\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{12\,b\cos(bx+a)^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*(4*cos(b*x + a)^2 - 1)*sqrt(
cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2*sin(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 41.62, size = 597, normalized size = 11.26

$$\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(6\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/8/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(6*(tan(1/2*b*x+
1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan
(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x
+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-3*(tan(1/2*b*x+1/2*a)*(t
an(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+
1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+6*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*
x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)
^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1
/2*2^(1/2))-3*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*
b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(
1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*(tan(1/2*b*x+1/2
*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4+2*tan(1/2*b*x+1/2*
a)^4*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)-2*(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2-2*(tan(1/2*b*x+1/2*a)^
3-tan(1/2*b*x+1/2*a))^(1/2))/tan(1/2*b*x+1/2*a)/(1+tan(1/2*b*x+1/2*a)^2)/(t
an(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 3.08, size = 108, normalized size = 2.04

$$-\frac{2\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)}{3\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*1i + exp(a*4i + b*x*4i)*1i + 1i))/(3*b*(exp
(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] 1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-4/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2
)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4304, 4303, 4292}

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b*Sin[2*a
+ 2*b*x]^(3/2)) + (8*Sin[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\
&= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\
&= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 52, normalized size = 0.66

$$\frac{\sqrt{\sin(2(a+bx))}\left(3\sec(a+bx)\left(\sec^2(a+bx)+9\right)-5\cot(a+bx)\csc(a+bx)\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] ((-5*Cot[a + b*x]*Csc[a + b*x] + 3*Sec[a + b*x]*(9 + Sec[a + b*x]^2))*Sqrt[
Sin[2*(a + b*x)]])/(120*b)
```

fricas [A] time = 0.43, size = 88, normalized size = 1.11

$$\frac{32\,\cos(bx+a)^5-32\,\cos(bx+a)^3+\sqrt{2}\left(32\,\cos(bx+a)^4-24\,\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{120\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/120*(32*cos(b*x + a)^5 - 32*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 24*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^5
- b*cos(b*x + a)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sin(2bx+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [B] time = 3.39, size = 131, normalized size = 1.66

$$\frac{4\,e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\sqrt{\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\left(2\,e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-3\,e^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+2\,e^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}+2\,e^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}+2\right)}{15\,b\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(2*exp(a*2i + b*x*2i) - 3*exp(a*4i + b*x*4i) + 2*exp(a*6i + b*x*
6i) + 2*exp(a*8i + b*x*8i) + 2))/(15*b*(exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i
 + b*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.80 $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/7*sin(b*x+a)/b/sin(2*b*x+2*a)^(7/2)-6/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2
)+8/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4304, 4303, 4291}

$$\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (6*Cos[a + b*x])/(35*b*Sin[2*a
+ 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) - (16*Cos[
a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{24}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [**A**] time = 0.15, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}\,(-10\cos(2(a+bx)) + 4\cos(4(a+bx)) + 4\cos(6(a+bx)) - 5)\csc^3(a+bx)\sec^4(a+bx)}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((-5 - 10*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^3*Sec[a + b*x]^4*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

fricas [**A**] time = 0.45, size = 113, normalized size = 1.08

$$-\frac{\sqrt{2}\left(128\cos(bx+a)^6 - 160\cos(bx+a)^4 + 20\cos(bx+a)^2 + 5\right)\sqrt{\cos(bx+a)\sin(bx+a)} + 128\left(\cos(bx+a\right.}{560\left(b\cos(bx+a)^6 - b\cos(bx+a)^4\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/560*(sqrt(2)*(128*cos(b*x + a)^6 - 160*cos(b*x + a)^4 + 20*cos(b*x + a)^
2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - cos(b*x + a)
^4)*sin(b*x + a))/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F(-1)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

mupad [B] time = 4.34, size = 351, normalized size = 3.34

$$-\frac{e^{a\,1i+b\,x\,1i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{7\,b\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4}+\frac{16\,e^{a\,3i+b\,x\,3i}\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{35\,b\left(e^{a\,2i+b\,x\,2i}-1\right)\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}-\frac{e^{a\,1i+b\,x\,1i}\left(\frac{1i}{7\,b}+\frac{e^{a\,2i+b\,x\,2i}\,8i}{35\,b}\right)\sqrt{\dots}}{\left(e^{a\,2i+b\,x\,2i}-1\right)^2\left(e^{a\,2\dots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (16*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i)) -
 (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*(1
i/(7*b) + (exp(a*2i + b*x*2i)*8i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (e
xp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i + b*x
*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(16/(35*b) - (44*exp(a*2i + b*x*2i))
/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(
(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x*2i)*1i + 1i)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.81 $\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{42b}$$

```
[Out] -5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b-1/18*sin(2*b*x+2
*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2641}

$$-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) - Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} - \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2bx$$
$$= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} - \sin^{\frac{9}{2}}$$
$$= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin}{14b}$$

Mathematica [A] time = 0.41, size = 96, normalized size = 0.98

$$\frac{-70\sin(2(a+bx)) - 156\sin(4(a+bx)) + 35\sin(6(a+bx)) + 18\sin(8(a+bx)) - 7\sin(10(a+bx)) + 240\sqrt{\sin}}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] + 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] - 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(\cos(bx+a)^2-1\right)\cos(2bx+2a)^2-\cos(bx+a)^2+1\right)\sin(2bx+2a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sin
(2*b*x + 2*a)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\sin^2(bx+a)\right)\left(\sin^{\frac{7}{2}}(2bx+2a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
[Out] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(2bx+2a)^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2 \sin(2\,a+2\,b\,x)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.82 $\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

```
[Out] -3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-1/14*sin(2*b*x+2
*a)^(7/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2639}

$$-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) - Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= -\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2bx}\\ &= \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}\end{aligned}$$

Mathematica [A] time = 0.22, size = 66, normalized size = 0.96

$$\frac{\sqrt{\sin(2(a+bx))}\,(-15\sin(2(a+bx))-14\sin(4(a+bx))+5\sin(6(a+bx)))+84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(-15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] + 5*Sin[6*(a + b*x)]))/(280*b)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(\cos\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)^{2}-\cos\left(bx+a\right)^{2}+1\right)\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sqr
t(sin(2*b*x + 2*a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 167.06, size = 278615779, normalized size = 4037909.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.83 $\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2*a
)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2641}

$$-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}\end{aligned}$$

Mathematica [A] time = 0.36, size = 76, normalized size = 1.10

$$\frac{-9\sin(2(a+bx))-10\sin(4(a+bx))+3\sin(6(a+bx))+20\sqrt{\sin(2(a+bx))}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] + 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^2-1\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 71.76, size = 172368074, normalized size = 2498088.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.84 $\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/6*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 34, normalized size = 0.85

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))-3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/6*(-3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/b
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sqrt{\sin(2bx+2a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(2\,bx + 2\,a\right)} \sin\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

maple [B] time = 5.71, size = 16549859, normalized size = 413746.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(2\,bx + 2\,a\right)} \sin\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\sin\left(a+b\,x\right)}^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 2641}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A] time = 0.25, size = 75, normalized size = 1.88

$$-\frac{2\sqrt{\sin(2(a+bx))}+\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*(2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - S
in[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]
])/b
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\cos(bx+a)^2-1}{\sqrt{\sin(2\,bx+2\,a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)/sqrt(sin(2*b*x + 2*a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 13.51, size = 53360209, normalized size = 1334005.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+b\,x)^2}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.86 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

```
[Out] 1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b+sin(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 2639}

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= \frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.91

$$\frac{\sqrt{\sin(2(a+bx))}\,\tan(a+bx)-E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*Tan[a + b*x])/(2*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^2-1\right)\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral((cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 -
1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 15.52, size = 83175028, normalized size = 1848333.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.87 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=48

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/3*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 2641}

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= \frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 83, normalized size = 1.73

$$\frac{\sqrt{\sin(2(a+bx))}\,\sec^2(a+bx)-\frac{\sqrt{2}\,(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sec[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a +
b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a
 + b*x)]])/(12*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)^2-1}{\left(\cos(2\,bx+2\,a)^2-1\right)\sqrt{\sin(2\,bx+2\,a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^2 - 1)/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*
a))), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 59.89, size = 123, normalized size = 2.56

$$\frac{\sqrt{1+\sin(2bx+2a)}\ \sqrt{-2\sin(2bx+2a)+2}\ \sqrt{-\sin(2bx+2a)}\ \operatorname{EllipticF}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)\sin(2bx+2}{12\sin(2bx+2a)^{\frac{3}{2}}\cos(2bx+2a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((1+sin(2*b*x+2*a))^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((1+sin(2*b*x+2*a))^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2+2*cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+b\,x)^2}{\sin(2\,a+2\,b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.88 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/5*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a)/
b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4296, 2636, 2639}

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} + \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.81, size = 66, normalized size = 0.86

$$-\frac{12E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\frac{4\sin^2(a+bx)(6\cos(2(a+bx))+3\cos(4(a+bx))+1)}{\sin^{\frac{5}{2}}(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/40*(12*EllipticE[a - Pi/4 + b*x, 2] + (4*(1 + 6*Cos[2*(a + b*x)] + 3*Cos
[4*(a + b*x)])*Sin[a + b*x]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^2-1\right)\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^4-2\,\cos\left(2\,bx+2\,a\right)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 -
 2*cos(2*b*x + 2*a)^2 + 1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(bx+a\right)}{\sin\left(2bx+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b\,x)^2}{\sin (2\,a+2\,b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.89 $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\sin^{\frac{3}{2}}(2a+$$

```
[Out] -7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b-7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-7/48*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/12*sin(b*x+a)*sin
(2*b*x+2*a)^(5/2)/b+7/32*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4298, 4302, 4301, 4306}

$$-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{48b}-\frac{7\sin^{-1}(\cos(a+b}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) - (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) + (7*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) - (7*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) - (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{12}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= -\frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} - \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{16}\int \cos(a$$

$$= \frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b} - \frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} - \frac{\sin(a+bx)}{}$$

$$= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b} - \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{}\right.}{64b}$$

Mathematica [A] time = 0.35, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(10\sin(a+bx)-9\sin(3(a+bx))+2\sin(5(a+bx)))-7\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\mathrm{lo}\right.}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(10*Sin[a + b*x] - 9
*Sin[3*(a + b*x)] + 2*Sin[5*(a + b*x)]))/3)/(64*b)
```

fricas [B] time = 0.47, size = 290, normalized size = 2.13

$$8\sqrt{2}\left(32\cos(bx+a)^4-60\cos(bx+a)^2+21\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)+42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(8*sqrt(2)*(32*cos(b*x + a)^4 - 60*cos(b*x + a)^2 + 21)*sqrt(cos(b*x
+ a)*sin(b*x + a))*sin(b*x + a) + 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(
b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 42*arctan(-(2*sqrt(2)*sqrt
(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
(4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 194.69, size = 322540527, normalized size = 2371621.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(a+b\,x\right)}^3\,{\sin\left(2\,a+2\,b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.90 $\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+\right.}{}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-1/8*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-5/16*cos(b*x+a)*sin(
2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 4302, 4305}

$$-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{16}\int \frac{}{\sqrt{\text{s}}}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)\right.}{32b}$$

Mathematica [A] time = 0.22, size = 86, normalized size = 0.78

$$\frac{2\sqrt{\sin(2(a+bx))}\,(\cos(3(a+bx))-6\cos(a+bx))+5\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\text{s}\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (5*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*(-6*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Si
n[2*(a + b*x)]])/(32*b)
```

fricas [B] time = 0.51, size = 281, normalized size = 2.55

$$8\sqrt{2}\left(4\cos(bx+a)^3-9\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)}{\cos(bx+a)^2+2\cos(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^3 - 9*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:ext_reduce Error: Bad Argument TypeEvaluatio
n time: 0.47Done
```

maple [B] time = 23.19, size = 47453403, normalized size = 431394.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(2bx + 2a)} \sin(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a + bx)^3 \sqrt{\sin(2a + 2bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.91 $\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos\right.}{8b}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-1/4*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 4306}

$$-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) - (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (Sin[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\right.}}{8b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 0.88

$$-\frac{2\sin(a+bx)\sqrt{\sin(2(a+bx))}+3\sin^{-1}(\cos(a+bx)-\sin(a+bx))+3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/8*(3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]]])/b
```

fricas [B] time = 0.49, size = 268, normalized size = 3.19

$$-\frac{8\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) - 6*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*s
qrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x
+ a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)
*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 46.94, size = 155804040, normalized size = 1854810.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{3}}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,x\right)}^{3}}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.92 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} \, dx$

Optimal. Leaf size=81

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] 1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+
2*a)^(1/2))/b+sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4294, 4308, 4305}

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4294

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Si
n[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} \, dx &= \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \csc(a+bx)\sqrt{\sin(2a+2bx)} \, dx \\
&= \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx \\
&= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}+
\end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.89

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))+2\sqrt{\sin(2(a+bx))}\sec(a+bx)-\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] + 2*Sec[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

fricas [B] time = 0.46, size = 296, normalized size = 3.65

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\cos(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)}}{\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*cos(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*c
os(b*x + a) - cos(b*x + a)*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1) -
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - 8*cos(b*x + a))/(b*cos(b*x + a
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 52.78, size = 149378290, normalized size = 1844176.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)^3}{\sin(2a+2bx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.93 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] 1/3*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4292}

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.96

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*Sin[2*(a + b*x)]^(3/2))
```

fricas [A] time = 0.45, size = 48, normalized size = 1.71

$$-\frac{\cos(bx+a)^2 - \sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)}{12\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(cos(b*x + a)^2 - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a
))/(b*cos(b*x + a)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 183.93, size = 727, normalized size = 25.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/48*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2
*a)^2-1)*(6*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-
tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2
))*tan(1/2*b*x+1/2*a)^6-3*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1
/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^6+18*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*t
an(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b
*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4-9*(tan(1/2*b*x+1/2*a)+
1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*Ellipt
icF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4+6*tan(1/
2*b*x+1/2*a)^8+18*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1
/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*
2^(1/2))*tan(1/2*b*x+1/2*a)^2-9*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*
x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-2*tan(1/2*b*x+1/2*a)^6+6*(tan(1
/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a)
)^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))-3*(tan(1/2*b*x+
1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)
*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*b*x+1/2*a)^
4-14*tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1
/2)/(1+tan(1/2*b*x+1/2*a)^2)^3/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1
/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 2.19, size = 85, normalized size = 3.04

$$-\frac{\sqrt{\sin(2\,a+2\,b\,x)}\,\left(2\,\sin(a+b\,x)+3\,\sin(3\,a+3\,b\,x)+\sin(5\,a+5\,b\,x)\right)}{6\,b\,\left(30\sin(a+b\,x)^2+12\sin(2\,a+2\,b\,x)^2+2\sin(3\,a+3\,b\,x)^2-32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(sin(2*a + 2*b*x)^(1/2)*(2*sin(a + b*x) + 3*sin(3*a + 3*b*x) + sin(5*a + 5
*b*x)))/(6*b*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2 + 30*sin(a + b*x
)^2 - 32))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/5*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)+1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 4292}

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2*
a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= \frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{1}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 35, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}\sec(a+bx)\left(\sec^2(a+bx)+4\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (Sec[a + b*x]*(4 + Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

fricas [A] time = 0.49, size = 55, normalized size = 1.00

$$\frac{4\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\cos\left(bx+a\right)^{2}+1\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{40\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a)*s
in(b*x + a)))/(b*cos(b*x + a)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}\left(bx+a\right)}{\sin\left(2bx+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{3}}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [B] time = 3.26, size = 88, normalized size = 1.60

$$\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*(3*exp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1))/(5*b*(exp(a*2i +
b*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/7*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)+2/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-4/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4296, 4304, 4291}

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (2*Sin[a + b*x])/(21*b*Sin[2*
a + 2*b*x]^(3/2)) - (4*Cos[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4}{21}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 0.68

$$-\frac{\sqrt{\sin(2(a+bx))}\,(12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc(a+bx)\sec^4(a+bx)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -1/336*((5 + 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]*Sec[a +
 b*x]^4*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [A] time = 0.43, size = 79, normalized size = 0.98

$$-\frac{32\cos(bx+a)^4\sin(bx+a) + \sqrt{2}\left(32\cos(bx+a)^4 - 8\cos(bx+a)^2 - 3\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{336\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/336*(32*cos(b*x + a)^4*sin(b*x + a) + sqrt(2)*(32*cos(b*x + a)^4 - 8*cos
(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^4*sin(b*x
 + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

mupad [B] time = 3.71, size = 300, normalized size = 3.70

$$-\frac{e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,5\mathrm{i}}{84\,b\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^2}+\frac{3\,e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{14\,b\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^3}-\frac{e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,2\mathrm{i}+b\,x\,2}}{2}}}{7\,b\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (3*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(14*b*(exp(a*2i + b*x*2i)*1i + 1i)^3) - (exp(a*1i + b*x*1i)*((e
xp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*5i)/(84*b*(exp
(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i
)^4) + (exp(a*1i + b*x*1i)*(5/(84*b) + (4*exp(a*2i + b*x*2i))/(21*b))*((exp
(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*
x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.96 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] 1/9*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)+1/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-4/45*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+8/45*sin(b*x+a)/b/sin(2*b*x+2*a)
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4296, 4304, 4303, 4292}

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) + Sin[a + b*x]/(15*b*Sin[2*a +
2*b*x]^(5/2)) - (4*Cos[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) + (8*Sin[a +
 b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
```

```
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{1}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8}{45}\int \frac{\sin(a+bx}{\sin^{\frac{3}{2}}(2a+2l}$$
$$= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx}}$$

Mathematica [A] time = 0.17, size = 62, normalized size = 0.58

$$\frac{\sqrt{\sin(2(a+bx))}\left(5\sec^5(a+bx)+17\sec^3(a+bx)+113\sec(a+bx)-15\cot(a+bx)\csc(a+bx)\right)}{1440b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
[Out] ((-15*Cot[a + b*x]*Csc[a + b*x] + 113*Sec[a + b*x] + 17*Sec[a + b*x]^3 + 5*
Sec[a + b*x]^5)*Sqrt[Sin[2*(a + b*x)]])/(1440*b)
```

fricas [A] time = 0.45, size = 98, normalized size = 0.92

$$\frac{128\cos(bx+a)^7 - 128\cos(bx+a)^5 + \sqrt{2}\left(128\cos(bx+a)^6 - 96\cos(bx+a)^4 - 12\cos(bx+a)^2 - 5\right)\sqrt{\cos}}{1440\left(b\cos(bx+a)^7 - b\cos(bx+a)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
[Out] 1/1440*(128*cos(b*x + a)^7 - 128*cos(b*x + a)^5 + sqrt(2)*(128*cos(b*x + a)
^6 - 96*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x +
 a)))/(b*cos(b*x + a)^7 - b*cos(b*x + a)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(bx+a)}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (bx + a)^3}{\sin (2\,bx + 2\,a)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

mupad [B] time = 5.16, size = 383, normalized size = 3.58

$$-\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,1i}{60\,b\,{\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}^3}-\frac{2\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,{\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}^4}+\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{9\,b\,{\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*1i)/(9*b*(exp(a*2i + b*x*2i)*1i + 1i)^5) - (2*exp(a*1i + b*x*1i)*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(9*b*(exp(a
*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/
2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(60*b*(exp(a*2i + b*x*2i)*1i + 1i)
^3) + (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i
)*1i)/2)^(1/2)*8i)/(45*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i +
1i)) - (exp(a*1i + b*x*1i)*(49/(180*b) - (19*exp(a*2i + b*x*2i))/(180*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) - 1)^2*(exp(a*2i + b*x*2i)*1i + 1i)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

3.97 $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{5\sin^{\frac{3}{2}}(2a+2\ldots}{1\ldots}$$

```
[Out] -5/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-5/12*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/3*sin(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b+5/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4308, 4301, 4302, 4306}

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{5\sin^{-1}(\cos(a+bx\ldots}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(12*b) + (Sin[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$= \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{3}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{4}\int \cos(a+$$

$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\,\mathrm{s}}{3}$$

$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\mathrm{s}}\right.}{16b}$$

Mathematica [A] time = 0.33, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\mathrm{lo}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(16*b)
```

fricas [B] time = 0.45, size = 290, normalized size = 2.13

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^4-12\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)-30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/192*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 19.86, size = 973, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] -16/5/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(6*(tan(1/2*b*
x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+
1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)
*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4-3
*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(
tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1
/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*
x+1/2*a)^4-12*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a
)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-
tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2
))*tan(1/2*b*x+1/2*a)^2+6*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1
/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)
+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2
),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+6*tan(1/2*b*x+1/2*a)^6*(tan(1/2*b*x+1/2
*a)^3-tan(1/2*b*x+1/2*a))^(1/2)+6*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1
)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*
x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)
+1)^(1/2),1/2*2^(1/2))-3*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/
2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+
2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2)
,1/2*2^(1/2))-4*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2
*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^4-12*tan(1/2*b*x+1/2*a)^4*(tan(1/2*b*x+1/2
*a)^3-tan(1/2*b*x+1/2*a))^(1/2)-4*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1
)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2+6*tan(1/2*b*x+1/2*a)^2
*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2))/(tan(1/2*b*x+1/2*a)*(tan(
1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(
1/2*b*x+1/2*a)+1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/(t
an(1/2*b*x+1/2*a)-1)/(tan(1/2*b*x+1/2*a)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx + a\right)\sin\left(2\,bx + 2\,a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.98 $\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+b\right.}{}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/4*cos(b*x+a)*sin(2*b
*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4308, 4301, 4302, 4305}

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(2*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{2}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{4}\int \frac{\text{co}}{\sqrt{\text{sir}}}$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{8b}$$

Mathematica [A] time = 0.20, size = 86, normalized size = 0.78

$$\frac{3\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(8*b)
```

fricas [B] time = 0.46, size = 281, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\text{s}}{\cos(bx+a)^2+2\cos(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*
x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a)
- sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x +
a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)
) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32
*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(
b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 20.55, size = 243, normalized size = 2.21

$$-\frac{8\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\text{EllipticF}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+}\right.\right.}{3b\sqrt{\tan}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -8/3/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-(t
an(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/
2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*tan(1/2*b
*x+1/2*a)^3+2*tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2
-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx + a\right)\sin\left(2\,bx + 2\,a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(2\,a+2\,b\,x\right)}^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.99 $\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=81

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+\right.}{2b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4308, 4301, 4306}

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= 2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\text{si}\right.}{2b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 70, normalized size = 0.86

$$-\frac{-2\sin(a+bx)\sqrt{\sin(2(a+bx))}+\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [B] time = 0.47, size = 266, normalized size = 3.28

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)+2\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(sq
rt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b
*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -
 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt(
2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a)
)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin
(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

maple [C] time = 10.36, size = 362, normalized size = 4.47

$$\frac{4\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 4/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)*(2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1)
)^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(
1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))-(
tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(ta
n(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2
*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*tan(1/2*b*
x+1/2*a)^2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2))/(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a
```

```
)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(2\,a+2\,b\,x)^{3/2}}{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x),x)
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.100 $\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

```
[Out] -arcsin(cos(b*x+a)-sin(b*x+a))/b+ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(1
/2))/b
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4308, 4305}

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*a + 2*b*x]]]/b
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(}\right.}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.98

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]/b
```

fricas [B] time = 0.49, size = 242, normalized size = 4.57

$$2\,\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right)-2\,\arctan\left(-\frac{2\,\sqrt{2}\,\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx\cdots}{\cos(bx+a)-\sin(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

maple [C] time = 8.59, size = 157, normalized size = 2.96

$$\frac{2\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\,\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\cdots\right.}{b\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 2/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)/(t
an(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)
+1)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{\sin(2a+2bx)}\,\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4292}

$$-\frac{\sqrt{\sin(2a+2bx)}\,\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 0.96

$$-\frac{\sqrt{\sin(2(a+bx))}\,\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b)
```

fricas [A] time = 0.45, size = 39, normalized size = 1.62

$$-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\sin(bx+a)}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

maple [C] time = 9.96, size = 308, normalized size = 12.83

$$\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(2\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 1/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1
/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1
/2*a)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(
1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2
*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+tan(
1/2*b*x+1/2*a)^2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)-(tan(1/2*b
*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2))/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*
a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [B] time = 0.31, size = 24, normalized size = 1.00

$$-\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2)),x)
```

```
[Out] -sin(2*a + 2*b*x)^(1/2)/(b*sin(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+4/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4308, 4303, 4292}

$$\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{2}\sec(a+bx)-\frac{1}{6}\cot(a+bx)\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ((-1/6*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/2)*Sqrt[Sin[2*(a + b*x)]]
)/b
```

fricas [A] time = 0.52, size = 74, normalized size = 1.40

$$\frac{4\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4\cos\left(bx+a\right)}{6\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*si
n(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

maple [C] time = 13.10, size = 194, normalized size = 3.66

$$-\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\right.\right.}{12b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\sqrt{\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] -1/12/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b
*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)-tan(1/2*b*x+1/2*a)^4+1)/(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b
*x+1/2*a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [B] time = 2.96, size = 103, normalized size = 1.94

$$\frac{4\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(1+e^{a\,4i+b\,x\,4i}-e^{a\,2i+b\,x\,2i}\right)}{3\,b\,{\left(e^{a\,2i+b\,x\,2i}-1\right)}^2\,\left(e^{a\,2i+b\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2)),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*4i + b*x*4i) - exp(a*2i + b*x*2i) + 1))/(3*b*(exp(a*2i +
b*x*2i) - 1)^2*(exp(a*2i + b*x*2i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.103 $\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)-16/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4308, 4303, 4304, 4291}

$$\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(15*b*Sin
[2*a + 2*b*x]^(3/2)) - (16*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = 2\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\left(3\csc^3(a+bx)+27\csc(a+bx)-5\tan(a+bx)\sec(a+bx)\right)}{60b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/60*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[a
 + b*x]*Tan[a + b*x]))/b
```

fricas [A] time = 0.56, size = 103, normalized size = 1.30

$$-\frac{\sqrt{2}\left(32\cos(bx+a)^4-40\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\cdots}{60\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/60*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a
)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*co
s(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

maple [C] time = 37.23, size = 481, normalized size = 6.09

$$-\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(24\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/80/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/tan(1/2*b*x+1/
2*a)^3*(24*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x
+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2
)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-
12*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+
1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*Ellipt
icF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^6-(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4+12*tan(1/2*b
*x+1/2*a)^4*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)-(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2-12*tan(1/2*b*x+1/
2*a)^2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)+(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)^2-1))^(1/2))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 3.36, size = 136, normalized size = 1.72

$$\frac{8\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(e^{a\,2i+b\,x\,2i}\,2i+e^{a\,4i+b\,x\,4i}\,3i+e^{a\,6i+b\,x\,6i}\,2i-e^{a\,8i+b\,x\,8i}\,2i-2i\right)}{15\,b\,{\left(e^{a\,2i+b\,x\,2i}-1\right)}^3\,{\left(e^{a\,2i+b\,x\,2i}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2)),x)
```

```
[Out] (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*
6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp
(a*2i + b*x*2i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] -2/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+12/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+32/35*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4308, 4303, 4304, 4292}

$$\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (12*Sin[a + b*x])/(35*b*Si
n[2*a + 2*b*x]^(5/2)) - (16*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (
32*Sin[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = 2\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{48}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}\,(-10\cos(2(a+bx)) - 4\cos(4(a+bx)) + 4\cos(6(a+bx)) + 5)\csc^4(a+bx)\sec^3(a+bx)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
+ b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(280*b)
```

fricas [A] time = 0.46, size = 118, normalized size = 1.12

$$\frac{128\cos(bx+a)^7 - 256\cos(bx+a)^5 + 128\cos(bx+a)^3 + \sqrt{2}\left(128\cos(bx+a)^6 - 224\cos(bx+a)^4 + 84\cos\right.}{280\left(b\cos(bx+a)^7 - 2b\cos(bx+a)^5 + b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/280*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(
2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(c
os(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sin(2bx+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [B] time = 4.13, size = 350, normalized size = 3.33

$$-\frac{2\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}+\frac{e^{a\,3i+b\,x\,3i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,32i}{35\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}-\frac{e^{a\,1i+b\,x\,1i}\,\left(\frac{2}{7\,b}-\frac{16\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\,\sqrt{\ }}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\,\left(e^{a\,2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(7/2)),x)
```

```
[Out] (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*32i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
- (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*(2
/(7*b) - (16*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (ex
p(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*
2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(32i/(35*b) + (exp(a*2i + b*x*2i)*88i
)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/
((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.105 $\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{2\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{7b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+\ldots}{7b}$$

```
[Out] -6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-2/7*cos(2*b*x+2*a)
*sin(2*b*x+2*a)^(7/2)/b+1/7*csc(b*x+a)^2*sin(2*b*x+2*a)^(11/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2639}

$$\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{2\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{7b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+\ldots}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x]^(3/2))/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(7/2))/(7*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + \frac{18}{7}\int \sin^{\frac{9}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + 2\int s$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + cs$$
$$= \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)s}{7b}$$

Mathematica [A] time = 0.25, size = 66, normalized size = 0.62

$$\frac{\sqrt{\sin(2(a+bx))}\,(15\sin(2(a+bx)) - 14\sin(4(a+bx)) - 5\sin(6(a+bx))) + 84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(70*b)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(2\,bx+2\,a\right)^4 - 2\,\cos\left(2\,bx+2\,a\right)^2 + 1\right)\csc\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*csc(b*x + a)^2*sqr
t(sin(2*b*x + 2*a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 29.08, size = 204, normalized size = 1.92

$$\frac{8\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{7}{2}}(2bx+2a)\right)}{56} - \frac{\sqrt{2}\left(6\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\text{EllipticE}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right) - 3\sqrt{1+\sin(2bx+}\right.}{80\cos(2bx+2a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] 8*2^(1/2)*(1/56*2^(1/2)*sin(2*b*x+2*a)^(7/2)-1/80*2^(1/2)*(6*(1+sin(2*b*x+2
*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((1
+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))-3*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*
x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((1+sin(2*b*x+2*a))^(1/2),
```

```
1/2*2^(1/2))-2*sin(2*b*x+2*a)^4+2*sin(2*b*x+2*a)^2)/cos(2*b*x+2*a)/sin(2*b*
x+2*a)^(1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{2} \sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.106 $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{2\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{5b}-\frac{2\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a}{5b}$$

```
[Out] -2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/5*csc(b*x+a)^2*s
in(2*b*x+2*a)^(9/2)/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{2\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{5b}-\frac{2\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(5*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + \frac{14}{5}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + 2\int \sin^{\frac{3}{2}}$$
$$= -\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \csc^2($$
$$= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin}{5b}$$

Mathematica [A] time = 0.27, size = 76, normalized size = 0.72

$$\frac{9\sin(2(a+bx)) - 10\sin(4(a+bx)) - 3\sin(6(a+bx)) + 20\sqrt{\sin(2(a+bx))}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{30b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(30*b*Sqrt[Sin[2*(a + b*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(2\,bx+2\,a\right)^2-1\right)\csc\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 25.25, size = 139, normalized size = 1.31

$$\frac{4\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{5}{2}}(2bx+2a)\right)}{20} + \frac{\sqrt{2}\left(\sqrt{1+\sin(2bx+2a)}\ \sqrt{-2\sin(2bx+2a)+2}\ \sqrt{-\sin(2bx+2a)}\ \text{EllipticF}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)+2\left(\sin^3(2bx+2a)\right)-2\,s}{24\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 4*2^(1/2)*(1/20*2^(1/2)*sin(2*b*x+2*a)^(5/2)+1/24*2^(1/2)*((1+sin(2*b*x+2*a
))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((1+s
```

```
in(2*b*x+2*a))^(1/2),1/2*2^(1/2))+2*sin(2*b*x+2*a)^3-2*sin(2*b*x+2*a))/cos(
2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.107 $\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=75

$$\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

```
[Out] -2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x
),2^(1/2))/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/3*csc(b*x+a)^2*sin
(2*b*x+2*a)^(7/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2639}

$$\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (2*EllipticE[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(
3/2))/(3*b) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= \frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}+\frac{10}{3}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx\\ &= -\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}+2\int \sqrt{\sin\\ &= \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a\end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 0.45

$$\frac{2\left(\sin^{\frac{3}{2}}(2(a+bx))+3E\left(a+bx-\frac{\pi}{4}\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
[Out] (2*(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2)))/(3*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(2\,bx+2\,a\right)^{2}-1\right)\csc\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 22.39, size = 137, normalized size = 1.83

$$\frac{2\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{3}{2}}(2bx+2a)\right)}{6}-\frac{\sqrt{2}\,\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\left(2\,\text{EllipticE}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1+\text{s}}\right.\right.}{4\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
[Out] 2*2^(1/2)*(1/6*2^(1/2)*sin(2*b*x+2*a)^(3/2)-1/4*2^(1/2)*(1+sin(2*b*x+2*a))^
(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*(2*EllipticE((1+s
in(2*b*x+2*a))^(1/2),1/2*2^(1/2))-EllipticF((1+sin(2*b*x+2*a))^(1/2),1/2*2^
(1/2)))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(2\,a+2\,b\,x\right)}^{5/2}}{{\sin\left(a+b\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.108 $\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

```
[Out] -2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x
),2^(1/2))/b+csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2)/b-2*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sqrt{\sin(2a+2bx)}\,\cos(2a+2bx)}{b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/b + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx=\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}+6\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$=-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}+2\int$$

$$=\frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin}{b}$$

Mathematica [A] time = 0.92, size = 73, normalized size = 1.04

$$\frac{2\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/b
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

maple [A] time = 15.64, size = 111, normalized size = 1.59

$$\frac{\sqrt{2}\left(\sqrt{2}\,\left(\sqrt{\sin}\,\left(2bx+2a\right)\right)+\frac{\sqrt{2}\,\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\operatorname{EllipticF}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)}{2\cos(2bx+2a)\sqrt{\sin(2bx+2a)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 2^(1/2)*(2^(1/2)*sin(2*b*x+2*a)^(1/2)+1/2*2^(1/2)*(1+sin(2*b*x+2*a))^(1/2)*
(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((1+sin(2*b*x+
2*a))^(1/2),1/2*2^(1/2))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.109 $\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

```
[Out] 2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))/b-csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 2639}

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2
))/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-2\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.13, size = 37, normalized size = 0.84

$$-\frac{2\left(E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\cot(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\csc\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 17.73, size = 176, normalized size = 4.00

$$\frac{2\sqrt{1+\sin\left(2bx+2a\right)}\,\sqrt{-2\sin\left(2bx+2a\right)+2}\,\sqrt{-\sin\left(2bx+2a\right)}\,\text{EllipticE}\left(\sqrt{1+\sin\left(2bx+2a\right)},\frac{\sqrt{2}}{2}\right)-\sqrt{1+}}{\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
[Out] 1/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2)*(2*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2
*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((1+sin(2*b*x+2*a))^(1/
2),1/2*2^(1/2))-(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(
2*b*x+2*a))^(1/2)*EllipticF((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))-2*cos(2*b
*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{{\sin\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2,x)
[Out] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.110 $\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=48

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

```
[Out] -2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/3*csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b
*x]])/(3*b)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}+\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}-\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.99, size = 82, normalized size = 1.71

$$-\frac{\sqrt{\sin(2(a+bx))}\csc^2(a+bx)+\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/3*(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos
[a + b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin
[2*(a + b*x)]])/b
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(bx+a\right)}{\sqrt{\sin\left(2bx+2a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\sin\left(a+b\,x\right)}^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)),x)
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.111 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx$

Optimal. Leaf size=77

$$-\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b-6/5*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)-1/5*csc(b*x+a)^2/b/
sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4300, 2636, 2639}

$$-\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (6*Cos[2*a + 2*b*x])/(5*b*Sqrt[Si
n[2*a + 2*b*x]]) - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx &= -\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}+\frac{6}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)} dx \\
&= -\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{6}{5}\int \sqrt{\sin(2a+2bx)}\, dx \\
&= -\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 64, normalized size = 0.83

$$\frac{\frac{2(-6\cos(2(a+bx))+3\cos(4(a+bx))+1)\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}-12E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(10*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\csc(bx+a)^2\sqrt{\sin(2bx+2a)}}{\cos(2bx+2a)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 - 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

maple [B] time = 59.94, size = 227, normalized size = 2.95

$$\frac{\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2bx+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\left(\sin^2(2bx+2a)\right)\text{EllipticE}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{}\cdots\right.}{5\cdots}\right)}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 1/8*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^(
5/2)*(6*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))-
3*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1
/2)*sin(2*b*x+2*a)^2*EllipticF((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))+6*sin(
2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)),x)
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.112 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$\frac{10F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -10/21*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-10/21*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)
^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2641}

$$\frac{10F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (10*EllipticF[a - Pi/4 + b*x, 2])/(21*b) - (10*Cos[2*a + 2*b*x])/(21*b*Sin[
2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{10}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{10}{21}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{10F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A] time = 0.47, size = 66, normalized size = 0.86

$$\frac{40F\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\left(-3\csc^4(a+bx)-13\csc^2(a+bx)+7\sec^2(a+bx)\right)}{84b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[a - Pi/4 + b*x, 2] + (-13*Csc[a + b*x]^2 - 3*Csc[a + b*x]^4 +
 7*Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(84*b)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\csc(bx+a)^2}{\left(\cos(2\,bx+2\,a)^2-1\right)\sqrt{\sin(2\,bx+2\,a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-csc(b*x + a)^2/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*a))),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

maple [A] time = 90.51, size = 154, normalized size = 2.00

$$\frac{\sqrt{2}\left(-\frac{16\sqrt{2}}{7\sin(2bx+2a)^{\frac{7}{2}}}+\frac{8\sqrt{2}\left(5\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\text{EllipticF}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)\left(\sin^3(2bx+2a)\right)+10\left(\sin\right.\right.}{21\sin(2bx+2a)^{\frac{7}{2}}\cos(2bx+2a)}\right.}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/16*2^(1/2)*(-16/7*2^(1/2)/sin(2*b*x+2*a)^(7/2)+8/21*2^(1/2)/sin(2*b*x+2*a
)^(7/2)*(5*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x
+2*a))^(1/2)*EllipticF((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)
^3+10*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-6)/cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)),x)
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
[Out] Timed out
```

3.113 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=106

$$-\frac{14E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

```
[Out] 14/15*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-14/45*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^
2/b/sin(2*b*x+2*a)^(5/2)-14/15*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4300, 2636, 2639}

$$-\frac{14E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-14*EllipticE[a - Pi/4 + b*x, 2])/(15*b) - (14*Cos[2*a + 2*b*x])/(45*b*Sin
[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2)) - (14*Co
s[2*a + 2*b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{14}{9}\int \frac{1}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{14}{15}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}} - \frac{14}{15}\int \sqrt{\sin(2a+2}$$
$$= -\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b} - \frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{14\cos(2a+2bx}{15b\sqrt{\sin(2a+2bx}}$$

Mathematica [A] time = 0.83, size = 85, normalized size = 0.80

$$-\frac{336E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\frac{(98\cos(2(a+bx))-28\cos(4(a+bx))-42\cos(6(a+bx))+21\cos(8(a+bx))-9)\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/360*(336*EllipticE[a - Pi/4 + b*x, 2] + ((-9 + 98*Cos[2*(a + b*x)] - 28*
Cos[4*(a + b*x)] - 42*Cos[6*(a + b*x)] + 21*Cos[8*(a + b*x)])*Csc[a + b*x]^
2)/Sin[2*(a + b*x)]^(5/2))/b
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^4-2\,\cos\left(2\,bx+2\,a\right)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 - 2*cos(
2*b*x + 2*a)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(bx+a\right)}{\sin\left(2bx+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx + a\right)^{2}}{\sin\left(2\,bx + 2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(a+b\,x\right)}^2\,{\sin\left(2\,a+2\,b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)),x)
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
[Out] Timed out
```

3.114 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=106

$$\frac{30F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] -30/77*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-18/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(7/2)-1/11*csc(b*x+a
)^2/b/sin(2*b*x+2*a)^(7/2)-30/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2641}

$$\frac{30F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (30*EllipticF[a - Pi/4 + b*x, 2])/(77*b) - (18*Cos[2*a + 2*b*x])/(77*b*Sin[
2*a + 2*b*x]^(7/2)) - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2)) - (30*Co
s[2*a + 2*b*x])/(77*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{18}{11}\int \frac{1}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$$
$$= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{90}{77}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{30}{77}\int \frac{1}{\sqrt{\sin(2a+2b}}$$
$$= \frac{30F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{77b} - \frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.36, size = 86, normalized size = 0.81

$$\frac{480F\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\left(-7\csc^6(a+bx)-32\csc^4(a+bx)-141\csc^2(a+bx)+11\sec^2(a+bx)\right(}{1232b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (480*EllipticF[a - Pi/4 + b*x, 2] + (-141*Csc[a + b*x]^2 - 32*Csc[a + b*x]^
4 - 7*Csc[a + b*x]^6 + 11*Sec[a + b*x]^2*(9 + Sec[a + b*x]^2))*Sqrt[Sin[2*(
a + b*x)]])/(1232*b)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2}{\left(\cos(2\,bx+2\,a)^4-2\,\cos(2\,bx+2\,a)^2+1\right)\sqrt{\sin(2\,bx+2\,a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*sq
rt(sin(2*b*x + 2*a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (bx+a)^2}{\sin (2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin (a+b\,x)^2\,\sin (2\,a+2\,b\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.115 $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=190

$$\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{14\sin^{\frac{5}{2}}(2a+2b}{15}$$

```
[Out] -7/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+7/6*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-14/15*cos(b*x+a)*sin(2
*b*x+2*a)^(5/2)/b+4/5*sin(b*x+a)*sin(2*b*x+2*a)^(7/2)/b+1/5*csc(b*x+a)^3*si
n(2*b*x+2*a)^(11/2)/b-7/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4300, 4308, 4301, 4302, 4305}

$$\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{15b}-\frac{7\sin^{-1}(\cos(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (7*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(6*b) - (14*Cos[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(15*b) + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(7
/2))/(5*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b} + \frac{16}{5}\int \csc(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$$
$$= \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b} + \frac{32}{5}\int \cos(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b} + \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b} + \frac{28}{5}\int \sin$$
$$= -\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b} + \frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b} + \csc^3(a$$
$$= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b} - \frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b} + 4\sin(a$$
$$= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b} - 14\cos($$
$$= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{8b}$$

Mathematica [A] time = 0.44, size = 100, normalized size = 0.53

$$\frac{7\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-\frac{2}{3}\sqrt{\sin(2(a+bx))}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (7*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(8*b)
```

fricas [A] time = 0.49, size = 291, normalized size = 1.53

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*sq
rt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b
*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*
x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(c
os(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - si
```

```
n(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4
*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 211.10, size = 441, normalized size = 2.32

$$-\frac{64\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] -64/21*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^6-3*
(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+
1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b
*x+1/2*a)^4+2*tan(1/2*b*x+1/2*a)^7+3*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1
/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1
/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+10*tan(1/2*b*x+1/2*a)^5-(t
an(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/
2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*
b*x+1/2*a)^3+2*tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^
2-1))^(1/2)/(tan(1/2*b*x+1/2*a)+1)^2/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*
a))^(1/2)/(tan(1/2*b*x+1/2*a)-1)^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^{3}\sin\left(2\,bx+2\,a\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.116 $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=164

$$\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{5\sin^{\frac{3}{2}}(2a+2b\ldots}{3\ldots}$$

[Out] -5/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(1/2))/b-5/3*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+4/3*sin(b*x+a)*sin(2*b*x+2*a)^(5/2)/b+1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2)/b+5/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b

Rubi [A] time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4300, 4308, 4301, 4302, 4306}

$$\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{3b}-\frac{5\sin^{-1}(\cos(a+bx)\ldots}{4b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]

[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(4*b) - (5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(4*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(3*b) + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2))/(3*b)

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + 4\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + 8\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + \frac{20}{3}\int \sin($$
$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{\csc^3(a-}{}$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{4\sin(a+}{}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{4b}$$

Mathematica [A] time = 0.23, size = 84, normalized size = 0.51

$$\frac{2\sqrt{\sin(2(a+bx))}\,(6\sin(a+bx)+\sin(3(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin
[3*(a + b*x)]))/(4*b)
```

fricas [A] time = 0.68, size = 280, normalized size = 1.71

$$8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-}{\cos(bx+a)^2+2\cos(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(
b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
 a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log(
-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*s
in(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
+ a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 81.76, size = 973, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 32/5*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*(tan(1/2*b*x+1
/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*El
lipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4-(tan
(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1
/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a)
)^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2
*a)^4-4*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^
(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/
2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan
(1/2*b*x+1/2*a)^2+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x
+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1
/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*
2^(1/2))*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)^6*(tan(1/2*b*x+1/2*a)^3-
tan(1/2*b*x+1/2*a))^(1/2)+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan
(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1
/2),1/2*2^(1/2))-(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/
2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)
*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(
1/2))+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^
(1/2)*tan(1/2*b*x+1/2*a)^4-4*tan(1/2*b*x+1/2*a)^4*(tan(1/2*b*x+1/2*a)^3-tan
(1/2*b*x+1/2*a))^(1/2)+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/
2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)^2*(tan(1/2
*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2))/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1
/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1
/2*a)+1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/(tan(1/2*b*
x+1/2*a)-1)/(tan(1/2*b*x+1/2*a)+1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(2\,a+2\,b\,x)^{7/2}}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.117 $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=127

$$\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{6\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{b}$$

```
[Out] -3*arcsin(cos(b*x+a)-sin(b*x+a))/b+3*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a
)^(1/2))/b+4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+csc(b*x+a)^3*sin(2*b*x+2*a)^
(7/2)/b-6*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4301, 4302, 4305}

$$\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{6\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (6*Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/b + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/b + (Csc[a + b*x]^3*Sin[2*a +
2*b*x]^(7/2))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b} + 8\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$= \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b} + 16\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= \frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b} + \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b} + 12\int \sin(a$$

$$= -\frac{6\cos(a+bx)\sqrt{\sin(2a+2bx)}}{b} + \frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b} + \frac{\csc^3(a$$

$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{b}$$

Mathematica [A] time = 0.14, size = 70, normalized size = 0.55

$$\frac{-3\sin^{-1}(\cos(a+bx)-\sin(a+bx)) + \sin^{\frac{3}{2}}(2(a+bx))\csc(a+bx) + 3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right. + c}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/b
```

fricas [B] time = 0.57, size = 268, normalized size = 2.11

$$8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) + 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(sq
rt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b
*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -
 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 41.22, size = 243, normalized size = 1.91

$$\frac{16\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1},\right.\right.}{3\sqrt{\tan^3\left(\frac{bx}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 16/3*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*El
lipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-(tan
(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*
a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))-tan(1/2*b*x+1
/2*a)^3-tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx+a\right)^3 \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.118 $\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=104

$$-\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{2\log\left(\sin(\right.}{}$$

```
[Out] 2*arcsin(cos(b*x+a)-sin(b*x+a))/b+2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)
^(1/2))/b-csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2)/b-4*sin(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4300, 4308, 4301, 4306}

$$-\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{2\log\left(\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (2*Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (4*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/
b - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} - \frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int \frac{\sin(}{\sqrt{\sin(2}}$$
$$= \frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b} + \frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin}\right.}{b}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 0.65

$$\frac{2\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)) - 2\sqrt{\sin(2(a+bx))}\csc(a+bx) + \log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
 Sqrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

fricas [B] time = 0.54, size = 295, normalized size = 2.84

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a) - 2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(b}}{\cos(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*s
in(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*si
n(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(
b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)^3\sin(2bx+2a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

maple [C] time = 86.50, size = 542, normalized size = 5.21

$$\frac{4\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(4\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 4*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(4*(tan(1/2*b*x+1/2*
a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*(ta
n(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(
1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))-2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/
2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*
a)^2-1))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*tan(1/
2*b*x+1/2*a)^2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*(tan(1/2*b*x
+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)+(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/
2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2
*a))^(1/2)*tan(1/2*b*x+1/2*a)^2-(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*
(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/
2))/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*
b*x+1/2*a)+1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(bx + a\right)^{3} \sin\left(2\,bx + 2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.119 $\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

```
[Out] -1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4292}

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/(3*b)
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.96

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/3*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

fricas [B] time = 0.46, size = 53, normalized size = 1.89

$$\frac{2\left(\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1\right)}{3\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
- 1)/(b*cos(b*x + a)^2 - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)^3 \sqrt{\sin(2bx+2a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

maple [C] time = 31.02, size = 192, normalized size = 6.86

$$\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\,\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(4\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\right.}{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 1/3*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)/tan(1/2*b*x+1/2*a)*(4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1
/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)
^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)+tan(1/2*b*x+1/2*a)^4-1)/(tan(1/2*b*x
+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1
/2*a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(bx+a)^3 \sqrt{\sin(2bx+2a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [B] time = 1.50, size = 95, normalized size = 3.39

$$\frac{4\sqrt{\sin(2a+2bx)}\,\left(4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)^2-6\sin\left(\frac{3a}{2}+\frac{3bx}{2}\right)^2+2\sin\left(\frac{5a}{2}+\frac{5bx}{2}\right)^2\right)}{3b\left(30\sin(a+bx)^2-12\sin(2a+2bx)^2+2\sin(3a+3bx)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^3,x)
```

```
[Out] (4*sin(2*a + 2*b*x)^(1/2)*(4*sin(a/2 + (b*x)/2)^2 - 6*sin((3*a)/2 + (3*b*x)
/2)^2 + 2*sin((5*a)/2 + (5*b*x)/2)^2))/(3*b*(2*sin(3*a + 3*b*x)^2 - 12*sin(
2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.120 $\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

```
[Out] -4/5*csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/5*csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2
)/b
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 4292}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[2
*a + 2*b*x]])/(5*b)
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}+\frac{4}{5}\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b}-\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 35, normalized size = 0.64

$$-\frac{\sqrt{\sin(2(a+bx))}\csc(a+bx)\left(\csc^2(a+bx)+4\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/5*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [A] time = 0.48, size = 76, normalized size = 1.38

$$-\frac{\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+4\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{5\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] -1/5*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(c
os(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{3}}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(bx+a\right)}{\sqrt{\sin\left(2bx+2a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
[Out] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{3}}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [B] time = 3.16, size = 93, normalized size = 1.69

$$-\frac{8\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(-e^{a\,2i+b\,x\,2i}\,3i+e^{a\,4i+b\,x\,4i}\,1i+1i\right)}{5\,b\,{\left(e^{a\,2i+b\,x\,2i}-1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2)),x)
[Out] -(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp
(a*2i + b*x*2i) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -16/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)^3/b/sin(2*b*x+2*a)^
(1/2)+32/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4300, 4308, 4303, 4292}

$$\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-16*Cos[a + b*x])/(21*b*Sin[2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^3/(7*b*Sqrt
[Sin[2*a + 2*b*x]]) + (32*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{8}{7}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{16}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{32}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.68

$$\frac{\sqrt{\sin(2(a+bx))}\,(-12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc^4(a+bx)\sec(a+bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(42*b)
```

fricas [A] time = 0.51, size = 104, normalized size = 1.28

$$\frac{32\cos(bx+a)^5 - 64\cos(bx+a)^3 + \sqrt{2}\left(32\cos(bx+a)^4 - 56\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(bx+a)} + }{42\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/42*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))/
(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

maple [C] time = 107.67, size = 222, normalized size = 2.74

$$-\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(-3\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+16\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\sqrt{-\text{ta}}\right.}{336\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] -1/336*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/
2*a)^2-1)/tan(1/2*b*x+1/2*a)^3*(-3*tan(1/2*b*x+1/2*a)^8+16*(tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*El
lipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^3-2*ta
n(1/2*b*x+1/2*a)^6+2*tan(1/2*b*x+1/2*a)^2+3)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b
*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [B] time = 3.70, size = 302, normalized size = 3.73

$$-\frac{10\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{21\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}+\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,12i}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}-\frac{8\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i}}{}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2)),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*12i)/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (10*exp(a*1i + b*x*1i)
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(21*b*(ex
p(a*2i + b*x*2i)*1i - 1i)^2) - (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)
*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i
)^4) - (exp(a*1i + b*x*1i)*(10i/(21*b) - (exp(a*2i + b*x*2i)*32i)/(21*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^3/b/sin(2*b*x+2*a)^(
3/2)+32/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-64/45*cos(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4303, 4304, 4291}

$$\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-8*Cos[a + b*x])/(15*b*Sin[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^3/(9*b*Sin[2
*a + 2*b*x]^(3/2)) + (32*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (64*
Cos[a + b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{3}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{64}{45}\int \frac{\cos(a+b}{\sin^{\frac{3}{2}}(2a+2}\\ &= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\left(5\csc^5(a+bx)+17\csc^3(a+bx)+113\csc(a+bx)-15\tan(a+bx)\sec(a+bx)\right)}{180b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/180*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*Cs
c[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

fricas [A] time = 0.61, size = 131, normalized size = 1.22

$$-\frac{\sqrt{2}\left(128\cos(bx+a)^6-288\cos(bx+a)^4+180\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\left(\cos(b\right.}{180\left(b\cos(bx+a)^6-2b\cos(bx+a)^4+b\cos(bx+a)^2\right)\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/180*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a)
^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(bx+a)}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 4.99, size = 383, normalized size = 3.58

$$-\frac{2\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{15\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}-\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,16i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}+\frac{8\,e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(5/2)),x)
```

```
[Out] (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((ex
p(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*16i)/(9*b*(exp(
a*2i + b*x*2i)*1i - 1i)^4) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1
i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(15*b*(exp(a*2i + b*x*2i)*1i - 1i)
^3) + (64*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x
*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i -
1i)) - (exp(a*1i + b*x*1i)*(98i/(45*b) + (exp(a*2i + b*x*2i)*38i)/(45*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.123 $\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\sin^2(a+bx)\right)}{b(m+4)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([2+1/2*m, 1/2-1/2*m],[3+1/2*m],sin(b*x
+a)^2)*sin(b*x+a)^3*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(4+m)
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4310, 2577}

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\sin^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(4
 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{3+m}(a+$$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\sin^2(a+bx)\right)\sin^3(a+bx)\sin^m(2a+}{b(4+m)}$$

Mathematica [C] time = 5.73, size = 602, normalized size = 7.17

$$\frac{}{b(m+2)\left(-2(m+4)\cos^2\left(\frac{1}{2}(a+bx)\right)F_1\left(\frac{m}{2}+1;-m,2m+4;\frac{m}{2}+2;\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (32*(4 + m)*(AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^6*Sin[(a + b*x)/2]^4*Sin[2*(a
+ b*x)]^m)/(b*(2 + m)*(-2*(4 + m)*AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(m*AppellF1[
2 + m/2, 1 - m, 3 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- m*AppellF1[2 + m/2, 1 - m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] + 3*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2] + 2*m*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[2 + m/2, -m, 5 + 2*m, 3 + m
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[2 + m/2, -m, 5
+ 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x
]) + (4 + m)*AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2]*(1 + Cos[a + b*x])))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

maple [F] time = 10.16, size = 0, normalized size = 0.00

$$\int \left(\sin^{3}\left(bx+a\right)\right)\left(\sin^{m}\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(a+b\,x\right)}^3\,{\sin\left(2\,a+2\,b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

3.124 $\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(a+bx)\right)}{b(m+3)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([3/2+1/2*m, 1/2-1/2*m],[5/2+1/2*m],sin
(b*x+a)^2)*sin(b*x+a)^2*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(3+m)
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4310, 2577}

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(3
 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{2+m}(a+b$$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(a+bx)\right)\sin^2(a+bx)\sin^m(2a+2b}{b(3+m)}$$

Mathematica [C] time = 3.51, size = 602, normalized size = 7.17

$$\frac{}{b(m+1)\left(-2(m+3)\cos^2\left(\frac{1}{2}(a+bx)\right)F_1\left(\frac{m+1}{2};-m,2m+3;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2(\cos(a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (16*(3 + m)*(AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^5*Sin[(a + b*x)/2]^3
*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(-2*(3 + m)*AppellF1[(1 + m)/2, -m, 3 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 +
2*(m*AppellF1[(3 + m)/2, 1 - m, 2*(1 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 3*AppellF1[(3 + m)/2, -m, 2*(2 + m),
 (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)
/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*
AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b
*x)/2]^2] + 2*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (3 + m)*AppellF1[(1 + m)/2
, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + C
os[a + b*x])))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

maple [F] time = 6.23, size = 0, normalized size = 0.00

$$\int \left(\sin^{2}\left(bx+a\right)\right)\left(\sin^{m}\left(2bx+2a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(a+b\,x\right)}^2\,{\sin\left(2\,a+2\,b\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

3.125 $\int \sin(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=82

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(a+bx)\right)}{b(m+2)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([1+1/2*m, 1/2-1/2*m],[2+1/2*m],sin(b*x
+a)^2)*sin(b*x+a)*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(2+m)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4310, 2577}

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(2 +
 m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{1+m}(a+$$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(a+bx)\right)\sin(a+bx)\sin^m(2a+2}{b(2+m)}$$

Mathematica [C] time = 0.26, size = 152, normalized size = 1.85

$$-\frac{i2^{-m-1}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{m+1}\left((1-2m)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(3-2m);e^{4i(a+bx)}\right)+(2m+1)e^{2i}\right.}{b\left(4m^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((-I)*2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I
)*(a + b*x)))^(1 + m)*((1 - 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m
)/4, E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1
[1, (5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

maple [F] time = 5.04, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\left(\sin^{m}\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

3.126 $\int \csc(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{m+2}{2};\sin^2(a+bx)\right)}{bm}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([1/2*m, 1/2-1/2*m],[1+1/2*m],sin(b*x+a
)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/m
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4310, 2577\}$

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{m+2}{2};\sin^2(a+bx)\right)}{bm}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2,
Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*m)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^m(2a+2bx)\,dx &= \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-1+m}(a\\ &= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2b}{bm}\end{aligned}$$

Mathematica [C] time = 0.87, size = 254, normalized size = 3.53

$$\frac{2(m+2)\cos^2\left(\frac{1}{2}(a+bx)\right)\sin^m(2(a+}{bm\left((m+2)(\cos(a+bx)+1)F_1\left(\frac{m}{2};-m,2m;\frac{m+2}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-4m\sin^2\left(\frac{1}{2}(a+bx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*(2 + m)*AppellF1[m/2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2]*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(b*m*((2 + m)*AppellF1[m/
2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a
+ b*x]) - 4*m*(AppellF1[(2 + m)/2, 1 - m, 2*m, (4 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[(2 + m)/2, -m, 1 + 2*m, (4 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[(a + b*x)/2]^2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx + 2\,a\right)^{m} \csc\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 4.34, size = 0, normalized size = 0.00

$$\int \csc\left(bx + a\right)\left(\sin^{m}\left(2bx + 2a\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
[Out] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx + 2\,a\right)^{m} \csc\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a + 2\,b\,x\right)^{m}}{\sin\left(a + b\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x),x)
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}\left(2a + 2bx\right) \csc\left(a + bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**m,x)
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x), x)
```

3.127 $\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(a+bx)\right)}{b(1-m)}$$

```
[Out] -(cos(b*x+a)^2)^(1/2-1/2*m)*csc(b*x+a)*hypergeom([1/2-1/2*m, -1/2+1/2*m],[1
/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(1-m)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4310, 2577}

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(a+bx)\right)}{b(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - m)/2, (
-1 + m)/2, (1 + m)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(
1 - m)))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-2+m}(a$$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(a+bx)\right)\sec}{b(1-m)}$$

Mathematica [C] time = 5.42, size = 938, normalized size = 11.04

$$\frac{}{b\left(m(m+1)F_1\left(\frac{m-1}{2};-m,2m;\frac{m+1}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(3\cos(a+bx)-2)\sec(a+bx)\cot^2\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*((-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + (1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2)*Csc[a + b*x]^2*Sin[
2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m
, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - (1 + m)*AppellF1[(-
1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Csc[
(a + b*x)/2]^2 + (-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2 - 2*(-1 + m)*m*(AppellF1
[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
 + 2*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])
*Sec[a + b*x] + m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])*Cot[(a + b*x)/2]^2*S
ec[a + b*x] + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2 - (2*(-1 + m)*m*(1 + m)*(
AppellF1[(3 + m)/2, 1 - m, 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] + 2*AppellF1[(3 + m)/2, -m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]^2)/(3 + m) + 2*
m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2]*Cot[(a + b*x)/2]*Tan[a + b*x] + 2*(-1 + m)*m*AppellF1[(1 +
 m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a
+ b*x)/2]*Tan[a + b*x]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\csc\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 4.28, size = 0, normalized size = 0.00

$$\int \left(\csc^{2}\left(bx+a\right)\right)\left(\sin^{m}\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\csc\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^m}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}{\left(2a+2bx \right)}\csc^{2}{\left(a+bx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**2, x)
```

3.128 $\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-2}{2};\frac{m}{2};\sin^2(a+bx)\right)}{b(2-m)}$$

```
[Out] -(cos(b*x+a)^2)^(1/2-1/2*m)*csc(b*x+a)^2*hypergeom([-1+1/2*m, 1/2-1/2*m],[1
/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(2-m)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4310, 2577}

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-2}{2};\frac{m}{2};\sin^2(a+bx)\right)}{b(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - m)/2,
 (-2 + m)/2, m/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(2 -
m)))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-3+m}(a+b$$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc^2(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sin^2(a+bx)\right)\sec(a+}{b(2-m)}$$

Mathematica [C] time = 19.22, size = 2308, normalized size = 27.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*
Cos[(a + b*x)/2]^2*Cot[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(-2 + m)*(2*
(AppellF1[m/2, 1 - m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2
] + 2*AppellF1[m/2, -m, 1 + 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2])*(-1 + Cos[a + b*x]) + AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))) + ((4 + m)*AppellF1[1 +
m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[a + b*x
]*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(2 + m)*((4 + m)*AppellF1[1 +
 m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Sec[a
 + b*x]) - 4*m*(AppellF1[2 + m/2, 1 - m, 2*m, 3 + m/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2] + 2*AppellF1[2 + m/2, -m, 1 + 2*m, 3 + m/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[a + b*x]*Sin[(a + b*x)/2]^2)) + ((4 + m
)*AppellF1[(2 + m)/2, -m, 1 + 2*m, (4 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*Sin[a + b*x]^2*Sin[2*(a + b*x)]^m)/(4*b*(2 + m)*(2*(m*AppellF1[
2 + m/2, 1 - m, 1 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
+ (1 + 2*m)*AppellF1[2 + m/2, -m, 2 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4 + m)*AppellF1[(2 + m)/2, -m, 1
+ 2*m, (4 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x
]))) + (2^(-3 + m)*Cot[(a + b*x)/2]*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*
x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*((2
+ m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2]*Tan[(a + b*x)/2]^2))/(b*m*(2 + m)*(Cos[a + b*x]*Sec[(a + b*x)/
2]^2)^m*((2^(-1 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a
+ b*x)/2] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-1/2*Cos[(a
+ b*x)/2] + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(a + b*x)
/2] + Sin[(3*(a + b*x))/2]))/2)*((2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2
 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/((2 +
 m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m) + (2^m*(Sec[(a + b*x)/2]^2)^(2*m)*
(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*
x)/2]*((2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/((2 + m)*(Cos[a + b*x]*Sec[(a +
 b*x)/2]^2)^m) - (2^(-1 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(
a + b*x)/2]^2)^(-1 - m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a +
b*x))/2]))^m*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Sec[(a + b*
x)/2]^2*Tan[(a + b*x)/2])*((2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2
, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/(2 + m) + (
2^(-1 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2]
+ Sin[(3*(a + b*x))/2]))^m*(-(m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])
+ (2 + m)*(-1/2*(m^2*AppellF1[1 + m/2, 1 - m, 2*m, 2 + m/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(1 + m/2) -
(m^2*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(1 + m/2)) - m*Tan[(a + b*x
)/2]^2*(-(((1 + m/2)*m*AppellF1[2 + m/2, 1 - m, 1 + 2*m, 3 + m/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(2 + m
/2)) - ((1 + m/2)*(1 + 2*m)*AppellF1[2 + m/2, -m, 2 + 2*m, 3 + m/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(2 +
 m/2))))/(m*(2 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\csc\left(bx+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 3.85, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(bx+a\right)\right)\left(\sin^{m}\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
[Out] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\csc\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{m}}{\sin\left(a+b\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3,x)
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}\left(2a+2bx\right)\csc^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**m,x)
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**3, x)
```

3.129 $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{128\cos^{15}(a+bx)}{15b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{128\cos^{9}(a+bx)}{9b}$$

```
[Out] -128/9*cos(b*x+a)^9/b+384/11*cos(b*x+a)^11/b-384/13*cos(b*x+a)^13/b+128/15*
cos(b*x+a)^15/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 270}

$$\frac{128\cos^{15}(a+bx)}{15b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{128\cos^{9}(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-128*Cos[a + b*x]^9)/(9*b) + (384*Cos[a + b*x]^11)/(11*b) - (384*Cos[a + b
*x]^13)/(13*b) + (128*Cos[a + b*x]^15)/(15*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^8(a+bx)\sin^7(a+bx)\,dx\\
&= -\frac{128\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{128\,\text{Subst}\left(\int \left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}(a+bx)}{15b}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 47, normalized size = 0.77

$$\frac{4\cos^9(a+bx)(10755\cos(2(a+bx))-3366\cos(4(a+bx))+429\cos(6(a+bx))-8330)}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-8330 + 10755*Cos[2*(a + b*x)] - 3366*Cos[4*(a + b*x)] +
 429*Cos[6*(a + b*x)]))/(6435*b)
```

fricas [A] time = 0.47, size = 46, normalized size = 0.75

$$\frac{128\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

giac [B] time = 2.90, size = 110, normalized size = 1.80

$$\frac{\cos\left(15\,bx+15\,a\right)}{1920\,b}+\frac{\cos\left(13\,bx+13\,a\right)}{1664\,b}-\frac{7\cos\left(11\,bx+11\,a\right)}{1408\,b}-\frac{7\cos\left(9\,bx+9\,a\right)}{1152\,b}+\frac{3\cos\left(7\,bx+7\,a\right)}{128\,b}+\frac{21\cos\left(5\,bx\right.}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] 1/1920*cos(15*b*x + 15*a)/b + 1/1664*cos(13*b*x + 13*a)/b - 7/1408*cos(11*b
*x + 11*a)/b - 7/1152*cos(9*b*x + 9*a)/b + 3/128*cos(7*b*x + 7*a)/b + 21/64
0*cos(5*b*x + 5*a)/b - 35/384*cos(3*b*x + 3*a)/b - 35/128*cos(b*x + a)/b
```

maple [B] time = 0.41, size = 111, normalized size = 1.82

$$-\frac{35\cos\left(bx+a\right)}{128b}-\frac{35\cos\left(3bx+3a\right)}{384b}+\frac{21\cos\left(5bx+5a\right)}{640b}+\frac{3\cos\left(7bx+7a\right)}{128b}-\frac{7\cos\left(9bx+9a\right)}{1152b}-\frac{7\cos\left(11bx+11a\right)}{1408b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] -35/128*cos(b*x+a)/b-35/384*cos(3*b*x+3*a)/b+21/640*cos(5*b*x+5*a)/b+3/128*
cos(7*b*x+7*a)/b-7/1152*cos(9*b*x+9*a)/b-7/1408*cos(11*b*x+11*a)/b+1/1664*c
os(13*b*x+13*a)/b+1/1920*cos(15*b*x+15*a)/b
```

maxima [A] time = 0.35, size = 91, normalized size = 1.49

$$\frac{429\cos\left(15\,bx+15\,a\right)+495\cos\left(13\,bx+13\,a\right)-4095\cos\left(11\,bx+11\,a\right)-5005\cos\left(9\,bx+9\,a\right)+19305\cos\left(7\right.}{823680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

mupad [B] time = 0.03, size = 46, normalized size = 0.75

$$-\frac{-\frac{128\cos(a+b\,x)^{15}}{15}+\frac{384\cos(a+b\,x)^{13}}{13}-\frac{384\cos(a+b\,x)^{11}}{11}+\frac{128\cos(a+b\,x)^{9}}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

```
[Out] -((128*cos(a + b*x)^9)/9 - (384*cos(a + b*x)^11)/11 + (384*cos(a + b*x)^13)
/13 - (128*cos(a + b*x)^15)/15)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

3.130 $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{64\sin^{13}(a+bx)}{13b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{3b}+\frac{64\sin^7(a+bx)}{7b}$$

```
[Out] 64/7*sin(b*x+a)^7/b-64/3*sin(b*x+a)^9/b+192/11*sin(b*x+a)^11/b-64/13*sin(b*
x+a)^13/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2564, 270}

$$-\frac{64\sin^{13}(a+bx)}{13b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{3b}+\frac{64\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (64*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(3*b) + (192*Sin[a + b*x]^1
1)/(11*b) - (64*Sin[a + b*x]^13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^7(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{64\,\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{64\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{3b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.29, size = 47, normalized size = 0.77

$$\frac{2\sin^7(a+bx)(6377\cos(2(a+bx))+1890\cos(4(a+bx))+231\cos(6(a+bx))+5230)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (2*(5230 + 6377*Cos[2*(a + b*x)] + 1890*Cos[4*(a + b*x)] + 231*Cos[6*(a + b
*x)])*Sin[a + b*x]^7)/(3003*b)
```

fricas [A] time = 0.51, size = 73, normalized size = 1.20

$$-\frac{64\left(231\,\cos\left(bx+a\right)^{12}-567\,\cos\left(bx+a\right)^{10}+371\,\cos\left(bx+a\right)^{8}-5\,\cos\left(bx+a\right)^{6}-6\,\cos\left(bx+a\right)^{4}-8\,\cos(\right.}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/b
```

giac [A] time = 1.73, size = 96, normalized size = 1.57

$$-\frac{\sin\left(13\,bx+13\,a\right)}{832\,b}-\frac{\sin\left(11\,bx+11\,a\right)}{704\,b}+\frac{\sin\left(9\,bx+9\,a\right)}{96\,b}+\frac{3\,\sin\left(7\,bx+7\,a\right)}{224\,b}-\frac{3\,\sin\left(5\,bx+5\,a\right)}{64\,b}-\frac{5\,\sin\left(3\,bx+3\right.}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1/832*sin(13*b*x + 13*a)/b - 1/704*sin(11*b*x + 11*a)/b + 1/96*sin(9*b*x +
 9*a)/b + 3/224*sin(7*b*x + 7*a)/b - 3/64*sin(5*b*x + 5*a)/b - 5/64*sin(3*b
*x + 3*a)/b + 5/16*sin(b*x + a)/b
```

maple [A] time = 1.76, size = 97, normalized size = 1.59

$$\frac{5\sin\left(bx+a\right)}{16b}-\frac{5\sin\left(3bx+3a\right)}{64b}-\frac{3\sin\left(5bx+5a\right)}{64b}+\frac{3\sin\left(7bx+7a\right)}{224b}+\frac{\sin\left(9bx+9a\right)}{96b}-\frac{\sin\left(11bx+11a\right)}{704b}-\frac{\sin\left(13\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] 5/16*sin(b*x+a)/b-5/64*sin(3*b*x+3*a)/b-3/64/b*sin(5*b*x+5*a)+3/224/b*sin(7
*b*x+7*a)+1/96/b*sin(9*b*x+9*a)-1/704/b*sin(11*b*x+11*a)-1/832/b*sin(13*b*x
+13*a)
```

maxima [A] time = 0.34, size = 80, normalized size = 1.31

$$-\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\sin\left(5\,b\right.}{192192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
 3*a) - 60060*sin(b*x + a))/b
```

mupad [B] time = 0.13, size = 45, normalized size = 0.74

$$\frac{-\frac{64\sin(a+b\,x)^{13}}{13}+\frac{192\sin(a+b\,x)^{11}}{11}-\frac{64\sin(a+b\,x)^{9}}{3}+\frac{64\sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

```
[Out] ((64*sin(a + b*x)^7)/7 - (64*sin(a + b*x)^9)/3 + (192*sin(a + b*x)^11)/11 -
 (64*sin(a + b*x)^13)/13)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

3.131 $\int \cos(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^7(a+bx)}{7b}$$

```
[Out] -32/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b-32/11*cos(b*x+a)^11/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 270}

$$-\frac{32\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b) - (32*Cos[a + b*x]^1
1)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^6(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{32\,\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{32\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.25, size = 37, normalized size = 0.80

$$\frac{4\cos^7(a+bx)(364\cos(2(a+bx))-63\cos(4(a+bx))-365)}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(693
*b)
```

fricas [A] time = 0.46, size = 36, normalized size = 0.78

$$-\frac{32\left(63\,\cos\left(bx+a\right)^{11}-154\,\cos\left(bx+a\right)^{9}+99\,\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

giac [B] time = 0.94, size = 82, normalized size = 1.78

$$-\frac{\cos\left(11\,bx+11\,a\right)}{352\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{288\,b}+\frac{5\,\cos\left(7\,bx+7\,a\right)}{224\,b}+\frac{\cos\left(5\,bx+5\,a\right)}{32\,b}-\frac{5\,\cos\left(3\,bx+3\,a\right)}{48\,b}-\frac{5\,\cos\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/352*cos(11*b*x + 11*a)/b - 1/288*cos(9*b*x + 9*a)/b + 5/224*cos(7*b*x +
7*a)/b + 1/32*cos(5*b*x + 5*a)/b - 5/48*cos(3*b*x + 3*a)/b - 5/16*cos(b*x +
 a)/b
```

maple [B] time = 0.46, size = 83, normalized size = 1.80

$$-\frac{5\cos\left(bx+a\right)}{16b}-\frac{5\cos\left(3bx+3a\right)}{48b}+\frac{\cos\left(5bx+5a\right)}{32b}+\frac{5\cos\left(7bx+7a\right)}{224b}-\frac{\cos\left(9bx+9a\right)}{288b}-\frac{\cos\left(11bx+11a\right)}{352b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/16*cos(b*x+a)/b-5/48*cos(3*b*x+3*a)/b+1/32*cos(5*b*x+5*a)/b+5/224*cos(7*
b*x+7*a)/b-1/288*cos(9*b*x+9*a)/b-1/352*cos(11*b*x+11*a)/b
```

maxima [A] time = 0.34, size = 69, normalized size = 1.50

$$-\frac{63\,\cos\left(11\,bx+11\,a\right)+77\,\cos\left(9\,bx+9\,a\right)-495\,\cos\left(7\,bx+7\,a\right)-693\,\cos\left(5\,bx+5\,a\right)+2310\,\cos\left(3\,bx+3\,a\right)}{22176\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

mupad [B] time = 0.15, size = 36, normalized size = 0.78

$$-\frac{32\,\left(63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^{9}+99\cos\left(a+b\,x\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -(32*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b)
```

sympy [A] time = 113.17, size = 199, normalized size = 4.33

$$\begin{cases} -\frac{151\sin(a+bx)\sin^5(2a+2bx)}{693b} - \frac{272\sin(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{693b} - \frac{128\sin(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{693b} - \frac{422\sin^4(2a+2bx)\ldots}{} \\ x\sin^5(2a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-151*sin(a + b*x)*sin(2*a + 2*b*x)**5/(693*b) - 272*sin(a + b*x)
*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(693*b) - 128*sin(a + b*x)*sin(2*a
 + 2*b*x)*cos(2*a + 2*b*x)**4/(693*b) - 422*sin(2*a + 2*b*x)**4*cos(a + b*x
)*cos(2*a + 2*b*x)/(693*b) - 608*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a +
 2*b*x)**3/(693*b) - 256*cos(a + b*x)*cos(2*a + 2*b*x)**5/(693*b), Ne(b, 0)
), (x*sin(2*a)**5*cos(a), True))
```

3.132 $\int \cos(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{16\sin^9(a+bx)}{9b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

```
[Out] 16/5*sin(b*x+a)^5/b-32/7*sin(b*x+a)^7/b+16/9*sin(b*x+a)^9/b
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = $\{4287, 2564, 270\}$

$$\frac{16\sin^9(a+bx)}{9b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (32*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^5(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\sin^5(a+bx)}{5b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 37, normalized size = 0.80

$$\frac{2\sin^5(a+bx)(220\cos(2(a+bx))+35\cos(4(a+bx))+249)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (2*(249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(315*
b)
```

fricas [A] time = 0.41, size = 53, normalized size = 1.15

$$\frac{16 \left(35 \cos\left(bx + a\right)^{8} - 50 \cos\left(bx + a\right)^{6} + 3 \cos\left(bx + a\right)^{4} + 4 \cos\left(bx + a\right)^{2} + 8\right) \sin\left(bx + a\right)}{315 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

giac [A] time = 2.92, size = 68, normalized size = 1.48

$$\frac{\sin\left(9 \, bx + 9 \, a\right)}{144 \, b} + \frac{\sin\left(7 \, bx + 7 \, a\right)}{112 \, b} - \frac{\sin\left(5 \, bx + 5 \, a\right)}{20 \, b} - \frac{\sin\left(3 \, bx + 3 \, a\right)}{12 \, b} + \frac{3 \, \sin\left(bx + a\right)}{8 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/144*sin(9*b*x + 9*a)/b + 1/112*sin(7*b*x + 7*a)/b - 1/20*sin(5*b*x + 5*a)
/b - 1/12*sin(3*b*x + 3*a)/b + 3/8*sin(b*x + a)/b
```

maple [A] time = 1.22, size = 69, normalized size = 1.50

$$\frac{3 \sin \left(bx +a \right)}{8b}-\frac{\sin \left(3bx +3a \right)}{12b}-\frac{\sin \left(5bx +5a \right)}{20b}+\frac{\sin \left(7bx +7a \right)}{112b}+\frac{\sin \left(9bx +9a \right)}{144b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/8*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)+1/112/b*sin(7*
b*x+7*a)+1/144/b*sin(9*b*x+9*a)
```

maxima [A] time = 0.34, size = 58, normalized size = 1.26

$$\frac{35 \, \sin\left(9 \, bx + 9 \, a\right) + 45 \, \sin\left(7 \, bx + 7 \, a\right) - 252 \, \sin\left(5 \, bx + 5 \, a\right) - 420 \, \sin\left(3 \, bx + 3 \, a\right) + 1890 \, \sin\left(bx + a\right)}{5040 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

mupad [B] time = 0.14, size = 36, normalized size = 0.78

$$\frac{16\,\left(35\,{\sin\left(a+b\,x\right)}^9-90\,{\sin\left(a+b\,x\right)}^7+63\,{\sin\left(a+b\,x\right)}^5\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

```
[Out] (16*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

sympy [A] time = 38.31, size = 162, normalized size = 3.52

$$\begin{cases} \frac{107 \sin(a+bx) \sin^4(2a+2bx)}{315b} + \frac{16 \sin(a+bx) \sin^2(2a+2bx) \cos^2(2a+2bx)}{21b} + \frac{128 \sin(a+bx) \cos^4(2a+2bx)}{315b} - \frac{104 \sin^3(2a+2bx) \cos(a+bx) \cos(2a}{315b} \\ x \sin^4(2a) \cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((107*sin(a + b*x)*sin(2*a + 2*b*x)**4/(315*b) + 16*sin(a + b*x)*s
in(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(21*b) + 128*sin(a + b*x)*cos(2*a +
2*b*x)**4/(315*b) - 104*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)/(
315*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(315*b), Ne(b
, 0)), (x*sin(2*a)**4*cos(a), True))
```

3.133 $\int \cos(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\cos^7(a+bx)}{7b}-\frac{8\cos^5(a+bx)}{5b}$$

```
[Out] -8/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 14}

$$\frac{8\cos^7(a+bx)}{7b}-\frac{8\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^4(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{8\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{8\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{8\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 27, normalized size = 0.87

$$\frac{4\cos^5(a+bx)(5\cos(2(a+bx))-9)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(35*b)
```

fricas [A] time = 0.45, size = 26, normalized size = 0.84

$$\frac{8 \left(5 \cos \left(bx+a\right)^{7}-7 \cos \left(bx+a\right)^{5}\right)}{35 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

giac [A] time = 0.29, size = 54, normalized size = 1.74

$$\frac{\cos \left(7 \, bx+7 \, a\right)}{56 \, b}+\frac{\cos \left(5 \, bx+5 \, a\right)}{40 \, b}-\frac{\cos \left(3 \, bx+3 \, a\right)}{8 \, b}-\frac{3 \, \cos \left(bx+a\right)}{8 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/56*cos(7*b*x + 7*a)/b + 1/40*cos(5*b*x + 5*a)/b - 1/8*cos(3*b*x + 3*a)/b
- 3/8*cos(b*x + a)/b
```

maple [A] time = 0.41, size = 55, normalized size = 1.77

$$-\frac{3 \cos \left(bx+a\right)}{8b}-\frac{\cos \left(3bx+3a\right)}{8b}+\frac{\cos \left(5bx+5a\right)}{40b}+\frac{\cos \left(7bx+7a\right)}{56b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/8*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b+1/40*cos(5*b*x+5*a)/b+1/56*cos(7*b*x
+7*a)/b
```

maxima [A] time = 0.33, size = 47, normalized size = 1.52

$$\frac{5 \, \cos \left(7 \, bx+7 \, a\right)+7 \, \cos \left(5 \, bx+5 \, a\right)-35 \, \cos \left(3 \, bx+3 \, a\right)-105 \, \cos \left(bx+a\right)}{280 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*
cos(b*x + a))/b
```

mupad [B] time = 0.03, size = 26, normalized size = 0.84

$$-\frac{8 \, \left(7 \cos \left(a+b\,x\right)^5-5 \cos \left(a+b\,x\right)^7\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

```
[Out] -(8*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

sympy [A] time = 11.27, size = 128, normalized size = 4.13

$$\begin{cases} -\frac{9 \sin \left(a+bx\right) \sin^{3} \left(2a+2bx\right)}{35b}-\frac{8 \sin \left(a+bx\right) \sin \left(2a+2bx\right) \cos^{2} \left(2a+2bx\right)}{35b}-\frac{22 \sin^{2} \left(2a+2bx\right) \cos \left(a+bx\right) \cos \left(2a+2bx\right)}{35b}-\frac{16 \cos \left(a+bx\right) \cos^{3} \left(2a+2bx\right.}{35b} \\ x \sin^{3} \left(2a\right) \cos \left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-9*sin(a + b*x)*sin(2*a + 2*b*x)**3/(35*b) - 8*sin(a + b*x)*sin(
2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(35*b) - 22*sin(2*a + 2*b*x)**2*cos(a + b*
x)*cos(2*a + 2*b*x)/(35*b) - 16*cos(a + b*x)*cos(2*a + 2*b*x)**3/(35*b), Ne
(b, 0)), (x*sin(2*a)**3*cos(a), True))
```

3.134 $\int \cos(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

```
[Out] 4/3*sin(b*x+a)^3/b-4/5*sin(b*x+a)^5/b
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2564, 14}

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (4*Sin[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^3(a+bx)\sin^2(a+bx)\,dx\\ &= \frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{4\,\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.87

$$\frac{2\sin^3(a+bx)(3\cos(2(a+bx))+7)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
[Out] (2*(7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(15*b)
```

fricas [A] time = 0.42, size = 33, normalized size = 1.06

$$-\frac{4\left(3\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] -4/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

giac [A] time = 0.46, size = 40, normalized size = 1.29

$$-\frac{\sin\left(5\,bx+5\,a\right)}{20\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{12\,b}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
[Out] -1/20*sin(5*b*x + 5*a)/b - 1/12*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

maple [A] time = 0.62, size = 41, normalized size = 1.32

$$\frac{\sin\left(bx+a\right)}{2b}-\frac{\sin\left(3bx+3a\right)}{12b}-\frac{\sin\left(5bx+5a\right)}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^2,x)
[Out] 1/2*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)
```

maxima [A] time = 0.33, size = 36, normalized size = 1.16

$$-\frac{3\sin\left(5\,bx+5\,a\right)+5\sin\left(3\,bx+3\,a\right)-30\sin\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] -1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

mupad [B] time = 0.03, size = 26, normalized size = 0.84

$$\frac{4\,\left(5\sin\left(a+b\,x\right)^{3}-3\sin\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^2,x)
[Out] (4*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

sympy [A] time = 3.12, size = 90, normalized size = 2.90

$$\begin{cases}\frac{7\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)}{15b}+\frac{8\sin\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{15b}-\frac{4\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{15b} & \text{for}\ b\neq 0\\ x\sin^{2}\left(2a\right)\cos\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((7*sin(a + b*x)*sin(2*a + 2*b*x)**2/(15*b) + 8*sin(a + b*x)*cos(2
*a + 2*b*x)**2/(15*b) - 4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)/(1
5*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a), True))
```

3.135 $\int \cos(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

```
[Out] -1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4284}

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] -Cos[a + b*x]/(2*b) - Cos[3*a + 3*b*x]/(6*b)
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.50

$$-\frac{2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(3*b)
```

fricas [A] time = 0.48, size = 13, normalized size = 0.43

$$-\frac{2\cos\left(bx+a\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*cos(b*x + a)^3/b
```

giac [A] time = 0.28, size = 26, normalized size = 0.87

$$-\frac{\cos\left(3\,bx+3\,a\right)}{6\,b}-\frac{\cos\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

maple [A] time = 0.22, size = 27, normalized size = 0.90

$$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] -1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

maxima [A] time = 0.32, size = 26, normalized size = 0.87

$$-\frac{\cos(3\,bx+3\,a)}{6\,b}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

mupad [B] time = 0.19, size = 43, normalized size = 1.43

$$\begin{cases} x\left(2\sin(a)-2\sin(a)^3\right) & \text{if } b=0 \\ -\frac{3\cos(a+b\,x)+\cos(3\,a+3\,b\,x)}{6\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x),x)
```

```
[Out] piecewise(b == 0, x*(2*sin(a) - 2*sin(a)^3), b ~= 0, -(3*cos(a + b*x) + cos
(3*a + 3*b*x))/(6*b))
```

sympy [A] time = 0.76, size = 53, normalized size = 1.77

$$\begin{cases} -\frac{\sin(a+bx)\sin(2a+2bx)}{3b}-\frac{2\cos(a+bx)\cos(2a+2bx)}{3b} & \text{for } b\neq 0 \\ x\sin(2a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-sin(a + b*x)*sin(2*a + 2*b*x)/(3*b) - 2*cos(a + b*x)*cos(2*a +
2*b*x)/(3*b), Ne(b, 0)), (x*sin(2*a)*cos(a), True))
```

3.136 $\int \cos(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] -1/2*arctanh(cos(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4287, 3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 42, normalized size = 3.00

$$\frac{1}{2}\left(\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] (-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b)/2
```

fricas [B] time = 0.60, size = 30, normalized size = 2.14

$$-\frac{\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

giac [A] time = 0.22, size = 16, normalized size = 1.14

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*b*x + 1/2*a)))/b
```

maple [A] time = 0.55, size = 22, normalized size = 1.57

$$\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a),x)
```

```
[Out] 1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.34, size = 84, normalized size = 6.00

$$-\frac{\log\left(\cos\left(bx\right)^{2} + 2\,\cos\left(bx\right)\cos(a) + \cos(a)^{2} + \sin\left(bx\right)^{2} - 2\,\sin\left(bx\right)\sin(a) + \sin(a)^{2}\right) - \log\left(\cos\left(bx\right)^{2} - 2\,\cos\left(\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(a) + sin(a)^2) - log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x),x)
```

```
[Out] -atanh(cos(a + b*x))/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

3.137 $\int \cos(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

```
[Out] 1/4*arctanh(sin(b*x+a))/b-1/4*csc(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4287, 2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(4*b) - Csc[a + b*x]/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^2(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}$$
$$= -\frac{\csc(a+bx)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{4b} - \frac{\csc(a+bx)}{4b}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 1.04

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.47, size = 50, normalized size = 1.79

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{8\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

giac [A] time = 0.27, size = 38, normalized size = 1.36

$$-\frac{\frac{2}{\sin(bx+a)}-\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2/sin(b*x + a) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) -
1)))/b
```

maple [A] time = 0.86, size = 34, normalized size = 1.21

$$-\frac{1}{4b\sin\left(bx+a\right)}+\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^2,x)
```

```
[Out] -1/4/b/sin(b*x+a)+1/4/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.46, size = 233, normalized size = 8.32

$$-\frac{\left(\cos\left(2\,bx+2\,a\right)^{2}+\sin\left(2\,bx+2\,a\right)^{2}-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^{2}+\cos(a)^{2}-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a}{\cos(bx+2\,a)^{2}+\cos(a)^{2}+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a}\right.}{8\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

mupad [B] time = 0.02, size = 26, normalized size = 0.93

$$\frac{\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{4\,b}-\frac{1}{4\,b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^2,x)
```

```
[Out] atanh(sin(a + b*x))/(4*b) - 1/(4*b*sin(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

3.138 $\int \cos(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{16b}-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

```
[Out] -3/16*arctanh(cos(b*x+a))/b+3/16*sec(b*x+a)/b-1/16*csc(b*x+a)^2*sec(b*x+a)/
b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4287, 2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{16b}-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(16*b) + (3*Sec[a + b*x])/(16*b) - (Csc[a + b*x]
^2*Sec[a + b*x])/(16*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{8b} \\
&= -\frac{\csc^2(a+bx)\sec(a+bx)}{16b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b} \\
&= \frac{3\sec(a+bx)}{16b} - \frac{\csc^2(a+bx)\sec(a+bx)}{16b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+\right.}{16b} \\
&= -\frac{3\tanh^{-1}(\cos(a+bx))}{16b} + \frac{3\sec(a+bx)}{16b} - \frac{\csc^2(a+bx)\sec(a+bx)}{16b}
\end{aligned}$$

Mathematica [B] time = 0.25, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx)) + 2\cos(3(a+bx)) + 3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right) - 3\cos(3(a+bx)\right.}{16b\left(\csc^2\left(\frac{1}{2}(a+bx)\right) - \sec\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(16*
b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

fricas [B] time = 0.47, size = 96, normalized size = 1.96

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\text{lo}}{32\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

giac [B] time = 0.41, size = 140, normalized size = 2.86

$$\frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
```

```
a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.95, size = 57, normalized size = 1.16

$$-\frac{1}{16b\sin(bx+a)^2\cos(bx+a)}+\frac{3}{16b\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16/b/sin(b*x+a)^2/cos(b*x+a)+3/16/b/cos(b*x+a)+3/16/b*ln(csc(b*x+a)-cot(
b*x+a))
```

maxima [B] time = 0.35, size = 974, normalized size = 19.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.08, size = 49, normalized size = 1.00

$$-\frac{3\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{16\,b}-\frac{\frac{3\cos\left(a+b\,x\right)^2}{16}-\frac{1}{8}}{b\,\left(\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

```
[Out] - (3*atanh(cos(a + b*x)))/(16*b) - ((3*cos(a + b*x)^2)/16 - 1/8)/(b*(cos(a
+ b*x) - cos(a + b*x)^3))
```

sympy [**F**(**-1**)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

3.139 $\int \cos(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{96b}-\frac{5\csc(a+bx)}{32b}+\frac{5\tanh^{-1}(\sin(a+bx))}{32b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

```
[Out] 5/32*arctanh(sin(b*x+a))/b-5/32*csc(b*x+a)/b-5/96*csc(b*x+a)^3/b+1/32*csc(b
*x+a)^3*sec(b*x+a)^2/b
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4287, 2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{96b}-\frac{5\csc(a+bx)}{32b}+\frac{5\tanh^{-1}(\sin(a+bx))}{32b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(32*b) - (5*Csc[a + b*x])/(32*b) - (5*Csc[a + b*x
]^3)/(96*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(32*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{32b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx\right.}{32b}$$
$$= -\frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{5\tanh^{-1}(\sin(a+bx))}{32b} - \frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec}{32b}$$

Mathematica [C] time = 0.02, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.47, size = 130, normalized size = 1.97

$$-\frac{30\cos(bx+a)^4 - 15\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a) + 15\left(\cos(bx+a)^4 -\right.}{192\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*
x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*co
s(b*x + a)^2)*sin(b*x + a))
```

giac [A] time = 0.44, size = 72, normalized size = 1.09

$$-\frac{\frac{6\sin(bx+a)}{\sin(bx+a)^2-1} + \frac{4\left(6\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 15\log(|\sin(bx+a)+1|) + 15\log(|\sin(bx+a)-1|)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/192*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(
b*x + a)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))
/b
```

maple [A] time = 1.11, size = 76, normalized size = 1.15

$$-\frac{1}{48b\sin\left(bx+a\right)^{3}\cos\left(bx+a\right)^{2}}+\frac{5}{96b\sin\left(bx+a\right)\cos\left(bx+a\right)^{2}}-\frac{5}{32b\sin\left(bx+a\right)}+\frac{5\ln\left(\sec\left(bx+a\right)+\tan\left(bx+\right.\right.}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^4,x)
```

```
[Out] -1/48/b/sin(b*x+a)^3/cos(b*x+a)^2+5/96/b/sin(b*x+a)/cos(b*x+a)^2-5/32/b/sin
(b*x+a)+5/32/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.53, size = 1780, normalized size = 26.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x
+ 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*
b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x +
3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*si
n(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
 - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b
*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(
2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x +
2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*
(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2
+ 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x
+ 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2
- 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20
*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x
+ a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*co
s(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
 7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*
b*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*
cos(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a)
- 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x
+ 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*s
in(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*si
n(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^
2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2
+ b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 +
4*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*
x + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a
) - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*
b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*
```

```
b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x +
8*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + b)
```

mupad [**B**] time = 0.10, size = 61, normalized size = 0.92

$$\frac{5\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{32\,b}-\frac{-\frac{5\,{\sin\left(a+b\,x\right)}^4}{32}+\frac{5\,{\sin\left(a+b\,x\right)}^2}{48}+\frac{1}{48}}{b\,\left({\sin\left(a+b\,x\right)}^3-{\sin\left(a+b\,x\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(32*b) - ((5*sin(a + b*x)^2)/48 - (5*sin(a + b*x)^4
)/32 + 1/48)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.140 $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{768b}+\frac{35\sec(a+bx)}{256b}-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

```
[Out] -35/256*arctanh(cos(b*x+a))/b+35/256*sec(b*x+a)/b+35/768*sec(b*x+a)^3/b-7/2
56*csc(b*x+a)^2*sec(b*x+a)^3/b-1/128*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4287, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{768b}+\frac{35\sec(a+bx)}{256b}-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(256*b) + (35*Sec[a + b*x])/(256*b) + (35*Sec[a
 + b*x]^3)/(768*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(128*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{128b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\,\text{Subst}\left(\int \frac{-}{-}\right.}{}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\,\text{Subst}\left(\int \left(\right.\right.}{}$$
$$= \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+}{}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{256b} + \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a+}{}$$

Mathematica [B] time = 0.48, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -1/768*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
 + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*C
os[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
 + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
 + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2
]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log
[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(
a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.48, size = 148, normalized size = 1.66

$$\frac{210\cos(bx+a)^6 - 350\cos(bx+a)^4 + 112\cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a\right.}{1536\left(b\cos(bx+a)^7 - 2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/1536*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*c
os(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
 + a)^3)
```

giac [B] time = 0.64, size = 209, normalized size = 2.35

$$\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+42}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/6144*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
 a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
1)^3 + 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.96, size = 99, normalized size = 1.11

$$-\frac{1}{128b\sin(bx+a)^4\cos(bx+a)^3}+\frac{7}{384b\sin(bx+a)^2\cos(bx+a)^3}-\frac{35}{768b\sin(bx+a)^2\cos(bx+a)}+\frac{35}{256b\cos(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)^3+7/384/b/sin(b*x+a)^2/cos(b*x+a)^3-35/768
/b/sin(b*x+a)^2/cos(b*x+a)+35/256/b/cos(b*x+a)+35/256/b*ln(csc(b*x+a)-cot(b
*x+a))
```

maxima [B] time = 0.50, size = 3846, normalized size = 43.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*
b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a)
+ cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329
*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*
b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*
a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b
*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*
x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*c
os(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4
*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x +
7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*
a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) -
15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x +
2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(
4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x
+ 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3
*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2
*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a
) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*
```

```
x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x +
8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(
10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b
*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 +
 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b
*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a
)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3
*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2
*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) -
3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x +
2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3
*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10*
a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) -
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(
4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*c
os(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2
*(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^
2 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2
*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b
*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9
*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b
*x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a
) - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*s
in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*
sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3
*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) -
3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a
)^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*
b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(
13*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b
*x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*
sin(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin
(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))
*sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204
*sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*
x + a))*sin(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a)
 - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(11*b*x +
 11*a) + 12*(329*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) + 329*sin(5*b*x +
5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*sin(10*b*x + 10*a) - 1316*(3
*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) - 7
0*sin(3*b*x + 3*a) + 105*sin(b*x + a))*sin(8*b*x + 8*a) + 816*(3*sin(6*b*x
+ 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) - 84*(47*s
in(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x + 6*a) +
 1316*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 420*(2*sin
(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 280*sin(3*b*x + 3*a)*sin
(2*b*x + 2*a) - 420*sin(2*b*x + 2*a)*sin(b*x + a) + 420*cos(b*x + a))/(b*co
s(14*b*x + 14*a)^2 + b*cos(12*b*x + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*
b*cos(8*b*x + 8*a)^2 + 9*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*
```

```
cos(2*b*x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*
sin(10*b*x + 10*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*
b*sin(4*b*x + 4*a)^2 + 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
+ 2*a)^2 - 2*(b*cos(12*b*x + 12*a) + 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x
 + 8*a) - 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a)
- b)*cos(14*b*x + 14*a) + 2*(3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a)
- 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos
(12*b*x + 12*a) - 6*(3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) - 3*b*cos(
4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 6*(3*b*cos(6*b*
x + 6*a) - 3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a)
- 6*(3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 6*(b
*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(1
2*b*x + 12*a) + 3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b
*x + 6*a) + 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) +
 2*(3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) +
3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 6*(3*b*sin(
8*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x +
2*a))*sin(10*b*x + 10*a) + 6*(3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) -
 b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 6*(3*b*sin(4*b*x + 4*a) + b*sin(2*b
*x + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.12, size = 78, normalized size = 0.88

$$\frac{\frac{35\cos(a+b\,x)^6}{256}-\frac{175\cos(a+b\,x)^4}{768}+\frac{7\cos(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{35\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

```
[Out] ((7*cos(a + b*x)^2)/96 - (175*cos(a + b*x)^4)/768 + (35*cos(a + b*x)^6)/256
 + 1/96)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*ata
nh(cos(a + b*x)))/(256*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.141 $\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{8\cos^{12}(a+bx)}{3b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{4\cos^{8}(a+bx)}{b}$$

```
[Out] -4*cos(b*x+a)^8/b+32/5*cos(b*x+a)^10/b-8/3*cos(b*x+a)^12/b
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4287, 2565, 266, 43}

$$-\frac{8\cos^{12}(a+bx)}{3b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{4\cos^{8}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-4*Cos[a + b*x]^8)/b + (32*Cos[a + b*x]^10)/(5*b) - (8*Cos[a + b*x]^12)/(3
*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^7(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{32\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(a+bx)\right)}{b}$$
$$= -\frac{16\,\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx, x, \cos^2(a+bx)\right)}{b}$$
$$= -\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}$$

Mathematica [A] time = 0.41, size = 68, normalized size = 1.55

$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))+5\cos(1\ldots}{3840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] -1/3840*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)]
 - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

fricas [A] time = 0.46, size = 36, normalized size = 0.82

$$-\frac{4\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^8\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

giac [B] time = 0.49, size = 85, normalized size = 1.93

$$-\frac{\cos(12\,bx+12\,a)}{768\,b}-\frac{\cos(10\,bx+10\,a)}{320\,b}+\frac{\cos(8\,bx+8\,a)}{128\,b}+\frac{5\,\cos(6\,bx+6\,a)}{192\,b}-\frac{5\,\cos(4\,bx+4\,a)}{256\,b}-\frac{5\,\cos(2\,bx+2\ldots}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/768*cos(12*b*x + 12*a)/b - 1/320*cos(10*b*x + 10*a)/b + 1/128*cos(8*b*x
 + 8*a)/b + 5/192*cos(6*b*x + 6*a)/b - 5/256*cos(4*b*x + 4*a)/b - 5/32*cos(2
*b*x + 2*a)/b
```

maple [B] time = 0.34, size = 86, normalized size = 1.95

$$-\frac{5\cos(2bx+2a)}{32b}-\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}+\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}-\frac{\cos(12bx+12a)}{768b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b-5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b+1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b-1/768*cos(12*b*x+12*a)/b
```

maxima [A] time = 0.34, size = 72, normalized size = 1.64

$$-\frac{5\cos\left(12\,bx+12\,a\right)+12\cos\left(10\,bx+10\,a\right)-30\cos\left(8\,bx+8\,a\right)-100\cos\left(6\,bx+6\,a\right)+75\cos\left(4\,bx+4\,a\right)}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/3840*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

mupad [B] time = 0.18, size = 35, normalized size = 0.80

$$-\frac{4\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -(4*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.142 $\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

```
[Out] 3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^3/b+1/20*sin(2*b*x+2*a)^5/b
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4285, 2635, 8, 2564, 30}

$$\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) + Sin[2*a + 2*b*x]^5/(20*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4285

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{1}{2}\int \sin^4(2a+2bx)\,dx + \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx$$

$$= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx + \frac{\text{Subst}\left(\int x^4\,d \ldots\right.}{\ldots}$$

$$= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\sin^5(2 \ldots}{2 \ldots}$$

$$= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\text{si}\ldots}{\ldots}$$

Mathematica [A] time = 0.19, size = 62, normalized size = 0.82

$$\frac{20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx)) + 120bx}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(640*b)
```

fricas [A] time = 0.46, size = 66, normalized size = 0.87

$$\frac{15\,bx + \left(128\,\cos(bx+a)^9 - 176\,\cos(bx+a)^7 + 8\,\cos(bx+a)^5 + 10\,\cos(bx+a)^3 + 15\,\cos(bx+a)\right)\sin(bx \ldots}{80\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.40, size = 74, normalized size = 0.97

$$\frac{3}{16}\,x + \frac{\sin(10\,bx + 10\,a)}{320\,b} + \frac{\sin(8\,bx + 8\,a)}{128\,b} - \frac{\sin(6\,bx + 6\,a)}{64\,b} - \frac{\sin(4\,bx + 4\,a)}{16\,b} + \frac{\sin(2\,bx + 2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x + 1/320*sin(10*b*x + 10*a)/b + 1/128*sin(8*b*x + 8*a)/b - 1/64*sin(6
*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b + 1/32*sin(2*b*x + 2*a)/b
```

maple [A] time = 1.65, size = 75, normalized size = 0.99

$$\frac{3x}{16} + \frac{\sin(2bx+2a)}{32b} - \frac{\sin(4bx+4a)}{16b} - \frac{\sin(6bx+6a)}{64b} + \frac{\sin(8bx+8a)}{128b} + \frac{\sin(10bx+10a)}{320b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/16*x+1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)+1/320/b*sin(10*b*x+10*a)
```

maxima [A] time = 0.34, size = 65, normalized size = 0.86

$$\frac{120\,bx + 2\,\sin(10\,bx + 10\,a) + 5\,\sin(8\,bx + 8\,a) - 10\,\sin(6\,bx + 6\,a) - 40\,\sin(4\,bx + 4\,a) + 20\,\sin(2\,bx + 2\,a \ldots}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

mupad [B] time = 1.73, size = 109, normalized size = 1.43

$$\frac{3x}{16}+\frac{\frac{3\tan(a+bx)^9}{16}+\frac{7\tan(a+bx)^7}{8}+\frac{8\tan(a+bx)^5}{5}-\frac{7\tan(a+bx)^3}{8}-\frac{3\tan(a+bx)}{16}}{b\left(\tan(a+bx)^{10}+5\tan(a+bx)^8+10\tan(a+bx)^6+10\tan(a+bx)^4+5\tan(a+bx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

```
[Out] (3*x)/16 + ((8*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/8 - (3*tan(a + b*x))/
16 + (7*tan(a + b*x)^7)/8 + (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 +
 1))
```

sympy [A] time = 116.23, size = 434, normalized size = 5.71

$$\begin{cases}\frac{3x\sin^2(a+bx)\sin^4(2a+2bx)}{16}+\frac{3x\sin^2(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{8}+\frac{3x\sin^2(a+bx)\cos^4(2a+2bx)}{16}+\frac{3x\sin^4(2a+2bx)\cos^2(a+bx)}{16}+\frac{3x}{} \\ x\sin^4(2a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a +
2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a + 2
*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(2*
a + 2*b*x)**4/16 + 7*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(
160*b) + 19*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480*b) +
sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) + 2*sin(a + b*x)*sin(2
*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) + 4*sin(a + b*x)*cos(
a + b*x)*cos(2*a + 2*b*x)**4/(15*b) - 57*sin(2*a + 2*b*x)**3*cos(a + b*x)**
2*cos(2*a + 2*b*x)/(160*b) - 109*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a +
 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**2, True))
```

3.143 $\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos^8(a+bx)}{b}-\frac{4\cos^6(a+bx)}{3b}$$

```
[Out] -4/3*cos(b*x+a)^6/b+cos(b*x+a)^8/b
```

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2565, 14}

$$\frac{\cos^8(a+bx)}{b}-\frac{4\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-4*Cos[a + b*x]^6)/(3*b) + Cos[a + b*x]^8/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^5(a+bx)\sin^3(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{8\,\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{4\cos^6(a+bx)}{3b}+\frac{\cos^8(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 48, normalized size = 1.71

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(384*b)
```

fricas [A] time = 0.65, size = 26, normalized size = 0.93

$$\frac{3\,\cos\left(bx+a\right)^{8}-4\,\cos\left(bx+a\right)^{6}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

giac [B] time = 0.32, size = 57, normalized size = 2.04

$$\frac{\cos\left(8\,bx+8\,a\right)}{128\,b}+\frac{\cos\left(6\,bx+6\,a\right)}{48\,b}-\frac{\cos\left(4\,bx+4\,a\right)}{32\,b}-\frac{3\,\cos\left(2\,bx+2\,a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/128*cos(8*b*x + 8*a)/b + 1/48*cos(6*b*x + 6*a)/b - 1/32*cos(4*b*x + 4*a)/
b - 3/16*cos(2*b*x + 2*a)/b
```

maple [B] time = 0.42, size = 58, normalized size = 2.07

$$-\frac{3\cos\left(2bx+2a\right)}{16b}-\frac{\cos\left(4bx+4a\right)}{32b}+\frac{\cos\left(6bx+6a\right)}{48b}+\frac{\cos\left(8bx+8a\right)}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b-1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b+1/128*co
s(8*b*x+8*a)/b
```

maxima [A] time = 0.34, size = 50, normalized size = 1.79

$$\frac{3\,\cos\left(8\,bx+8\,a\right)+8\,\cos\left(6\,bx+6\,a\right)-12\,\cos\left(4\,bx+4\,a\right)-72\,\cos\left(2\,bx+2\,a\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/384*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*c
os(2*b*x + 2*a))/b
```

mupad [B] time = 0.14, size = 26, normalized size = 0.93

$$-\frac{\frac{4\cos\left(a+b\,x\right)^6}{3}-\cos\left(a+b\,x\right)^8}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
```

```
[Out] -((4*cos(a + b*x)^6)/3 - cos(a + b*x)^8)/b
```

sympy [A] time = 39.28, size = 359, normalized size = 12.82

$$\begin{cases} -\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} - \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} - \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} - \frac{3x\sin(\ldots}{} \\ x\sin^3(2a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 - 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 - 3*x*sin(a + b*x)*sin(2*a + 2*b*
x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 - 3*x*sin(a + b*x)*cos(a + b*x)*cos(2
*a + 2*b*x)**3/8 + 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 + 3*x*sin(2*a
 + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*cos(2*a
+ 2*b*x)**3/(96*b) - 3*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b)
 - sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) - s
in(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(2*b) - 31*cos(a + b*x)
**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(2*a)**3*cos(a)**2, True))
```

3.144 $\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

```
[Out] 1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b+1/12*sin(2*b*x+2*a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4285, 2635, 8, 2564, 30}

$$\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) + Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4285

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx &= \frac{1}{2}\int \sin^2(2a+2bx)\,dx+\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\int 1\,dx}{4}+\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\sin(2a+2bx)\right)}{4b}\\ &= \frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\sin^3(2a+2bx)}{12b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 40, normalized size = 0.82

$$-\frac{-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))-12bx}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] -1/48*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)]
)/b
```

fricas [A] time = 0.50, size = 47, normalized size = 0.96

$$\frac{3\,bx-\left(8\,\cos\left(bx+a\right)^{5}-2\,\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

giac [A] time = 0.19, size = 46, normalized size = 0.94

$$\frac{1}{4}\,x-\frac{\sin\left(6\,bx+6\,a\right)}{48\,b}-\frac{\sin\left(4\,bx+4\,a\right)}{16\,b}+\frac{\sin\left(2\,bx+2\,a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x - 1/48*sin(6*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b + 1/16*sin(2*b*x
+ 2*a)/b
```

maple [A] time = 0.96, size = 47, normalized size = 0.96

$$\frac{x}{4}+\frac{\sin\left(2bx+2a\right)}{16b}-\frac{\sin\left(4bx+4a\right)}{16b}-\frac{\sin\left(6bx+6a\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

```
[Out] 1/4*x+1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/48/b*sin(6*b*x+6*a)
```

maxima [A] time = 0.34, size = 43, normalized size = 0.88

$$\frac{12\,bx-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

mupad [B] time = 0.31, size = 43, normalized size = 0.88

$$\frac{x}{4}-\frac{\frac{\sin(4\,a+4\,b\,x)}{16}-\frac{\sin(2\,a+2\,b\,x)}{16}+\frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
```

```
[Out] x/4 - (sin(4*a + 4*b*x)/16 - sin(2*a + 2*b*x)/16 + sin(6*a + 6*b*x)/48)/b
```

sympy [A] time = 11.68, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{\sin^2(a+bx)\sin(2a+2}{24b} \\ x\sin^2(2a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 + sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b
*x)/(24*b) + sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) + sin(a +
b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) - 7*sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a)**2, True))
```

3.145 $\int \cos^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^4(a+bx)}{2b}$$

[Out] -1/2*cos(b*x+a)^4/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4287, 2565, 30\}$

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]

[Out] -Cos[a + b*x]^4/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos^3(a+bx)\sin(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^4(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]

[Out] -1/2*Cos[a + b*x]^4/b

fricas [A] time = 0.52, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/2*cos(b*x + a)^4/b
```

giac [B] time = 0.50, size = 29, normalized size = 1.93

$$-\frac{\cos\left(4\,bx+4\,a\right)}{16\,b}-\frac{\cos\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/16*cos(4*b*x + 4*a)/b - 1/4*cos(2*b*x + 2*a)/b
```

maple [B] time = 0.24, size = 30, normalized size = 2.00

$$-\frac{\cos\left(2bx+2a\right)}{4b}-\frac{\cos\left(4bx+4a\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a),x)
```

```
[Out] -1/4*cos(2*b*x+2*a)/b-1/16*cos(4*b*x+4*a)/b
```

maxima [A] time = 0.33, size = 26, normalized size = 1.73

$$-\frac{\cos\left(4\,bx+4\,a\right)+4\,\cos\left(2\,bx+2\,a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/16*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

mupad [B] time = 0.15, size = 13, normalized size = 0.87

$$-\frac{\cos\left(a+b\,x\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x),x)
```

```
[Out] -cos(a + b*x)^4/(2*b)
```

sympy [A] time = 3.24, size = 131, normalized size = 8.73

$$\begin{cases} -\frac{x\sin^2(a+bx)\sin(2a+2bx)}{4}-\frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2}+\frac{x\sin(2a+2bx)\cos^2(a+bx)}{4}-\frac{\sin(a+bx)\sin(2a+2bx)\cos(a+bx)}{4b}-\frac{\cos^2(\ldots}{} \\ x\sin(2a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 - x*sin(a + b*x)*cos(a + b
*x)*cos(2*a + 2*b*x)/2 + x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*x
)*sin(2*a + 2*b*x)*cos(a + b*x)/(4*b) - cos(a + b*x)**2*cos(2*a + 2*b*x)/(2
*b), Ne(b, 0)), (x*sin(2*a)*cos(a)**2, True))
```

3.146 $\int \cos^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$\frac{\log(\sin(a+bx))}{2b}$$

```
[Out] 1/2*ln(sin(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4287, 3475\}$

$$\frac{\log(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/(2*b)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \cot(a+bx)\,dx\\ &= \frac{\log(\sin(a+bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.57

$$\frac{\log(\tan(a+bx))+\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/(2*b)
```

fricas [A] time = 0.52, size = 14, normalized size = 1.00

$$\frac{\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*log(1/2*sin(b*x + a))/b
```

giac [B] time = 0.22, size = 56, normalized size = 4.00

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.64, size = 13, normalized size = 0.93

$$\frac{\ln(\sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a),x)
```

```
[Out] 1/2*ln(sin(b*x+a))/b
```

maxima [B] time = 0.33, size = 82, normalized size = 5.86

$$\frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos(bx)^2 - 2\cos\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.16, size = 14, normalized size = 1.00

$$\frac{\ln\left(\sin(a+b\,x)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x),x)
```

```
[Out] log(sin(a + b*x)^2)/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

3.147 $\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\cot(a+bx)}{4b}$$

[Out] -1/4*cot(b*x+a)/b

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{4287, 3767, 8\}$

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]

[Out] -Cot[a + b*x]/(4*b)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^2(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int 1\,dx, x, \cot(a+bx)\right)}{4b}\\
&= -\frac{\cot(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]

[Out] -1/4*Cot[a + b*x]/b

fricas [**A**] time = 0.52, size = 19, normalized size = 1.46

$$-\frac{\cos\left(bx+a\right)}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] -1/4*cos(b*x + a)/(b*sin(b*x + a))
```

giac [**A**] time = 0.23, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="giac")
[Out] -1/4/(b*tan(b*x + a))
```

maple [**A**] time = 1.13, size = 12, normalized size = 0.92

$$-\frac{\cot\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x)
[Out] -1/4*cot(b*x+a)/b
```

maxima [**B**] time = 0.33, size = 53, normalized size = 4.08

$$-\frac{\sin\left(2\,bx+2\,a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] -1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*co
s(2*b*x + 2*a) + b)
```

mupad [**B**] time = 0.14, size = 11, normalized size = 0.85

$$-\frac{\cot\left(a+b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
[Out] -cot(a + b*x)/(4*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**2,x)
[Out] Timed out
```

3.148 $\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\tan(a+bx))}{8b}-\frac{\cot^2(a+bx)}{16b}$$

```
[Out] -1/16*cot(b*x+a)^2/b+1/8*ln(tan(b*x+a))/b
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2620, 14}

$$\frac{\log(\tan(a+bx))}{8b}-\frac{\cot^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(16*b) + Log[Tan[a + b*x]]/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\sec(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan(a+bx)\right)}{8b}\\
&= -\frac{\cot^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/16*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/b
```

fricas [B] time = 0.58, size = 65, normalized size = 2.17

$$-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\cos\left(bx+a\right)^{2}\right)-\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-1}{16\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.36, size = 119, normalized size = 3.97

$$-\frac{\frac{\left(\frac{4\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-1\right)\left(\cos(bx+a)+1\right)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-4\log\left(\frac{\left|-\cos(bx+a)+1\right|}{\left|\cos(bx+a)+1\right|}\right)+8\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/64*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) - 1)))/b
```

maple [A] time = 1.08, size = 27, normalized size = 0.90

$$-\frac{1}{16b\sin\left(bx+a\right)^{2}}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16/b/sin(b*x+a)^2+1/8*ln(tan(b*x+a))/b
```

maxima [B] time = 0.35, size = 656, normalized size = 21.87

$$\frac{4\,\cos\left(4\,bx+4\,a\right)\cos\left(2\,bx+2\,a\right)-8\,\cos\left(2\,bx+2\,a\right)^{2}+\left(2\left(2\,\cos\left(2\,bx+2\,a\right)-1\right)\cos\left(4\,bx+4\,a\right)-\cos\left(4\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*co
s(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2
*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2
*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b
*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
```

```
2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*
cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

mupad [B] time = 0.18, size = 36, normalized size = 1.20

$$-\frac{\frac{\ln(\cos(a+b\,x))}{8}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}+\frac{1}{16\sin(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
[Out] -(log(cos(a + b*x))/8 - log(sin(a + b*x)^2)/16 + 1/(16*sin(a + b*x)^2))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**3,x)
[Out] Timed out
```

3.149 $\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan(a+bx)}{16b}-\frac{\cot^3(a+bx)}{48b}-\frac{\cot(a+bx)}{8b}$$

```
[Out] -1/8*cot(b*x+a)/b-1/48*cot(b*x+a)^3/b+1/16*tan(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2620, 270}

$$\frac{\tan(a+bx)}{16b}-\frac{\cot^3(a+bx)}{48b}-\frac{\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -Cot[a + b*x]/(8*b) - Cot[a + b*x]^3/(48*b) + Tan[a + b*x]/(16*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^4(a+bx)\sec^2(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,\tan(a+bx)\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(a+bx)\right)}{16b}\\ &= -\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.14

$$\frac{\tan(a+bx)}{16b}-\frac{5\cot(a+bx)}{48b}-\frac{\cot(a+bx)\csc^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-5*Cot[a + b*x])/(48*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(48*b) + Tan[a + b
*x]/(16*b)
```

fricas [A] time = 0.46, size = 54, normalized size = 1.29

$$-\frac{8 \cos(bx+a)^4 - 12 \cos(bx+a)^2 + 3}{48 \left(b\cos(bx+a)^3 - b\cos(bx+a)\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

giac [A] time = 0.50, size = 35, normalized size = 0.83

$$-\frac{\frac{6 \tan(bx+a)^2+1}{\tan(bx+a)^3} - 3 \tan(bx+a)}{48 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/48*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

maple [A] time = 1.62, size = 51, normalized size = 1.21

$$\frac{-\frac{1}{3\sin(bx+a)^3\cos(bx+a)} + \frac{4}{3\sin(bx+a)\cos(bx+a)} - \frac{8\cot(bx+a)}{3}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x)
```

```
[Out] 1/16/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+
a))
```

maxima [B] time = 0.34, size = 308, normalized size = 7.33

$$\frac{(2 \cos(2}{3\left(b\cos(8bx+8a)^2 + 4b\cos(6bx+6a)^2 + 4b\cos(2bx+2a)^2 + b\sin(8bx+8a)^2 + 4b\sin(6bx+6a)^2 - 8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

mupad [B] time = 0.18, size = 37, normalized size = 0.88

$$\frac{\tan(a+b\,x)}{16\,b}-\frac{\frac{\tan(a+b\,x)^2}{8}+\frac{1}{48}}{b\tan(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
[Out] tan(a + b*x)/(16*b) - (tan(a + b*x)^2/8 + 1/48)/(b*tan(a + b*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**4,x)
[Out] Timed out
```

3.150 $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}-\frac{3\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

```
[Out] -3/64*cot(b*x+a)^2/b-1/128*cot(b*x+a)^4/b+3/32*ln(tan(b*x+a))/b+1/64*tan(b*
x+a)^2/b
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4287, 2620, 266, 43}

$$\frac{\tan^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}-\frac{3\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(64*b) - Cot[a + b*x]^4/(128*b) + (3*Log[Tan[a + b*x]])
/(32*b) + Tan[a + b*x]^2/(64*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{3\cot^2(a+bx)}{64b} - \frac{\cot^4(a+bx)}{128b} + \frac{3\log(\tan(a+bx))}{32b} + \frac{\tan^2(a+bx)}{64b}$$

Mathematica [A] time = 0.35, size = 54, normalized size = 0.90

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -1/128*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[S
in[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

fricas [B] time = 0.47, size = 138, normalized size = 2.30

$$\frac{6\cos(bx+a)^4 - 9\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 6\,(}{128\left(b\cos(bx+a)^6 - 2b\cos(bx+a)^4 + b\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
 a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

giac [B] time = 0.74, size = 232, normalized size = 3.87

$$\frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2} + 96\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/2048*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(co
s(b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 111*(cos(b*
x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a)
+ 1)^3 + 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 1)/((cos(b*x + a) -
 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 + 96*
```

```
log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 192*log(abs(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 1.26, size = 69, normalized size = 1.15

$$-\frac{1}{128b\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{128b\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{64b\sin(bx+a)^2}+\frac{3\ln(\tan(bx+a))}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)^2+3/128/b/sin(b*x+a)^2/cos(b*x+a)^2-3/64/b
/sin(b*x+a)^2+3/32*ln(tan(b*x+a))/b
```

maxima [B] time = 0.44, size = 3188, normalized size = 53.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
```

```
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(10*b*x + 10*a) + b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) - b)*cos(12*b*x + 12*a) + 4*(b*cos(8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 2*(4
*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*
b*x + 8*a) - 8*(b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) + 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a)
 - 2*(2*b*sin(10*b*x + 10*a) + b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) +
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 4*(b*sin(8*
b*x + 8*a) - 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 2*(4*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 8*(b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.14, size = 82, normalized size = 1.37

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b}+\frac{\frac{3\cos(a+b\,x)^4}{64}-\frac{9\cos(a+b\,x)^2}{128}+\frac{1}{64}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a +
 b*x)^4)/64 - (9*cos(a + b*x)^2)/128 + 1/64)/(b*(cos(a + b*x)^2 - 2*cos(a +
 b*x)^4 + cos(a + b*x)^6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.151 $\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^{13}(a+bx)}{13b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^9(a+bx)}{9b}$$

```
[Out] -32/9*cos(b*x+a)^9/b+64/11*cos(b*x+a)^11/b-32/13*cos(b*x+a)^13/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2565, 270}

$$-\frac{32\cos^{13}(a+bx)}{13b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^9)/(9*b) + (64*Cos[a + b*x]^11)/(11*b) - (32*Cos[a + b*x]
^13)/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^8(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{32\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{32\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.39, size = 37, normalized size = 0.80

$$\frac{4\cos^9(a+bx)(540\cos(2(a+bx))-99\cos(4(a+bx))-505)}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-505 + 540*Cos[2*(a + b*x)] - 99*Cos[4*(a + b*x)]))/(128
7*b)
```

fricas [A] time = 0.45, size = 36, normalized size = 0.78

$$-\frac{32\left(99\,\cos\left(bx+a\right)^{13}-234\,\cos\left(bx+a\right)^{11}+143\,\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

giac [B] time = 0.60, size = 96, normalized size = 2.09

$$-\frac{\cos\left(13\,bx+13\,a\right)}{1664\,b}-\frac{3\,\cos\left(11\,bx+11\,a\right)}{1408\,b}+\frac{\cos\left(9\,bx+9\,a\right)}{576\,b}+\frac{\cos\left(7\,bx+7\,a\right)}{64\,b}+\frac{\cos\left(5\,bx+5\,a\right)}{128\,b}-\frac{25\,\cos\left(3\,bx+3\,a\right.}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/1664*cos(13*b*x + 13*a)/b - 3/1408*cos(11*b*x + 11*a)/b + 1/576*cos(9*b*
x + 9*a)/b + 1/64*cos(7*b*x + 7*a)/b + 1/128*cos(5*b*x + 5*a)/b - 25/384*co
s(3*b*x + 3*a)/b - 5/32*cos(b*x + a)/b
```

maple [B] time = 0.40, size = 97, normalized size = 2.11

$$-\frac{5\cos\left(bx+a\right)}{32b}-\frac{25\cos\left(3bx+3a\right)}{384b}+\frac{\cos\left(5bx+5a\right)}{128b}+\frac{\cos\left(7bx+7a\right)}{64b}+\frac{\cos\left(9bx+9a\right)}{576b}-\frac{3\cos\left(11bx+11a\right)}{1408b}-\frac{\cos\left(1\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(b*x+a)/b-25/384*cos(3*b*x+3*a)/b+1/128*cos(5*b*x+5*a)/b+1/64*cos(
7*b*x+7*a)/b+1/576*cos(9*b*x+9*a)/b-3/1408*cos(11*b*x+11*a)/b-1/1664*cos(13
*b*x+13*a)/b
```

maxima [A] time = 0.34, size = 80, normalized size = 1.74

$$-\frac{99\,\cos\left(13\,bx+13\,a\right)+351\,\cos\left(11\,bx+11\,a\right)-286\,\cos\left(9\,bx+9\,a\right)-2574\,\cos\left(7\,bx+7\,a\right)-1287\,\cos\left(5\,bx+\right.}{164736\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/164736*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3
*a) + 25740*cos(b*x + a))/b
```

mupad [B] time = 0.07, size = 36, normalized size = 0.78

$$-\frac{32\,\left(99\cos\left(a+b\,x\right)^{13}-234\cos\left(a+b\,x\right)^{11}+143\cos\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -(32*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(1287
*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.152 $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{16\sin^{11}(a+bx)}{11b}+\frac{16\sin^9(a+bx)}{3b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

```
[Out] 16/5*sin(b*x+a)^5/b-48/7*sin(b*x+a)^7/b+16/3*sin(b*x+a)^9/b-16/11*sin(b*x+a
)^11/b
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2564, 270}

$$-\frac{16\sin^{11}(a+bx)}{11b}+\frac{16\sin^9(a+bx)}{3b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (48*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(3*b) - (16*Sin[a + b*x]^11)/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^7(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{16\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{16\sin^5(a+bx)}{5b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{3b}-\frac{16\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.21, size = 47, normalized size = 0.77

$$\frac{\sin^5(a+bx)(3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx))+3042)}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(2310*b)
```

fricas [A] time = 0.48, size = 63, normalized size = 1.03

$$\frac{16\left(105 \cos\left(bx+a\right)^{10} - 140 \cos\left(bx+a\right)^{8} + 5 \cos\left(bx+a\right)^{6} + 6 \cos\left(bx+a\right)^{4} + 8 \cos\left(bx+a\right)^{2} + 16\right) \sin\left(bx\right.}{1155\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*co
s(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

giac [A] time = 0.37, size = 82, normalized size = 1.34

$$\frac{\sin\left(11\, bx + 11\, a\right)}{704\, b} + \frac{\sin\left(9\, bx + 9\, a\right)}{192\, b} - \frac{\sin\left(7\, bx + 7\, a\right)}{448\, b} - \frac{11 \sin\left(5\, bx + 5\, a\right)}{320\, b} - \frac{\sin\left(3\, bx + 3\, a\right)}{32\, b} + \frac{7 \sin\left(bx + a\right)}{32\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/704*sin(11*b*x + 11*a)/b + 1/192*sin(9*b*x + 9*a)/b - 1/448*sin(7*b*x + 7
*a)/b - 11/320*sin(5*b*x + 5*a)/b - 1/32*sin(3*b*x + 3*a)/b + 7/32*sin(b*x
+ a)/b
```

maple [A] time = 1.25, size = 83, normalized size = 1.36

$$\frac{7 \sin\left(bx + a\right)}{32b} - \frac{\sin\left(3bx + 3a\right)}{32b} - \frac{11 \sin\left(5bx + 5a\right)}{320b} - \frac{\sin\left(7bx + 7a\right)}{448b} + \frac{\sin\left(9bx + 9a\right)}{192b} + \frac{\sin\left(11bx + 11a\right)}{704b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] 7/32*sin(b*x+a)/b-1/32*sin(3*b*x+3*a)/b-11/320/b*sin(5*b*x+5*a)-1/448/b*sin
(7*b*x+7*a)+1/192/b*sin(9*b*x+9*a)+1/704/b*sin(11*b*x+11*a)
```

maxima [A] time = 0.34, size = 69, normalized size = 1.13

$$\frac{105 \sin\left(11\, bx + 11\, a\right) + 385 \sin\left(9\, bx + 9\, a\right) - 165 \sin\left(7\, bx + 7\, a\right) - 2541 \sin\left(5\, bx + 5\, a\right) - 2310 \sin\left(3\, bx + 3\right.}{73920\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*
a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

mupad [B] time = 0.06, size = 45, normalized size = 0.74

$$\frac{-\frac{16 \sin(a+b\,x)^{11}}{11} + \frac{16 \sin(a+b\,x)^{9}}{3} - \frac{48 \sin(a+b\,x)^{7}}{7} + \frac{16 \sin(a+b\,x)^{5}}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
```

```
[Out] ((16*sin(a + b*x)^5)/5 - (48*sin(a + b*x)^7)/7 + (16*sin(a + b*x)^9)/3 - (1
6*sin(a + b*x)^11)/11)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.153 $\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\cos^9(a+bx)}{9b}-\frac{8\cos^7(a+bx)}{7b}$$

```
[Out] -8/7*cos(b*x+a)^7/b+8/9*cos(b*x+a)^9/b
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 2565, 14}

$$\frac{8\cos^9(a+bx)}{9b}-\frac{8\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^7)/(7*b) + (8*Cos[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^6(a+bx)\sin^3(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{8\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{8\cos^7(a+bx)}{7b}+\frac{8\cos^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 27, normalized size = 0.87

$$\frac{4\cos^7(a+bx)(7\cos(2(a+bx))-11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

fricas [A] time = 0.42, size = 26, normalized size = 0.84

$$\frac{8\left(7\cos\left(bx+a\right)^{9}-9\cos\left(bx+a\right)^{7}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

giac [A] time = 0.37, size = 54, normalized size = 1.74

$$\frac{\cos\left(9\,bx+9\,a\right)}{288\,b}+\frac{3\cos\left(7\,bx+7\,a\right)}{224\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{12\,b}-\frac{3\cos\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/288*cos(9*b*x + 9*a)/b + 3/224*cos(7*b*x + 7*a)/b - 1/12*cos(3*b*x + 3*a)
/b - 3/16*cos(b*x + a)/b
```

maple [A] time = 0.45, size = 55, normalized size = 1.77

$$-\frac{3\cos\left(bx+a\right)}{16b}-\frac{\cos\left(3bx+3a\right)}{12b}+\frac{3\cos\left(7bx+7a\right)}{224b}+\frac{\cos\left(9bx+9a\right)}{288b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+3/224*cos(7*b*x+7*a)/b+1/288*cos(9
*b*x+9*a)/b
```

maxima [A] time = 0.33, size = 47, normalized size = 1.52

$$\frac{7\cos\left(9\,bx+9\,a\right)+27\cos\left(7\,bx+7\,a\right)-168\cos\left(3\,bx+3\,a\right)-378\cos\left(bx+a\right)}{2016\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/2016*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 3
78*cos(b*x + a))/b
```

mupad [B] time = 0.15, size = 26, normalized size = 0.84

$$-\frac{8\left(9\cos\left(a+b\,x\right)^{7}-7\cos\left(a+b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
```

```
[Out] -(8*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

sympy [A] time = 113.55, size = 284, normalized size = 9.16

$$\begin{cases}-\frac{94\sin^{3}\left(a+bx\right)\sin^{3}\left(2a+2bx\right)}{315b}-\frac{32\sin^{3}\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{105b}-\frac{4\sin^{2}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{7b}-\frac{64\sin^{2}\left(a+b\right.}{} \\ x\sin^{3}\left(2a\right)\cos^{3}\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-94*sin(a + b*x)**3*sin(2*a + 2*b*x)**3/(315*b) - 32*sin(a + b*x
)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(105*b) - 4*sin(a + b*x)**2*sin(2
*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)**2*cos
(a + b*x)*cos(2*a + 2*b*x)**3/(105*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x)**3
*cos(a + b*x)**2/(105*b) + 8*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*
cos(2*a + 2*b*x)**2/(35*b) - 46*sin(2*a + 2*b*x)**2*cos(a + b*x)**3*cos(2*a
 + 2*b*x)/(105*b) - 16*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b), Ne(b, 0)
), (x*sin(2*a)**3*cos(a)**3, True))
```

3.154 $\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{4\sin^7(a+bx)}{7b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^3(a+bx)}{3b}$$

```
[Out] 4/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b+4/7*sin(b*x+a)^7/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{4287, 2564, 270\}$

$$\frac{4\sin^7(a+bx)}{7b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b) + (4*Sin[a + b*x]^7)/(7
*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^5(a+bx)\sin^2(a+bx)\,dx\\
&= \frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{4\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{4\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 37, normalized size = 0.80

$$\frac{\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(210*b)
```

fricas [A] time = 0.41, size = 43, normalized size = 0.93

$$-\frac{4\left(15\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*
x + a)/b
```

giac [A] time = 0.23, size = 54, normalized size = 1.17

$$-\frac{\sin\left(7\,bx+7\,a\right)}{112\,b}-\frac{3\,\sin\left(5\,bx+5\,a\right)}{80\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{48\,b}+\frac{5\,\sin\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/112*sin(7*b*x + 7*a)/b - 3/80*sin(5*b*x + 5*a)/b - 1/48*sin(3*b*x + 3*a)
/b + 5/16*sin(b*x + a)/b
```

maple [A] time = 0.99, size = 55, normalized size = 1.20

$$\frac{5\sin\left(bx+a\right)}{16b}-\frac{\sin\left(3bx+3a\right)}{48b}-\frac{3\sin\left(5bx+5a\right)}{80b}-\frac{\sin\left(7bx+7a\right)}{112b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] 5/16*sin(b*x+a)/b-1/48*sin(3*b*x+3*a)/b-3/80/b*sin(5*b*x+5*a)-1/112/b*sin(7
*b*x+7*a)
```

maxima [A] time = 0.33, size = 47, normalized size = 1.02

$$-\frac{15\sin\left(7\,bx+7\,a\right)+63\sin\left(5\,bx+5\,a\right)+35\sin\left(3\,bx+3\,a\right)-525\sin\left(bx+a\right)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

mupad [B] time = 0.16, size = 36, normalized size = 0.78

$$\frac{4\,\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

```
[Out] (4*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

sympy [A] time = 38.32, size = 202, normalized size = 4.39

$$\begin{cases} \frac{38\sin^3(a+bx)\sin^2(2a+2bx)}{105b} + \frac{32\sin^3(a+bx)\cos^2(2a+2bx)}{105b} + \frac{8\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{35b} + \frac{11\sin(a+bx)\sin^2(2a+2bx)}{35b} \\ x\sin^2(2a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((38*sin(a + b*x)**3*sin(2*a + 2*b*x)**2/(105*b) + 32*sin(a + b*x)
**3*cos(2*a + 2*b*x)**2/(105*b) + 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(a
+ b*x)*cos(2*a + 2*b*x)/(35*b) + 11*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a
+ b*x)**2/(35*b) + 24*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(35*
b) - 12*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)/(35*b), Ne(b, 0))
, (x*sin(2*a)**2*cos(a)**3, True))
```

3.155 $\int \cos^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{2\cos^5(a+bx)}{5b}$$

```
[Out] -2/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4287, 2565, 30\}$

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos^4(a+bx)\sin(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int x^4\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{2\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

fricas [A] time = 0.45, size = 13, normalized size = 0.87

$$-\frac{2\cos\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] -2/5*cos(b*x + a)^5/b
```

giac [B] time = 0.18, size = 40, normalized size = 2.67

$$-\frac{\cos\left(5\,bx+5\,a\right)}{40\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{8\,b}-\frac{\cos\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
[Out] -1/40*cos(5*b*x + 5*a)/b - 1/8*cos(3*b*x + 3*a)/b - 1/4*cos(b*x + a)/b
```

maple [B] time = 0.17, size = 41, normalized size = 2.73

$$-\frac{\cos\left(bx+a\right)}{4b}-\frac{\cos\left(3bx+3a\right)}{8b}-\frac{\cos\left(5bx+5a\right)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a),x)
[Out] -1/4*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b-1/40*cos(5*b*x+5*a)/b
```

maxima [B] time = 0.33, size = 34, normalized size = 2.27

$$-\frac{\cos\left(5\,bx+5\,a\right)+5\,\cos\left(3\,bx+3\,a\right)+10\,\cos\left(bx+a\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
[Out] -1/40*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

mupad [B] time = 0.14, size = 13, normalized size = 0.87

$$-\frac{2\cos\left(a+b\,x\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x),x)
[Out] -(2*cos(a + b*x)^5)/(5*b)
```

sympy [A] time = 11.23, size = 117, normalized size = 7.80

$$\begin{cases} -\frac{2\sin^{3}\left(a+bx\right)\sin\left(2a+2bx\right)}{5b}-\frac{4\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{5b}+\frac{\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{5b}-\frac{2\cos^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{5b} & \text{for} \\ x\sin\left(2a\right)\cos^{3}\left(a\right) & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*sin(2*a + 2*b*x)/(5*b) - 4*sin(a + b*x)**2*co
s(a + b*x)*cos(2*a + 2*b*x)/(5*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b
*x)**2/(5*b) - 2*cos(a + b*x)**3*cos(2*a + 2*b*x)/(5*b), Ne(b, 0)), (x*sin(
2*a)*cos(a)**3, True))
```

3.156 $\int \cos^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos(a+bx)}{2b}-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] -1/2*arctanh(cos(b*x+a))/b+1/2*cos(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4287, 2592, 321, 206}

$$\frac{\cos(a+bx)}{2b}-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b) + Cos[a + b*x]/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \cos(a+bx)\cot(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= \frac{\cos(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{2b} + \frac{\cos(a+bx)}{2b}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.64

$$\frac{1}{2}\left(\frac{\cos(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
[Out] (Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)/2
```

fricas [A] time = 0.49, size = 38, normalized size = 1.36

$$\frac{2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 1/4*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

giac [B] time = 0.69, size = 57, normalized size = 2.04

$$-\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="giac")
[Out] -1/4*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a)
 + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.56, size = 34, normalized size = 1.21

$$\frac{\cos(bx+a)}{2b} + \frac{\ln(\csc(bx+a) - \cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a),x)
[Out] 1/2*cos(b*x+a)/b+1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.34, size = 92, normalized size = 3.29

$$\frac{2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.05, size = 22, normalized size = 0.79

$$\frac{\frac{\cos(a+b\,x)}{2} - \frac{\mathrm{atanh}(\cos(a+b\,x))}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x),x)
```

```
[Out] (cos(a + b*x)/2 - atanh(cos(a + b*x))/2)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

3.157 $\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\csc(a+bx)}{4b}$$

```
[Out] -1/4*csc(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{4287, 2606, 8\}$

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -Csc[a + b*x]/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \cot(a+bx)\csc(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{4b}\\
&= -\frac{\csc(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*Csc[a + b*x]/b
```

fricas [**A**] time = 0.46, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] -1/4/(b*sin(b*x + a))
```

giac [**A**] time = 0.25, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="giac")
[Out] -1/4/(b*sin(b*x + a))
```

maple [**A**] time = 0.47, size = 14, normalized size = 1.08

$$-\frac{1}{4b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x)
[Out] -1/4/b/sin(b*x+a)
```

maxima [**B**] time = 0.34, size = 84, normalized size = 6.46

$$-\frac{\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\sin\left(bx+a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] -1/2*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b
*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a
) + b)
```

mupad [**B**] time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
[Out] -1/(4*b*sin(a + b*x))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**2,x)
[Out] Timed out
```

3.158 $\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

```
[Out] -1/16*arctanh(cos(b*x+a))/b-1/16*cot(b*x+a)*csc(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4287, 3768, 3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(16*b) - (Cot[a + b*x]*Csc[a + b*x])/(16*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\,dx\\
&= -\frac{\cot(a+bx)\csc(a+bx)}{16b}+\frac{1}{16}\int \csc(a+bx)\,dx\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 79, normalized size = 2.32

$$\frac{1}{8}\left(-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-1/8*Csc[(a + b*x)/2]^2/b - Log[Cos[(a + b*x)/2]]/(2*b) + Log[Sin[(a + b*x
)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b))/8
```

fricas [B] time = 0.48, size = 72, normalized size = 2.12

$$-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-2\cos\left(bx+a\right)}{32\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 -
 1)*log(-1/2*cos(b*x + a) + 1/2) - 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.66, size = 92, normalized size = 2.71

$$-\frac{\frac{\left(\frac{2\left(\cos\left(bx+a\right)-1\right)}{\cos\left(bx+a\right)+1}-1\right)\left(\cos\left(bx+a\right)+1\right)}{\cos\left(bx+a\right)-1}+\frac{\cos\left(bx+a\right)-1}{\cos\left(bx+a\right)+1}-2\log\left(\frac{\left|-\cos\left(bx+a\right)+1\right|}{\left|\cos\left(bx+a\right)+1\right|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/64*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.86, size = 40, normalized size = 1.18

$$-\frac{\cot\left(bx+a\right)\csc\left(bx+a\right)}{16b}+\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16*cot(b*x+a)*csc(b*x+a)/b+1/16/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.35, size = 558, normalized size = 16.41

$$\frac{4\left(\cos\left(3\,bx+3\,a\right)+\cos\left(bx+a\right)\right)\cos\left(4\,bx+4\,a\right)-4\left(2\cos\left(2\,bx+2\,a\right)-1\right)\cos\left(3\,bx+3\,a\right)-8\cos\left(2\,bx+2\,a\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) - 4*(2*cos(2*b*x
 + 2*a) - 1)*cos(3*b*x + 3*a) - 8*cos(2*b*x + 2*a)*cos(b*x + a) + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b*x + 2
*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin
(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^
2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(sin(3*b*x + 3*a) + sin(b*x
 + a))*sin(4*b*x + 4*a) - 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x
```

```
 + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*
x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4
*b*cos(2*b*x + 2*a) + b)
```

mupad [**B**] time = 0.06, size = 36, normalized size = 1.06

$$\frac{\cos\left(a+b\,x\right)}{16\,b\,\left({\cos\left(a+b\,x\right)}^2-1\right)}-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

```
[Out] cos(a + b*x)/(16*b*(cos(a + b*x)^2 - 1)) - atanh(cos(a + b*x))/(16*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

3.159 $\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\csc^3(a+bx)}{48b}-\frac{\csc(a+bx)}{16b}+\frac{\tanh^{-1}(\sin(a+bx))}{16b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b-1/16*csc(b*x+a)/b-1/48*csc(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4287, 2621, 302, 207}

$$-\frac{\csc^3(a+bx)}{48b}-\frac{\csc(a+bx)}{16b}+\frac{\tanh^{-1}(\sin(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) - Csc[a + b*x]/(16*b) - Csc[a + b*x]^3/(48*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{\csc(a+bx)}{16b} - \frac{\csc^3(a+bx)}{48b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{16b} - \frac{\csc(a+bx)}{16b} - \frac{\csc^3(a+bx)}{48b}$$

Mathematica [C] time = 0.02, size = 31, normalized size = 0.72

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.45, size = 94, normalized size = 2.19

$$\frac{3\left(\cos(bx+a)^2-1\right)\log(\sin(bx+a)+1)\sin(bx+a)-3\left(\cos(bx+a)^2-1\right)\log(-\sin(bx+a)+1)\sin(bx+}{96\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.93, size = 52, normalized size = 1.21

$$-\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log(|\sin(bx+a)+1|) + 3\log(|\sin(bx+a)-1|)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1
)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 1.21, size = 47, normalized size = 1.09

$$-\frac{1}{48b\sin(bx+a)^3} - \frac{1}{16b\sin(bx+a)} + \frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x)
```

```
[Out] -1/48/b/sin(b*x+a)^3-1/16/b/sin(b*x+a)+1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 0.66, size = 834, normalized size = 19.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

mupad [B] time = 0.06, size = 38, normalized size = 0.88

$$\frac{\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b}-\frac{\frac{{\sin\left(a+b\,x\right)}^2}{16}+\frac{1}{48}}{b\,{\sin\left(a+b\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

```
[Out] atanh(sin(a + b*x))/(16*b) - (sin(a + b*x)^2/16 + 1/48)/(b*sin(a + b*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

3.160 $\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{256b}-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}$$

```
[Out] -15/256*arctanh(cos(b*x+a))/b+15/256*sec(b*x+a)/b-5/256*csc(b*x+a)^2*sec(b*
x+a)/b-1/128*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4287, 2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{256b}-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(256*b) + (15*Sec[a + b*x])/(256*b) - (5*Csc[a
+ b*x]^2*Sec[a + b*x])/(256*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(128*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{128b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\right.}{25}$$
$$= \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{15}{}$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{256b} + \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\text{cs}}{}$$

Mathematica [B] time = 0.33, size = 195, normalized size = 2.79

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{2048b} - \frac{7\csc^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{2048b} + \frac{7\sec^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{256b} - \frac{15\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-7*Csc[(a + b*x)/2]^2)/(1024*b) - Csc[(a + b*x)/2]^4/(2048*b) - (15*Log[Co
s[(a + b*x)/2]])/(256*b) + (15*Log[Sin[(a + b*x)/2]])/(256*b) + (7*Sec[(a +
 b*x)/2]^2)/(1024*b) + Sec[(a + b*x)/2]^4/(2048*b) + Sin[(a + b*x)/2]/(32*b
*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - Sin[(a + b*x)/2]/(32*b*(Cos[(a +
b*x)/2] + Sin[(a + b*x)/2]))
```

fricas [B] time = 0.44, size = 132, normalized size = 1.89

$$\frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)}{512\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/512*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
- 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*co
s(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

giac [B] time = 0.82, size = 163, normalized size = 2.33

$$\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + 60\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/2048*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
 1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(abs(-cos(b
*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 1.12, size = 78, normalized size = 1.11

$$-\frac{1}{128b\sin\left(bx+a\right)^{4}\cos\left(bx+a\right)}-\frac{5}{256b\sin\left(bx+a\right)^{2}\cos\left(bx+a\right)}+\frac{15}{256b\cos\left(bx+a\right)}+\frac{15\ln\left(\csc\left(bx+a\right)-\cot\right.}{256b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)-5/256/b/sin(b*x+a)^2/cos(b*x+a)+15/256/b/c
os(b*x+a)+15/256/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [B] time = 0.41, size = 2237, normalized size = 31.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/512*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*
x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1
)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos
(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
 + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*co
s(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) +
72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*c
os(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a
) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(
8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
 - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2
*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x
+ 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
 - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*c
os(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*
b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) -
sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a
) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b
*x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
 6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*co
s(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x +
10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos
(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*
a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(
4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^
2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*
b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2
*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
```

```
 - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*s
in(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
 - 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin
(2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x +
5*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin
(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
 8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x
+ 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40
*(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3
*a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))
/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2
+ 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2
- 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*
cos(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*
b*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*
x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x +
4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
 - b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2
*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b
*x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
 + 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + b)
```

mupad [B] time = 0.19, size = 66, normalized size = 0.94

$$\frac{\frac{15\cos(a+b\,x)^4}{256}-\frac{25\cos(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

```
[Out] ((15*cos(a + b*x)^4)/256 - (25*cos(a + b*x)^2)/256 + 1/32)/(b*(cos(a + b*x)
 - 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(256*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

3.161 $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sin^{\frac{3}{2}}(2a+2\ldots}{2\ldots}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-5/24*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/6*sin(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b+5/16*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4301, 4302, 4306}

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{24b}-\frac{5\sin^{-1}(\cos(a+bx\ldots}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) + (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{8}\int \cos(a+$$

$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+bx)\,\mathrm{s}}{}$$

$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\mathrm{s}}\right.}{32b}$$

Mathematica [A] time = 0.34, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\mathrm{lo}\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(32*b)
```

fricas [B] time = 0.50, size = 290, normalized size = 2.13

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^4-12\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)-30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 91.92, size = 199880292, normalized size = 1469708.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(2\,a+2\,b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
[Out] Timed out
```

3.162 $\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{8b}+\frac{3\log\left(\sin(a+bx\right.}{}$$

```
[Out] -3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+1/4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/8*cos(b*x+a)*sin(2
*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4301, 4302, 4305}

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\sqrt{\sin(2a+2bx)}\,\cos(a+bx)}{8b}+\frac{3\log\left(\sin(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) + (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{8b}+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{8}\int \frac{\cos(a}{\sqrt{\sin(2}}\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{s}\right.}{16b}\end{aligned}$$

Mathematica [A] time = 0.19, size = 86, normalized size = 0.78

$$\frac{3\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(16*b)
```

fricas [B] time = 0.50, size = 281, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-s}{\cos(bx+a)^2+2\cos(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*
x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a)
- sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x +
a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)
) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32
*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(
b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

maple [B] time = 24.52, size = 74316382, normalized size = 675603.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\sin(2a+2bx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.163 $\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+\right.}{4b}$$

```
[Out] -1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4301, 4306}

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{s\right.}{4b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 0.83

$$-\frac{-2\sin(a+bx)\sqrt{\sin(2(a+bx))}+\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+c\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [B] time = 0.53, size = 266, normalized size = 3.17

$$8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)+2\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt
(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a
))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*si
n(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 2.75, size = 5342284, normalized size = 63598.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sqrt{\sin(2\,a+2\,b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4305}

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2b}\right.}{2b}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.90

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]])/(2*b)
```

fricas [B] time = 0.44, size = 242, normalized size = 4.17

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx}{\cos(bx+a)-\sin(bx+a)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx + a)}{\sqrt{\sin(2\,bx + 2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 3.18, size = 16154758, normalized size = 278530.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx + a)}{\sqrt{\sin(2\,bx + 2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a + b\,x)}{\sqrt{\sin(2\,a + 2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=24

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4291}

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.96

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]]))
```

fricas [A] time = 0.50, size = 39, normalized size = 1.62

$$-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)} + \sin(bx+a)}{2\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x +
a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

maple [B] time = 10.74, size = 55902198, normalized size = 2329258.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [B] time = 0.19, size = 24, normalized size = 1.00

$$-\frac{\sqrt{\sin(2a+2bx)}}{2b\,\sin(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
[Out] -sin(2*a + 2*b*x)^(1/2)/(2*b*sin(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.166 $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+2/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4303, 4292}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -Cos[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{4}\sec(a+bx)-\frac{1}{12}\cot(a+bx)\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] ((-1/12*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/4)*Sqrt[Sin[2*(a + b*x)]
])/b
```

fricas [A] time = 0.59, size = 74, normalized size = 1.40

$$\frac{4\,\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\,\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4\,\cos\left(bx+a\right)}{12\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*s
in(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

maple [C] time = 33.70, size = 194, normalized size = 3.66

$$-\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\cdots\right.\right.}{24b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sqrt{\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/24/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b
*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)-tan(1/2*b*x+1/2*a)^4+1)/(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b
*x+1/2*a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 3.17, size = 104, normalized size = 1.96

$$-\frac{2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(3\,\cos\left(a+b\,x\right)-6\,\cos\left(3\,a+3\,b\,x\right)+4\,\cos\left(5\,a+5\,b\,x\right)-\cos\left(7\,a+7\,b\,x\right)\right)}{3\,b\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+4\,\cos\left(4\,a+4\,b\,x\right)-4\,\cos\left(6\,a+6\,b\,x\right)+\cos\left(8\,a+8\,b\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(2*sin(2*a + 2*b*x)^(1/2)*(3*cos(a + b*x) - 6*cos(3*a + 3*b*x) + 4*cos(5*a
 + 5*b*x) - cos(7*a + 7*b*x)))/(3*b*(4*cos(2*a + 2*b*x) + 4*cos(4*a + 4*b*x
) - 4*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 5))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.167 $\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+4/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4303, 4304, 4291}

$$\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(15*b*Sin[2*a
 + 2*b*x]^(3/2)) - (8*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\left(3\csc^3(a+bx)+27\csc(a+bx)-5\tan(a+bx)\sec(a+bx)\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/120*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[
a + b*x]*Tan[a + b*x]))/b
```

fricas [A] time = 0.46, size = 103, normalized size = 1.30

$$-\frac{\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-40\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)}{120\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/120*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*c
os(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2bx+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [B] time = 3.23, size = 136, normalized size = 1.72

$$\frac{4\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,2{}\mathrm{i}+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,3{}\mathrm{i}+{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}\,2{}\mathrm{i}-{\mathrm{e}}^{a\,8{}\mathrm{i}+b\,x\,8{}\mathrm{i}}\,2{}\mathrm{i}-2{}\mathrm{i}\right)}{15\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}^3\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*
6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp
(a*2i + b*x*2i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.168 $\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] -1/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+6/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/
2)-8/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+16/35*sin(b*x+a)/b/sin(2*b*x+2*a)
^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4303, 4304, 4292}

$$\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (6*Sin[a + b*x])/(35*b*Sin[2*a
 + 2*b*x]^(5/2)) - (8*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (16*Sin
[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{24}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}\,(-10\cos(2(a+bx)) - 4\cos(4(a+bx)) + 4\cos(6(a+bx)) + 5)\csc^4(a+bx)\sec^3(a+bx)}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
+ b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

fricas [A] time = 0.43, size = 118, normalized size = 1.12

$$\frac{128\,\cos(bx+a)^7 - 256\,\cos(bx+a)^5 + 128\,\cos(bx+a)^3 + \sqrt{2}\left(128\,\cos(bx+a)^6 - 224\,\cos(bx+a)^4 + 84\,\cos\right.}{560\left(b\cos(bx+a)^7 - 2\,b\cos(bx+a)^5 + b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/560*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(
2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(c
os(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

mupad [B] time = 3.87, size = 350, normalized size = 3.33

$$-\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}+\frac{e^{a\,3i+b\,x\,3i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,16i}{35\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}-\frac{e^{a\,1i+b\,x\,1i}\,\left(\frac{1}{7\,b}-\frac{8\,e^{a\,2i+b\,x\,2i}}{35\,b}\right)\,\sqrt{e^{-a}}}{\left(e^{a\,2i+b\,x\,2i}+1\right)^2\,\left(e^{a\,2i+b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*16i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
- (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*(1/(
7*b) - (8*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a
*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)
*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(16i/(35*b) + (exp(a*2i + b*x*2i)*44i)/(
35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((e
xp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.169 $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

```
[Out] -5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/18*sin(2*b*x+2
*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = $\{4297, 2635, 2641\}$

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
 + 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) + Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$

$$= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2bx$$

$$= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \text{sin}$$

$$= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\,\text{si}}{14b}$$

Mathematica [A] time = 0.39, size = 96, normalized size = 0.98

$$\frac{70\sin(2(a+bx)) - 156\sin(4(a+bx)) - 35\sin(6(a+bx)) + 18\sin(8(a+bx)) + 7\sin(10(a+bx)) + 240\sqrt{\sin(}}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] - 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] + 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(2\,bx+2\,a\right)^2\cos\left(bx+a\right)^2-\cos\left(bx+a\right)^2\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sin(2*b*x +
2*a)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(bx+a\right)\right)\left(\sin^{\frac{7}{2}}\left(2bx+2a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^2\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (a+b\,x)^2\,\sin (2\,a+2\,b\,x)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.170 $\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

```
[Out] -3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/14*sin(2*b*x+2
*a)^(7/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 2635, 2639}

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) + Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2b\\ &= \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 66, normalized size = 0.96

$$\frac{\sqrt{\sin(2(a+bx))}\,(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))+84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(280*b)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(2\,bx+2\,a\right)^{2}\cos\left(bx+a\right)^{2}-\cos\left(bx+a\right)^{2}\right)\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sqrt(sin(2*b
*x + 2*a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 196.39, size = 336654856, normalized size = 4879055.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,x\right)}^2\,{\sin\left(2\,a+2\,b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.171 $\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2*a
)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 2635, 2641}

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,d\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}\end{aligned}$$

Mathematica [A] time = 0.35, size = 76, normalized size = 1.10

$$\frac{9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))+20\sqrt{\sin(2(a+bx))}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 61.35, size = 174364216, normalized size = 2527017.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.172 $\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/6*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4297, 2639}

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.85

$$\frac{\sin^{\frac{3}{2}}(2(a+bx))+3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(bx+a\right)^2\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)^2 \sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 7.57, size = 24858211, normalized size = 621455.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)^2 \sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(a+b\,x)^2\,\sqrt{\sin(2\,a+2\,b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4297, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= \frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A] time = 0.89, size = 76, normalized size = 1.90

$$\frac{2\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/(4
*b)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)^2}{\sqrt{\sin(2bx+2a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 15.39, size = 59624230, normalized size = 1490605.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a+b\,x)^2}{\sqrt{\sin(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.174 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=46

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b-cos(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 2639}

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 39, normalized size = 0.85

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral(-cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 - 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

maple [B] time = 17.33, size = 94273592, normalized size = 2049425.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(a+b\,x\right)}^2}{{\sin\left(2\,a+2\,b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.175 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$

Optimal. Leaf size=48

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/3*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 2641}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A] time = 1.00, size = 82, normalized size = 1.71

$$-\frac{\sqrt{\sin(2(a+bx))}\csc^2(a+bx)+\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))F\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/12*(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Co
s[a + b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Si
n[2*(a + b*x)]])/b
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(bx+a\right)^{2}}{\left(\cos\left(2\,bx+2\,a\right)^{2}-1\right)\sqrt{\sin\left(2\,bx+2\,a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-cos(b*x + a)^2/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*a))),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

maple [A] time = 59.05, size = 123, normalized size = 2.56

$$\frac{\sqrt{1+\sin\left(2bx+2a\right)}\ \sqrt{-2\sin\left(2bx+2a\right)+2}\ \sqrt{-\sin\left(2bx+2a\right)}\ \operatorname{EllipticF}\left(\sqrt{1+\sin\left(2bx+2a\right)},\frac{\sqrt{2}}{2}\right)\sin\left(2bx+2\right.}{12\sin\left(2bx+2a\right)^{\frac{3}{2}}\cos\left(2bx+2a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((1+sin(2*b*x+2*a))^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((1+sin(2*b*x+2*a))^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(a+b\,x\right)}^2}{{\sin\left(2\,a+2\,b\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.176 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/5*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a)/
b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4295, 2636, 2639}

$$-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.56, size = 64, normalized size = 0.83

$$\frac{\frac{2(-6\cos(2(a+bx))+3\cos(4(a+bx))+1)\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))} - 12E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(40*b)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)^2\sqrt{\sin(2bx+2a)}}{\cos(2bx+2a)^4 - 2\cos(2bx+2a)^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 - 2*cos(
2*b*x + 2*a)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2}{\sin(2bx+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

maple [B] time = 212.85, size = 227, normalized size = 2.95

$$\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2bx+2a)^{\frac{5}{2}}} + \frac{4\sqrt{2}\left(6\sqrt{1+\sin(2bx+2a)}\,\sqrt{-2\sin(2bx+2a)+2}\,\sqrt{-\sin(2bx+2a)}\,\left(\sin^2(2bx+2a)\right)\text{EllipticE}\left(\sqrt{1+\sin(2bx+2a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{}\right.}{5}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/32*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2
```

```
*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))
-3*(1+sin(2*b*x+2*a))^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(
1/2)*sin(2*b*x+2*a)^2*EllipticF((1+sin(2*b*x+2*a))^(1/2),1/2*2^(1/2))+6*sin
(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.177 $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{7\sqrt{\sin(2a+2}}{3}$$

```
[Out] -7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+7/48*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/12*cos(b*x+a)*sin
(2*b*x+2*a)^(5/2)/b-7/32*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4297, 4301, 4302, 4305}

$$\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\sqrt{\sin(2a+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) + (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) + (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{12}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{16}\int \sin(a+$$
$$= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{32b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \cos(a+b$$
$$= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{}\right.}{64b}$$

Mathematica [A] time = 0.34, size = 99, normalized size = 0.73

$$\frac{-7\sin^{-1}(\cos(a+bx)-\sin(a+bx)) - \frac{2}{3}\sqrt{\sin(2(a+bx))}\,(10\cos(a+bx)+9\cos(3(a+bx))+2\cos(5(a+bx))) +}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 7*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(64*b)
```

fricas [B] time = 0.46, size = 291, normalized size = 2.14

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5 - 4\cos(bx+a)^3 - 7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)} - 42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/768*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\cos^3(bx+a)\right)\left(\sin^{\frac{3}{2}}(2bx+2a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx + a\right)^{3} \sin\left(2\,bx + 2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^3\,{\sin\left(2\,a+2\,b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

3.178 $\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}-\frac{5\log\left(\sin(a+b\right.}{}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+1/8*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+5/16*sin(b*x+a)*sin(
2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4297, 4301, 4306}

$$\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\sin(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{16}\int \frac{s}{\sqrt{s}}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)\right.}{32b}$$

Mathematica [A] time = 0.19, size = 84, normalized size = 0.76

$$\frac{2\sqrt{\sin(2(a+bx))}\,(6\sin(a+bx)+\sin(3(a+bx))) - 5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)) + \log\left(\sin(a+bx)+\right.\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin
[3*(a + b*x)]))/(32*b)
```

fricas [B] time = 0.56, size = 280, normalized size = 2.55

$$8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a) + 10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-}{\cos(bx+a)^2+2\cos(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:ext_reduce Error: Bad Argument TypeEvaluatio
n time: 0.48Done
```

maple [B] time = 32.66, size = 88680892, normalized size = 806189.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^{3}\sqrt{\sin\left(2\,bx+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^{3}\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.179 $\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\mathrm{cc}\right.}{8b}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4297, 4305}

$$-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\mathrm{cc}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) + (Cos[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\right.}}{8b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.87

$$\frac{-3\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\sin^{\frac{3}{2}}(2(a+bx))\csc(a+bx)+3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\mathrm{c}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/(8*b)
```

fricas [B] time = 0.73, size = 268, normalized size = 3.19

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sq
rt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
 a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*
sin(b*x + a) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

maple [B] time = 69.30, size = 191110780, normalized size = 2275128.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)^3}{\sqrt{\sin(2a+2bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
[Out] Timed out
```

3.180 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] 1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+
2*a)^(1/2))/b-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4293, 4307, 4306}

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4293

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4307

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \sec(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}-\end{aligned}$$

Mathematica [A] time = 0.09, size = 70, normalized size = 0.85

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))-2\sqrt{\sin(2(a+bx))}\csc(a+bx)+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

fricas [B] time = 0.54, size = 295, normalized size = 3.60

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)s}}{\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*s
in(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos
(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
 a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

maple [B] time = 32.12, size = 179323229, normalized size = 2186868.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(a+b\,x\right)^3}{{\sin \left(2\,a+2\,b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

3.181 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

[Out] -1/3*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4291}

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]

[Out] -Cos[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.96

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))\csc^3(a+bx)}{24b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]

[Out] -1/24*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b

fricas [B] time = 0.64, size = 53, normalized size = 1.89

$$\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1}{12\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")

[Out] 1/12*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^2 - b)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

maple [C] time = 167.45, size = 192, normalized size = 6.86

$$\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(4\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{24\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/24*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*
a)^2-1)/tan(1/2*b*x+1/2*a)*(4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+
1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)+tan(1/2*b*x+1/2*a)^4-1)/(tan(1/2*b*
x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+
1/2*a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

mupad [B] time = 1.17, size = 94, normalized size = 3.36

$$\frac{\sqrt{\sin(2\,a+2\,b\,x)}\left(\frac{2\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{3}-\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+\frac{\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2}{3}\right)}{b\left(30\sin(a+b\,x)^2-12\sin(2\,a+2\,b\,x)^2+2\sin(3\,a+3\,b\,x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] (sin(2*a + 2*b*x)^(1/2)*((2*sin(a/2 + (b*x)/2)^2)/3 - sin((3*a)/2 + (3*b*x)
/2)^2 + sin((5*a)/2 + (5*b*x)/2)^2/3))/(b*(2*sin(3*a + 3*b*x)^2 - 12*sin(2*
a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

3.182 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/5*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 4291}

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -Cos[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{1}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 35, normalized size = 0.64

$$-\frac{\sqrt{\sin(2(a+bx))}\,\csc(a+bx)\left(\csc^2(a+bx)+4\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/40*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

fricas [A] time = 0.47, size = 76, normalized size = 1.38

$$-\frac{\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+4\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{40\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(
cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{3}}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{3}\left(bx+a\right)}{\sin\left(2bx+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{3}}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

mupad [B] time = 3.07, size = 93, normalized size = 1.69

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] -(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp(a
*2i + b*x*2i) - 1)^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

3.183 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/7*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)-2/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(
3/2)+4/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4295, 4303, 4292}

$$\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -Cos[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (2*Cos[a + b*x])/(21*b*Sin[2
*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.68

$$\frac{\sqrt{\sin(2(a+bx))}\,(-12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc^4(a+bx)\sec(a+bx)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

fricas [A] time = 0.52, size = 104, normalized size = 1.28

$$\frac{32\cos(bx+a)^5 - 64\cos(bx+a)^3 + \sqrt{2}\left(32\cos(bx+a)^4 - 56\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{336\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/336*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))
/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

mupad [B] time = 3.66, size = 302, normalized size = 3.73

$$-\frac{5\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{84\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^2}+\frac{{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,3{}\mathrm{i}}{14\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^3}-\frac{{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{7\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*3i)/(14*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (5*exp(a*1i + b*x*1i)*
((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(84*b*(exp
(a*2i + b*x*2i)*1i - 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4
) - (exp(a*1i + b*x*1i)*(5i/(84*b) - (exp(a*2i + b*x*2i)*4i)/(21*b))*((exp(
- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x
*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

3.184 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/9*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)-1/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)+4/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-8/45*cos(b*x+a)/b/sin(2*b*x+2*a
)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4295, 4303, 4304, 4291}

$$\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -Cos[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) - Cos[a + b*x]/(15*b*Sin[2*a +
 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (8*Cos[a
+ b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} + \frac{1}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8}{45}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}$$
$$= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.09, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\left(5\csc^5(a+bx)+17\csc^3(a+bx)+113\csc(a+bx)-15\tan(a+bx)\sec(a+bx)\right)}{1440b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -1/1440*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*C
sc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

fricas [A] time = 0.48, size = 131, normalized size = 1.22

$$-\frac{\sqrt{2}\left(128\cos(bx+a)^6-288\cos(bx+a)^4+180\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\left(\cos(bx+\right.}{1440\left(b\cos(bx+a)^6-2b\cos(bx+a)^4+b\cos(bx+a)^2\right)\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/1440*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a
)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x +
a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(bx+a)}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

```
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{3}}{\sin\left(2\,bx+2\,a\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

mupad [B] time = 4.88, size = 383, normalized size = 3.58

$$-\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}}{60\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^{3}}-\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,2i}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^{4}}+\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1}{2}}}{9\,b\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((exp(
- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*2i)/(9*b*(exp(a*2
i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2
- (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(60*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) +
 (8*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)) -
 (exp(a*1i + b*x*1i)*(49i/(180*b) + (exp(a*2i + b*x*2i)*19i)/(180*b))*((exp
(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*
x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

3.185 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4305}

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

fricas [B] time = 0.49, size = 137, normalized size = 4.42

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-s}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

maple [C] time = 0.81, size = 98, normalized size = 3.16

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] int(cos(x)/sin(2*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.186 $\int \csc(x)\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=25

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -arcsin(cos(x)-sin(x))+ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4308, 4305}

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc(x)\sqrt{\sin(2x)}\,dx &= 2\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx\\ &= -\sin^{-1}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

fricas [B] time = 0.45, size = 137, normalized size = 5.48

$$\frac{1}{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{2}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-s}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/2*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/4*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(x)\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

maple [C] time = 2.46, size = 99, normalized size = 3.96

$$\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(2*x)^(1/2),x)
```

```
[Out] 2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(x)\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\sin(2x)}}{\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(1/2)/sin(x),x)
```

```
[Out] int(sin(2*x)^(1/2)/sin(x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.187 $\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(a+bx)\right)}{b(m+4)}$$

```
[Out] -cos(b*x+a)^3*cot(b*x+a)*hypergeom([2+1/2*m, 1/2-1/2*m],[3+1/2*m],cos(b*x+a
)^2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(4+m)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4309, 2576}

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(4 + m)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{3+m}(a+bx)\sin^m(a+b$$

$$= -\frac{\cos^3(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}}{b(4+m)}$$

Mathematica [C] time = 13.25, size = 2472, normalized size = 29.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(1 + m)*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2*(2 + m), (3 + m)/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + m)
/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[a
 + b*x]^3*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] +
 Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(1 + m)*(
Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*((2^m*(6*AppellF1[(1 + m)/2, -m, 2*(1 +
m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m
)/2, -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - A
ppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(1 + 2*m)*(Cos[(a + b*x)/2]*(
-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m)/((1 + m)*(Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)^m) + (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 +
m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m,
2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(
1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x
)/2] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-1/2*Cos[(a + b*x
)/2] + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(a + b*x)/2] +
 Sin[(3*(a + b*x))/2]))/2)*Tan[(a + b*x)/2])/((1 + m)*(Cos[a + b*x]*Sec[(a
+ b*x)/2]^2)^m) + (2^(2 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2
*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1
 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] -
 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)
/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2]^2)/((1 + m)*(Cos[a + b*x]*S
ec[(a + b*x)/2]^2)^m) - (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m),
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2,
 -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - Appel
lF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(a + b*x)
/2]^2)^(-1 - m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]
))^m*Tan[(a + b*x)/2]*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Se
c[(a + b*x)/2]^2*Tan[(a + b*x)/2]))/(1 + m) + (2^(1 + m)*(Sec[(a + b*x)/2]^
2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Ta
n[(a + b*x)/2]*((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 1 + 2*m, 1 + (3 +
 m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a +
 b*x)/2])/(3 + m) + ((1 + m)*(1 + 2*m)*AppellF1[1 + (1 + m)/2, -m, 2 + 2*m,
 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2
*Tan[(a + b*x)/2])/(3 + m) - 12*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m
, 3 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a +
 b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - ((1 + m)*(3 + 2*m)*AppellF1[1 + (1
+ m)/2, -m, 4 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2
]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 6*(-((m*(1 + m)*AppellF1[
1 + (1 + m)/2, 1 - m, 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - (2*(1 + m)^2*
AppellF1[1 + (1 + m)/2, -m, 1 + 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 8*(
-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 2*(2 + m), 1 + (3 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3
 + m)) - (2*(1 + m)*(2 + m)*AppellF1[1 + (1 + m)/2, -m, 1 + 2*(2 + m), 1 +
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[
(a + b*x)/2])/(3 + m))))/((1 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

maple [F] time = 6.18, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(bx+a\right)\right)\left(\sin^{m}\left(2bx+2a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^{3}\,\sin\left(2\,a+2\,b\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**m,x)
[Out] Timed out
```

3.188 $\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(a+bx)\right)}{b(m+3)}$$

```
[Out] -cos(b*x+a)^2*cot(b*x+a)*hypergeom([3/2+1/2*m, 1/2-1/2*m],[5/2+1/2*m],cos(b
*x+a)^2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(3+m)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4309, 2576}

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(3 + m)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{2+m}(a+bx)\sin^m(a$$

$$= -\frac{\cos^2(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)}{b(3+m)}$$

Mathematica [C] time = 7.81, size = 890, normalized size = 10.47

$$\frac{}{b(m+1)\left(8(m+3)F_1\left(\frac{m+1}{2};-m,2(m+1);\frac{m+3}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\frac{1}{2}(a+bx)\right)-2(m\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (4*(3 + m)*(4*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(1 + m)/2, -m, 3 + 2*m,
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos
[a + b*x]^2*Sin[(a + b*x)/2]*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(8*(3 + m)*Appe
llF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Cos[(a + b*x)/2]^2 - 2*(3 + m)*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3
+ m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 - 8*(3
+ m)*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(4*m*AppellF1[(3 + m)/2, 1 - m, 2*(1
+ m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[(3 +
 m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 4*m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[(3 + m)/2, -m, 2*(1 + m), (5 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -m, 2*(1 + m), (
5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(3 + m)/2,
 -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*m*A
ppellF1[(3 + m)/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 8*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 8*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x])))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx + 2\,a\right)^{m} \cos\left(bx + a\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx + 2\,a\right)^{m} \cos\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

maple [F] time = 5.90, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(bx + a\right)\right)\left(\sin^{m}\left(2bx + 2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx + 2\,a\right)^{m} \cos\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)^2 \sin(2a+2bx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**m,x)
[Out] Timed out
```

3.189 $\int \cos(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(a+bx)\right)}{b(m+2)}$$

```
[Out] -cos(b*x+a)*cot(b*x+a)*hypergeom([1+1/2*m, 1/2-1/2*m],[2+1/2*m],cos(b*x+a)^
2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(2+m)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4309, 2576}

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 + m
)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*(2
 + m)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^m(2a+2bx)\,dx &= \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{1+m}(a+bx)\sin^m(a+bx\\ &= -\frac{\cos(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\,\mathrm{s}}{b(2+m)}\end{aligned}$$

Mathematica [C] time = 0.26, size = 149, normalized size = 1.80

$$\frac{2^{-m-1}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{m+1}\left((2m-1)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(3-2m);e^{4i(a+bx)}\right)+(2m+1)e^{2i(a+bx)}\right.}{b\left(4m^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I)*(a
+ b*x)))^(1 + m)*((-1 + 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m)/4,
 E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1[1,
(5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

maple [F] time = 5.36, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\left(\sin^{m}\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(2\,bx+2\,a\right)^{m}\cos\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sin\left(2\,a+2\,b\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

3.190 $\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^9(a+bx)}{9b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^5(a+bx)}{5b}$$

```
[Out] -4/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b-4/9*cos(b*x+a)^9/b
```

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4312, 2565, 270}

$$-\frac{4\cos^9(a+bx)}{9b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b) - (4*Cos[a + b*x]^9)/(
9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4312

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((f_.)*sin[(a_.) + (b_.)*(x_)])^(
n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] :> Dist[2^p/(e^p*f^p), Int[(e
*Cos[a + b*x])^(m + p)*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^4(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{4\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 37, normalized size = 0.80

$$\frac{\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{630b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(630*b
)
```

fricas [A] time = 0.53, size = 36, normalized size = 0.78

$$-\frac{4\left(35\,\cos\left(bx+a\right)^{9}-90\,\cos\left(bx+a\right)^{7}+63\,\cos\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

giac [A] time = 0.64, size = 68, normalized size = 1.48

$$-\frac{\cos\left(9\,bx+9\,a\right)}{576\,b}+\frac{\cos\left(7\,bx+7\,a\right)}{448\,b}+\frac{\cos\left(5\,bx+5\,a\right)}{80\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{48\,b}-\frac{3\,\cos\left(bx+a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/576*cos(9*b*x + 9*a)/b + 1/448*cos(7*b*x + 7*a)/b + 1/80*cos(5*b*x + 5*a
)/b - 1/48*cos(3*b*x + 3*a)/b - 3/32*cos(b*x + a)/b
```

maple [A] time = 0.43, size = 69, normalized size = 1.50

$$-\frac{3\cos\left(bx+a\right)}{32b}-\frac{\cos\left(3bx+3a\right)}{48b}+\frac{\cos\left(5bx+5a\right)}{80b}+\frac{\cos\left(7bx+7a\right)}{448b}-\frac{\cos\left(9bx+9a\right)}{576b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] -3/32*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+1/80*cos(5*b*x+5*a)/b+1/448*cos(7*
b*x+7*a)/b-1/576*cos(9*b*x+9*a)/b
```

maxima [A] time = 0.34, size = 58, normalized size = 1.26

$$-\frac{35\,\cos\left(9\,bx+9\,a\right)-45\,\cos\left(7\,bx+7\,a\right)-252\,\cos\left(5\,bx+5\,a\right)+420\,\cos\left(3\,bx+3\,a\right)+1890\,\cos\left(bx+a\right)}{20160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/20160*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

mupad [B] time = 0.20, size = 36, normalized size = 0.78

$$-\frac{4\,\left(35\cos\left(a+b\,x\right)^9-90\cos\left(a+b\,x\right)^7+63\cos\left(a+b\,x\right)^5\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
```

```
[Out] -(4*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

3.191 $\int \sin(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=293

$$-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)}{b-dn}$$

```
[Out] -2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(
I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d*x
+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n+b)
*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-
d*n-b)/d],[1+1/2*(-d*n-b)/d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(
d*n+b)
```

Rubi [A] time = 0.85, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4553, 2285, 2253, 2252, 2251}

$$-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)}{b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c +
d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/
d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
```

```
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)dx$$
$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c-}\right.$$
$$= \left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+dx)}\left(i-ie^{2ic}\right.$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(i-i\right.\right.$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(1-e\right.\right.$$
$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+}\right.}{b-dn}$$

Mathematica [A] time = 0.90, size = 209, normalized size = 0.71

$$\frac{2^{-n-1}e^{-ix(b+d)}\left(-1+e^{2i(c+dx)}\right)\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^n\left(e^{idx}(\cos(a)-i\sin(a))(b-dn)\,{}_2F_1\left(1,\frac{1}{2}\left(-\frac{b}{d}+n+2\right);-\right.\right.}{(b-dn)(b+dn)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

```
[Out] (2^(-1 - n)*(-1 + E^((2*I)*(c + d*x)))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E
^(I*(c + d*x)))^n*(E^(I*d*x)*(b - d*n)*Hypergeometric2F1[1, (2 - b/d + n)/2
, -1/2*(b + d*(-2 + n))/d, E^((2*I)*(c + d*x))]*(Cos[a] - I*Sin[a]) + E^(I*
(2*b + d)*x)*(b + d*n)*Hypergeometric2F1[1, (b + d*(2 + n))/(2*d), (2 + b/d
 - n)/2, E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a])))/(E^(I*(b + d)*x)*(b - d
*n)*(b + d*n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(dx+c\right)^n\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)^n\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

maple [F] time = 14.32, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\left(\sin^{n}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^n,x)
[Out] int(sin(b*x+a)*sin(d*x+c)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)^{n}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)\,{\sin\left(c+d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^n,x)
[Out] int(sin(a + b*x)*sin(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**n,x)
[Out] Timed out
```

3.192 $\int \sin(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=91

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2637}

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] -Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)-\frac{3}{8}\cos(a+c+(b+d)\right.\\ &= -\left(\frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \text{co}\\ &= -\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\end{aligned}$$

Mathematica [A] time = 0.54, size = 86, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\sin(a+bx-3c-3dx)}{b-3d}+\frac{3\sin(a+bx-c-dx)}{b-d}+\frac{\sin(a+bx+3c+3dx)}{b+3d}-\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] (-(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sin[a - c + b*x - d*x])/(b -
d) + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sin[a + c + (b + d)*x])/(b +
 d))/8
```

fricas [A] time = 0.57, size = 122, normalized size = 1.34

$$-\frac{3\left(\left(b^{2}d-d^{3}\right)\cos\left(dx+c\right)^{3}-\left(b^{2}d-3\,d^{3}\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)-\left(\left(b^{3}-bd^{2}\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^{2}-\right.}{b^{4}-10\,b^{2}d^{2}+9\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(3*((b^2*d - d^3)*cos(d*x + c)^3 - (b^2*d - 3*d^3)*cos(d*x + c))*sin(b*x +
 a) - ((b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c)^2 - (b^3 - 7*b*d^2)*cos(b*x
+ a))*sin(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

giac [A] time = 0.75, size = 84, normalized size = 0.92

$$\frac{\sin\left(bx+3\,dx+a+3\,c\right)}{8\,\left(b+3\,d\right)}-\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,\left(b+d\right)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,\left(b-d\right)}-\frac{\sin\left(bx-3\,dx+a-3\,c\right)}{8\,\left(b-3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) - 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

maple [A] time = 0.86, size = 84, normalized size = 0.92

$$-\frac{\sin\left(a-3c+\left(b-3d\right)x\right)}{8\left(b-3d\right)}+\frac{3\sin\left(a-c+\left(b-d\right)x\right)}{8\left(b-d\right)}-\frac{3\sin\left(a+c+\left(b+d\right)x\right)}{8\left(b+d\right)}+\frac{\sin\left(a+3c+\left(b+3d\right)x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^3,x)
```

```
[Out] -1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [B] time = 0.41, size = 916, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) - 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) + 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
```

```
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

mupad [B] time = 1.57, size = 311, normalized size = 3.42

$$-e^{a\,1i-c\,3i+b\,x\,1i-d\,x\,3i}\,\left(\frac{b+3\,d}{b^2\,16i-d^2\,144i}+\frac{e^{-a\,2i-b\,x\,2i}\,\left(b-3\,d\right)}{b^2\,16i-d^2\,144i}\right)+e^{a\,1i+c\,3i+b\,x\,1i+d\,x\,3i}\,\left(\frac{b-3\,d}{b^2\,16i-d^2\,144i}+\frac{e^{-a\,2i-b\,x\,2i}\,\left(b\right.}{b^2\,16i-d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] exp(a*1i + c*3i + b*x*1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) + (exp(-
 a*2i - b*x*2i)*(b + 3*d))/(b^2*16i - d^2*144i)) - exp(a*1i - c*3i + b*x*1i
 - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) + (exp(- a*2i - b*x*2i)*(b - 3*d
))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/
(b^2*16i - d^2*16i) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^2*16i
)) - exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*16i) +
(exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

sympy [A] time = 32.89, size = 921, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 + sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a - 3*d
*x)/(24*d) + sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, -3*d)
), (3*x*sin(a - d*x)*sin(c + d*x)**3/8 + 3*x*sin(a - d*x)*sin(c + d*x)*cos(
c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*x*cos(a
 - d*x)*cos(c + d*x)**3/8 + 3*sin(a - d*x)*cos(c + d*x)**3/(8*d) + 5*sin(c
+ d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)**2/
(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*s
in(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*
x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + 3*sin(a + d*x)*cos(c + d*x)**3/
(8*d) - 5*sin(c + d*x)**3*cos(a + d*x)/(8*d) - 3*sin(c + d*x)*cos(a + d*x)*
cos(c + d*x)**2/(4*d), Eq(b, d)), (x*sin(a + 3*d*x)*sin(c + d*x)**3/8 - 3*x
*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a
+ 3*d*x)*cos(c + d*x)/8 - x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + sin(a + 3*d*
x)*cos(c + d*x)**3/(8*d) - 7*sin(c + d*x)**3*cos(a + 3*d*x)/(24*d) - sin(c
+ d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)), (-b**3*sin(c + d*
x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 3*b**2*d*sin(a + b*x)*s
in(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 7*b*d**2*sin(c
 + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sin(c + d
*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) - 9*d**3*si
n(a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*
d**3*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

3.193 $\int \sin(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

```
[Out] -1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2638}

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

```
[Out] -Cos[a + b*x]/(2*b) + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)-\frac{1}{4}\sin(a-2c+(b-2d)x)-\frac{1}{4}\sin(a+2c+(b+2d)x)\right) dx\\ &= -\left(\frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx\right)-\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \text{s}\\ &= -\frac{\cos(a+bx)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A] time = 0.71, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cos(a+bx-2c-2dx)}{b-2d}+\frac{\cos(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}-\frac{2\cos(a)\cos(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b + Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

fricas [A] time = 0.43, size = 71, normalized size = 1.15

$$\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-\left(b^2-2\,d^2\right)\cos(bx+a)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
+ c) - (b^2 - 2*d^2)*cos(b*x + a))/(b^3 - 4*b*d^2)
```

giac [A] time = 0.19, size = 56, normalized size = 0.90

$$\frac{\cos\left(bx+2\,dx+a+2\,c\right)}{4\,(b+2\,d)}+\frac{\cos\left(bx-2\,dx+a-2\,c\right)}{4\,(b-2\,d)}-\frac{\cos\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*cos(b*x - 2*d*x + a - 2*c)/(
b - 2*d) - 1/2*cos(b*x + a)/b
```

maple [A] time = 0.20, size = 57, normalized size = 0.92

$$-\frac{\cos\left(bx+a\right)}{2b}+\frac{\cos\left(a-2c+(b-2d)\,x\right)}{4b-8d}+\frac{\cos\left(a+2c+(b+2d)\,x\right)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^2,x)
```

```
[Out] -1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

maxima [B] time = 0.35, size = 414, normalized size = 6.68

$$\left(b^2\cos\left(2\,c\right)-2\,bd\cos\left(2\,c\right)\right)\cos\left((b+2\,d)x+a+4\,c\right)+\left(b^2\cos\left(2\,c\right)-2\,bd\cos\left(2\,c\right)\right)\cos\left((b+2\,d)x+a\right)+\left(b^2\,\mathrm{cc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos(
2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c
))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
2*d)*x - a) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) - 2*(b^2
*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin(
2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b +
 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
- (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*sin(2*c) -
 4*d^2*sin(2*c))*sin(b*x + a + 2*c) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*sin
(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*sin
(2*c)^2)*d^2)
```

mupad [B] time = 0.74, size = 98, normalized size = 1.58

$$-\frac{d\,\left(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\cos\left(a+2\,c\right.}{16\,b\,d^2-4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] - (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^
2*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 -
4*b^3) - cos(a + b*x)/(2*b)
```

<u>**sympy**</u> **[A]** time = 6.70, size = 408, normalized size = 6.58

$$\begin{cases} x\sin(a)\sin^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ \frac{x\sin(a-2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}-\frac{3\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+\cos(a-2 \\ \frac{x\sin(a+2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{3\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}-\cos(a+ \\ -\frac{b^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (x*sin(a - 2*d*x)*sin(c + d*x)**2/4 - x*sin(a - 2*d*x)*cos(c + d*x)**2/4 -
 x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - 3*sin(a - 2*d*x)*sin(c + d*
x)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2*d)),
 (x*sin(a + 2*d*x)*sin(c + d*x)**2/4 - x*sin(a + 2*d*x)*cos(c + d*x)**2/4 +
 x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - 3*sin(a + 2*d*x)*sin(c + d*
x)*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, 2*d)),
(-b**2*sin(c + d*x)**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*b*d*sin(a + b*x)*
sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(a
+ b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*
d**2), True))
```

3.194 $\int \sin(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4569, 2637}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x],x]
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)-\frac{1}{2}\cos(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx-\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\ &= \frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 43, normalized size = 1.00

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x],x]
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

fricas [A] time = 0.43, size = 42, normalized size = 0.98

$$\frac{d\cos(dx+c)\sin(bx+a)-b\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (d*cos(d*x + c)*sin(b*x + a) - b*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

giac [A] time = 0.36, size = 40, normalized size = 0.93

$$-\frac{\sin(bx+dx+a+c)}{2(b+d)}+\frac{\sin(bx-dx+a-c)}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

maple [A] time = 0.42, size = 40, normalized size = 0.93

$$\frac{\sin(a-c+(b-d)x)}{2b-2d}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

maxima [A] time = 0.32, size = 40, normalized size = 0.93

$$-\frac{\sin(bx+dx+a+c)}{2(b+d)}-\frac{\sin(-bx+dx-a+c)}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

mupad [B] time = 1.06, size = 84, normalized size = 1.95

$$\frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2}+\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}-\frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2}-\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x),x)
```

```
[Out] (d*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (b*
(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

sympy [A] time = 1.49, size = 153, normalized size = 3.56

$$\begin{cases} x\sin(a)\sin(c) & \text{for}\ b=0\wedge d=0 \\ \frac{x\sin(a-dx)\sin(c+dx)}{2}-\frac{x\cos(a-dx)\cos(c+dx)}{2}-\frac{\sin(a-dx)\cos(c+dx)}{2d} & \text{for}\ b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2}+\frac{x\cos(a+dx)\cos(c+dx)}{2}-\frac{\sin(c+dx)\cos(a+dx)}{2d} & \text{for}\ b=d \\ -\frac{b\sin(c+dx)\cos(a+bx)}{b^2-d^2}+\frac{d\sin(a+bx)\cos(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*sin(c + d
*x)/2 - x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 -
sin(c + d*x)*cos(a + d*x)/(2*d), Eq(b, d)), (-b*sin(c + d*x)*cos(a + b*x)/(
b**2 - d**2) + d*sin(a + b*x)*cos(c + d*x)/(b**2 - d**2), True))
```

3.195 $\int \csc(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

```
[Out] x*cos(a-c)+ln(sin(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4582, 3475, 8}

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]*Sin[a + b*x],x]
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \csc(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int 1\,dx+\sin(a-c)\int \cot(c+bx)\,dx\\ &= x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.15, size = 26, normalized size = 1.00

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]*Sin[a + b*x],x]
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

fricas [A] time = 0.42, size = 31, normalized size = 1.19

$$\frac{bx\cos(-a+c)-\log\left(\frac{1}{2}\sin(bx+c)\right)\sin(-a+c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b*((tan(a/2)^2*tan(c/2)-tan(a/2)*t
an(c/2)^2+tan(a/2)-tan(c/2))/(tan(a/2)^2*tan(c/2)^2+tan(a/2)^2+tan(c/2)^2+1
)*ln(abs(tan((b*x+c)/2)))+(-tan(a/2)^2*tan(c/2)+tan(a/2)*tan(c/2)^2-tan(a/2
)+tan(c/2))/(tan(a/2)^2*tan(c/2)^2+tan(a/2)^2+tan(c/2)^2+1)*ln(tan((b*x+c)/
2)^2+1)+(2*tan(a/2)^2*tan(c/2)^2-2*tan(a/2)^2+8*tan(a/2)*tan(c/2)-2*tan(c/2
)^2+2)*1/2/(tan(a/2)^2*tan(c/2)^2+tan(a/2)^2+tan(c/2)^2+1)*(b*x+c)/2)
```

maple [B] time = 1.55, size = 325, normalized size = 12.50

$$-\frac{\ln\left(\tan\left(bx+a\right)\cos(a)\cos(c)+\tan\left(bx+a\right)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c)\right)\cos(a)\sin(c)}{b\left(\left(\cos^{2}(a)\right)\left(\cos^{2}(c)\right)+\left(\cos^{2}(a)\right)\left(\sin^{2}(c)\right)+\left(\cos^{2}(c)\right)\left(\sin^{2}(a)\right)+\left(\sin^{2}(a)\right)\left(\sin^{2}(c)\right)\right)}+\frac{\ln\left(\tan\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)*sin(b*x+a),x)
```

```
[Out] -1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)
^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(
a)*cos(c))*cos(a)*sin(c)+1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*
sin(a)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*s
in(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*cos(c)+1/2/b/(cos(c)^2+sin(c)^2)/
(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*sin(c)-1/2/b/(cos(c)^2+sin(c)
^2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*cos(c)+1/b/(cos(c)^2+sin(
c)^2)/(cos(a)^2+sin(a)^2)*arctan(tan(b*x+a))*cos(a)*cos(c)+1/b/(cos(c)^2+si
n(c)^2)/(cos(a)^2+sin(a)^2)*arctan(tan(b*x+a))*sin(a)*sin(c)
```

maxima [B] time = 0.40, size = 108, normalized size = 4.15

$$\frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos(c)+\cos(c)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin(c)+\sin(c)^{2}\right)\sin\left(-a+\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*c
os(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(
-a + c))/b
```

mupad [B] time = 0.87, size = 111, normalized size = 4.27

$$x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+\frac{\ln\left(-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x),x)
```

```
[Out] x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(a
*1i - c*1i)/2) + (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*((exp(c*1i - a
*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

sympy [B] time = 8.20, size = 333, normalized size = 12.81

$$\left(\begin{cases} 0 & \text{f} \\ x & \text{f} \\ 0 & \text{f} \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} & \text{o} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*tan
(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) + t
an(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*ta
n(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)*
*2 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log(sin(b
*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 + b
) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c/
2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2
)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b)
 + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)*
*2 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

3.196 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b-csc(b*x+c)*sin(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4582, 2606, 8, 3770}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}-\frac{\sin(a-c)\,\text{Subst}(\int 1\,dx,x,\csc(c+bx))}{b}\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.10, size = 90, normalized size = 2.50

$$-\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc[c
 + b*x]*Sin[a - c])/b
```

fricas [A] time = 0.51, size = 71, normalized size = 1.97

$$-\frac{\cos\left(-a+c\right)\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-\cos\left(-a+c\right)\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-2\sin\left(-a+c\right)}{2\,b\sin\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*lo
g(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b*((-tan((b*x+c)/2)*tan(a/2)^2*tan
(c/2)+tan((b*x+c)/2)*tan(a/2)*tan(c/2)^2-tan((b*x+c)/2)*tan(a/2)+tan((b*x+c
)/2)*tan(c/2))/(2*tan(a/2)^2*tan(c/2)^2+2*tan(a/2)^2+2*tan(c/2)^2+2)+(-tan(
(b*x+c)/2)*tan(a/2)^2*tan(c/2)^2+tan((b*x+c)/2)*tan(a/2)^2-4*tan((b*x+c)/2)
*tan(a/2)*tan(c/2)+tan((b*x+c)/2)*tan(c/2)^2-tan((b*x+c)/2)-tan(a/2)^2*tan(
c/2)+tan(a/2)*tan(c/2)^2-tan(a/2)+tan(c/2))/(2*tan(a/2)^2*tan(c/2)^2+2*tan(
a/2)^2+2*tan(c/2)^2+2)/tan((b*x+c)/2)+(tan(a/2)^2*tan(c/2)^2-tan(a/2)^2+4*t
an(a/2)*tan(c/2)-tan(c/2)^2+1)/(2*tan(a/2)^2*tan(c/2)^2+2*tan(a/2)^2+2*tan(
c/2)^2+2)*ln(abs(tan((b*x+c)/2))))
```

maple [B] time = 2.23, size = 890, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] -8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)
^2*sin(c)^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x
+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*si
n(c)+cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)-8/b/(-4*
cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)
^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2
+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(
a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)-8/b/(-4*cos(a)^2*
cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos
(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+2*tan(1/
2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(a)*sin(c)
-sin(a)*cos(c))*cos(a)*sin(c)+8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2
-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a
)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2
*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*cos(c
```

```
)-8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a
)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a
)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x
+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c
)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*cos(a)*cos(c)-8/b/(-4*cos(a
)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(
-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(
1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a
)*cos(c)+2*sin(a)*sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*si
n(a)^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*sin(c)
```

maxima [B] time = 0.36, size = 454, normalized size = 12.61

$$-\frac{2\left(\cos\left(bx+2a\right)-\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)-2\cos\left(bx+2a\right)\cos(a)+2\cos\left(bx+2c\right)\cos(a)+\left(\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a +
c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a
+ 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2
)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c) - 2*co
s(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a + 2*c)^2
- 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(-a
+ c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2*b*x + a +
 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
 + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*
b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

mupad [B] time = 5.21, size = 252, normalized size = 7.00

$$-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\right.}{2\,b\,\sqrt{e^{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^2,x)
```

```
[Out] (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c
*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*
2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x*1i
)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*
2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/(2*b*exp
(a*2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(
a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

sympy [B] time = 101.78, size = 3266, normalized size = 90.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (log(tan(b*x/2))/b, Eq(c,
0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan
```

```
(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/
2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*
tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(t
an(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3
*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*
tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/
2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)
*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + tan(b*x/2))*ta
n(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**
2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) -
 log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)
**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
 b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(b
*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)
**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**4
*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan
(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(
b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) +
b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*
tan(c/2) - b*tan(b*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(b*tan(
c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/
2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - 2*log(tan(b*x/2) - 1/tan(c/
2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*
x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x
/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*t
an(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*
x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/2
)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 +
 b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1
/tan(c/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 + tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/
2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2
)) - 2*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2
 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - 2
*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c
/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - tan(b*x/2)
/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 +
b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)), True))*cos(a) + Piec
ewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (-1/(b*sin(b*x)), Eq(c, 0)), (x/sin(c)*
*2, Eq(b, 0)), (4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(2*b*ta
n(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*
b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) +
4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2)**5*tan
(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2
*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/2
) + tan(b*x/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*t
an(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/
2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2
*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2
*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(
c/2)*tan(b*x/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)/(2
*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4
 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2
)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2)
**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(
c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log(
tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(
```

```
c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2
*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log(tan(b*x/2) - 1/tan(c/2))*
tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*
x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2
 - 2*b*tan(c/2)*tan(b*x/2)) + tan(c/2)**6*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b
*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*t
an(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(c/2)**5/(
2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**
4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/
2)) - tan(c/2)**4*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*
tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c
/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**5
*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2
)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2
)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2
)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b
*x/2)) + tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2
)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 -
2*b*tan(c/2)*tan(b*x/2)), True))*sin(a)
```

3.197 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

```
[Out] -cos(a-c)*cot(b*x+c)/b-1/2*csc(b*x+c)^2*sin(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3767, 8}

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc^2(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx\\ &= -\frac{\cos(a-c)\,\text{Subst}\left(\int 1\,dx,x,\cot(c+bx)\right)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int x\,dx,x,\csc(c+\right.}{b}\\ &= -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 35, normalized size = 0.90

$$\frac{\csc(c)\csc^2(bx+c)(\cos(a)-\cos(a-c)\cos(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] ((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

fricas [A] time = 0.43, size = 47, normalized size = 1.21

$$\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)-\sin(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c)
^2 - b)
```

giac [B] time = 0.21, size = 145, normalized size = 3.72

$$-\frac{\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+c)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)^2\tan\left(\right.}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*ta
n(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)*t
an(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(1/2
*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b
*x + c)^2)
```

maple [B] time = 2.34, size = 120, normalized size = 3.08

$$\frac{-\frac{1}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c))}-\frac{}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^2(\tan(bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(-1/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(sin(a)*cos(c)-cos(a)*sin(
c))/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*si
n(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2)
```

maxima [B] time = 0.34, size = 399, normalized size = 10.23

$$\frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (si
n(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(
2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2*cos
(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos(2*a)
 + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.198 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\csc^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}$$

```
[Out] -1/2*arctanh(cos(b*x+c))*cos(a-c)/b-1/2*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1/
3*csc(b*x+c)^3*sin(a-c)/b
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3768, 3770}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\csc^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c
+ b*x])/(2*b) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^3(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^3(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a-$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc^3(}{}$$

Mathematica [A] time = 0.58, size = 67, normalized size = 1.00

$$-\frac{2\sin(a-c)\csc^3(bx+c) + 3\cos(a-c)\cot(bx+c)\csc(bx+c) + 6\cos(a-c)\tanh^{-1}\left(\cos(c) - \sin(c)\tan\left(\frac{bx}{2}\right)\right.}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -1/6*(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Cot
[c + b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/b
```

fricas [B] time = 0.65, size = 141, normalized size = 2.10

$$\frac{6\cos(bx+c)\cos(-a+c)\sin(bx+c) - 3\left(\cos(bx+c)^2\cos(-a+c) - \cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)}{12\left(b\cos(bx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x + c
)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) -
4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b*((-64/3*tan((b*x+c)/2)^3*tan(a/2
)^6*tan(c/2)^5-128/3*tan((b*x+c)/2)^3*tan(a/2)^6*tan(c/2)^3-64/3*tan((b*x+c
)/2)^3*tan(a/2)^6*tan(c/2)+64/3*tan((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^6+64/3
*tan((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^4-64/3*tan((b*x+c)/2)^3*tan(a/2)^5*ta
n(c/2)^2-64/3*tan((b*x+c)/2)^3*tan(a/2)^5-64/3*tan((b*x+c)/2)^3*tan(a/2)^4*
tan(c/2)^5-128/3*tan((b*x+c)/2)^3*tan(a/2)^4*tan(c/2)^3-64/3*tan((b*x+c)/2)
^3*tan(a/2)^4*tan(c/2)+128/3*tan((b*x+c)/2)^3*tan(a/2)^3*tan(c/2)^6+128/3*t
an((b*x+c)/2)^3*tan(a/2)^3*tan(c/2)^4-128/3*tan((b*x+c)/2)^3*tan(a/2)^3*tan
(c/2)^2-128/3*tan((b*x+c)/2)^3*tan(a/2)^3+64/3*tan((b*x+c)/2)^3*tan(a/2)^2*
tan(c/2)^5+128/3*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)^3+64/3*tan((b*x+c)/2)
^3*tan(a/2)^2*tan(c/2)+64/3*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)^6+64/3*tan((
b*x+c)/2)^3*tan(a/2)*tan(c/2)^4-64/3*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)^2-6
4/3*tan((b*x+c)/2)^3*tan(a/2)+64/3*tan((b*x+c)/2)^3*tan(c/2)^5+128/3*tan((b
*x+c)/2)^3*tan(c/2)^3+64/3*tan((b*x+c)/2)^3*tan(c/2)+32*tan((b*x+c)/2)^2*ta
n(a/2)^6*tan(c/2)^6+32*tan((b*x+c)/2)^2*tan(a/2)^6*tan(c/2)^4-32*tan((b*x+c
```

```
)/2)^2*tan(a/2)^6*tan(c/2)^2-32*tan((b*x+c)/2)^2*tan(a/2)^6+128*tan((b*x+c)
/2)^2*tan(a/2)^5*tan(c/2)^5+256*tan((b*x+c)/2)^2*tan(a/2)^5*tan(c/2)^3+128*
tan((b*x+c)/2)^2*tan(a/2)^5*tan(c/2)+32*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2
)^6+32*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2)^4-32*tan((b*x+c)/2)^2*tan(a/2)^
4*tan(c/2)^2-32*tan((b*x+c)/2)^2*tan(a/2)^4+256*tan((b*x+c)/2)^2*tan(a/2)^3
*tan(c/2)^5+512*tan((b*x+c)/2)^2*tan(a/2)^3*tan(c/2)^3+256*tan((b*x+c)/2)^2
*tan(a/2)^3*tan(c/2)-32*tan((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)^6-32*tan((b*x+
c)/2)^2*tan(a/2)^2*tan(c/2)^4+32*tan((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)^2+32*
tan((b*x+c)/2)^2*tan(a/2)^2+128*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)^5+256*ta
n((b*x+c)/2)^2*tan(a/2)*tan(c/2)^3+128*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)-3
2*tan((b*x+c)/2)^2*tan(c/2)^6-32*tan((b*x+c)/2)^2*tan(c/2)^4+32*tan((b*x+c)
/2)^2*tan(c/2)^2+32*tan((b*x+c)/2)^2-64*tan((b*x+c)/2)*tan(a/2)^6*tan(c/2)^
5-128*tan((b*x+c)/2)*tan(a/2)^6*tan(c/2)^3-64*tan((b*x+c)/2)*tan(a/2)^6*tan
(c/2)+64*tan((b*x+c)/2)*tan(a/2)^5*tan(c/2)^6+64*tan((b*x+c)/2)*tan(a/2)^5*
tan(c/2)^4-64*tan((b*x+c)/2)*tan(a/2)^5*tan(c/2)^2-64*tan((b*x+c)/2)*tan(a/
2)^5-64*tan((b*x+c)/2)*tan(a/2)^4*tan(c/2)^5-128*tan((b*x+c)/2)*tan(a/2)^4*
tan(c/2)^3-64*tan((b*x+c)/2)*tan(a/2)^4*tan(c/2)+128*tan((b*x+c)/2)*tan(a/2
)^3*tan(c/2)^6+128*tan((b*x+c)/2)*tan(a/2)^3*tan(c/2)^4-128*tan((b*x+c)/2)*
tan(a/2)^3*tan(c/2)^2-128*tan((b*x+c)/2)*tan(a/2)^3+64*tan((b*x+c)/2)*tan(a
/2)^2*tan(c/2)^5+128*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)^3+64*tan((b*x+c)/2)
*tan(a/2)^2*tan(c/2)+64*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^6+64*tan((b*x+c)/2
)*tan(a/2)*tan(c/2)^4-64*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^2-64*tan((b*x+c)/
2)*tan(a/2)+64*tan((b*x+c)/2)*tan(c/2)^5+128*tan((b*x+c)/2)*tan(c/2)^3+64*t
an((b*x+c)/2)*tan(c/2))/(512*tan(a/2)^6*tan(c/2)^6+1536*tan(a/2)^6*tan(c/2)
^4+1536*tan(a/2)^6*tan(c/2)^2+512*tan(a/2)^6+1536*tan(a/2)^4*tan(c/2)^6+460
8*tan(a/2)^4*tan(c/2)^4+4608*tan(a/2)^4*tan(c/2)^2+1536*tan(a/2)^4+1536*tan
(a/2)^2*tan(c/2)^6+4608*tan(a/2)^2*tan(c/2)^4+4608*tan(a/2)^2*tan(c/2)^2+15
36*tan(a/2)^2+512*tan(c/2)^6+1536*tan(c/2)^4+1536*tan(c/2)^2+512)+(-22*tan(
(b*x+c)/2)^3*tan(a/2)^2*tan(c/2)^2+22*tan((b*x+c)/2)^3*tan(a/2)^2-88*tan((b
*x+c)/2)^3*tan(a/2)*tan(c/2)+22*tan((b*x+c)/2)^3*tan(c/2)^2-22*tan((b*x+c)/
2)^3-6*tan((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)+6*tan((b*x+c)/2)^2*tan(a/2)*tan
(c/2)^2-6*tan((b*x+c)/2)^2*tan(a/2)+6*tan((b*x+c)/2)^2*tan(c/2)-3*tan((b*x+
c)/2)*tan(a/2)^2*tan(c/2)^2+3*tan((b*x+c)/2)*tan(a/2)^2-12*tan((b*x+c)/2)*t
an(a/2)*tan(c/2)+3*tan((b*x+c)/2)*tan(c/2)^2-3*tan((b*x+c)/2)-2*tan(a/2)^2*
tan(c/2)+2*tan(a/2)*tan(c/2)^2-2*tan(a/2)+2*tan(c/2))/(48*tan(a/2)^2*tan(c/
2)^2+48*tan(a/2)^2+48*tan(c/2)^2+48)/tan((b*x+c)/2)^3+(tan(a/2)^2*tan(c/2)^
2-tan(a/2)^2+4*tan(a/2)*tan(c/2)-tan(c/2)^2+1)/(4*tan(a/2)^2*tan(c/2)^2+4*t
an(a/2)^2+4*tan(c/2)^2+4)*ln(abs(tan((b*x+c)/2))))
```

maple [B] time = 4.90, size = 14880, normalized size = 222.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^4*sin(b*x+a),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.42, size = 1773, normalized size = 26.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
+ 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*b*
x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(a))
*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c
) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b*x +
```

```
4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16*(3*co
s(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a +
 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos
(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*(
cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos(-a + c) +
 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*cos(a)*cos(-
a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin(4*b*x + a +
 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)*sin(2*b*x +
a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*cos(2*b*x + a +
 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 6*(3*cos(2*b
*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos
(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x + a + 4*c) - 3*c
os(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x + a + 6*c)
- 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(4*b*x + a
 + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin
(b*x)*sin(c) + sin(c)^2) + 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*
b*x + a + 4*c)^2*cos(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos
(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 +
 9*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^
2 - 6*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*c
os(-a + c) - 3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6
*b*x + a + 6*c) - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c
))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos
(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) +
 cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(3*sin(5*b*x + 2
*a + 4*c) + 3*sin(5*b*x + 6*c) + 8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4
*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) - 6*(3*sin(
4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c)
- 6*(3*sin(4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x +
6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a)
- 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c) + 16*(3*sin(2*b*x + a + 2*c) - sin
(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*
x + 4*c) - 18*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c) + 6*si
n(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*sin(a))/(b*cos(6*b*x + a + 6*c)^2 +
9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*x + a + 2*c)^2 - 6*b*cos(2*b*x + a
 + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*
b*sin(2*b*x + a + 2*c)^2 - 6*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + si
n(a)^2)*b - 2*(3*b*cos(4*b*x + a + 4*c) - 3*b*cos(2*b*x + a + 2*c) + b*cos(
a))*cos(6*b*x + a + 6*c) - 6*(3*b*cos(2*b*x + a + 2*c) - b*cos(a))*cos(4*b*
x + a + 4*c) - 2*(3*b*sin(4*b*x + a + 4*c) - 3*b*sin(2*b*x + a + 2*c) + b*s
in(a))*sin(6*b*x + a + 6*c) - 6*(3*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(4
*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^4,x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.199 $\int \csc^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(a-c)\cot^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

```
[Out] -cos(a-c)*cot(b*x+c)/b-1/3*cos(a-c)*cot(b*x+c)^3/b-1/4*csc(b*x+c)^4*sin(a-c
)/b
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4582, 2606, 30, 3767}

$$-\frac{\cos(a-c)\cot^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Cos[a - c]*Cot[c + b*x]^3)/(3*b) - (Csc[c
 + b*x]^4*Sin[a - c])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \csc^5(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc^4(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc^4(c+bx)\,dx\\ &= -\frac{\cos(a-c)\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(c+bx)\right)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int x^3\,dx,\right.}{b}\\ &= -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\cos(a-c)\cot^3(c+bx)}{3b}-\frac{\csc^4(c+bx)\sin(a-c)}{4b}\end{aligned}$$

Mathematica [A] time = 0.38, size = 58, normalized size = 0.97

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^4(bx+c)(\cos(a-c)(\cos(4bx+3c)-4\cos(2bx+c))+3\cos(a))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] ((3*Cos[a] + Cos[a - c]*(-4*Cos[c + 2*b*x] + Cos[3*c + 4*b*x]))*Csc[c/2]*Cs
c[c + b*x]^4*Sec[c/2])/(24*b)
```

fricas [A] time = 0.57, size = 75, normalized size = 1.25

$$\frac{4\left(2\cos\left(bx+c\right)^3\cos\left(-a+c\right)-3\cos\left(bx+c\right)\cos\left(-a+c\right)\right)\sin\left(bx+c\right)+3\sin\left(-a+c\right)}{12\left(b\cos\left(bx+c\right)^4-2\,b\cos\left(bx+c\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) - 3*cos(b*x + c)*cos(-a + c))*sin(b*x
 + c) + 3*sin(-a + c))/(b*cos(b*x + c)^4 - 2*b*cos(b*x + c)^2 + b)
```

giac [B] time = 3.25, size = 301, normalized size = 5.02

$$-\frac{6\tan\left(bx+c\right)^3\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-6\tan\left(bx+c\right)^3\tan\left(\frac{1}{2}a\right)^2+24\tan\left(bx+c\right)^3\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+6\tan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x + c)^3*tan(1/2
*a)^2 + 24*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(b*x + c)^2*tan(1/2*
a)^2*tan(1/2*c) - 6*tan(b*x + c)^3*tan(1/2*c)^2 - 6*tan(b*x + c)^2*tan(1/2*
a)*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)
^3 + 6*tan(b*x + c)^2*tan(1/2*a) - 2*tan(b*x + c)*tan(1/2*a)^2 - 6*tan(b*x
+ c)^2*tan(1/2*c) + 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 3*tan(1/2*a)^2*t
an(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 - 3*tan(1/2*a)*tan(1/2*c)^2 + 2*tan
(b*x + c) + 3*tan(1/2*a) - 3*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(
1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b*x + c)^4)
```

maple [B] time = 4.64, size = 321, normalized size = 5.35

$$\frac{(\cos(a)\sin(c)-\sin(a)\cos(c))\left(\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(a)\right)\left(\sin^2(c)\right)+\left(\cos^2(c)\right)\left(\sin^2(a)\right)+\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)}{4(\cos(a)\cos(c)+\sin(a)\sin(c))^4(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c))^4}-\frac{}{(\cos(a)\cos(c)+\sin(a)\sin(c))^4(\tan(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^5*sin(b*x+a),x)
```

```
[Out] 1/b*(1/4*(cos(a)*sin(c)-sin(a)*cos(c))*(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2
+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(
b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^
4-1/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*si
n(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(-3*cos(a)*sin(c)+3*sin(a)*cos
(c))/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2-1/3*(cos(a)^2*cos(c)^2+3*cos(a)
```

```
^2*sin(c)^2-4*cos(a)*cos(c)*sin(a)*sin(c)+3*cos(c)^2*sin(a)^2+sin(a)^2*sin(
c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*
sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^3)
```

maxima [B] time = 0.35, size = 1076, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*((6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x +
4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) - 4*(6*sin(4*b*x + 2*a + 4
*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*
cos(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a + 3*c) - sin(a + c))*cos(4*b*x +
2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin
(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(2*b*x
+ 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(2*b*x +
 a + 3*c) - sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*cos(a + c)
 - (6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c
) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*c)
 - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin
(6*b*x + a + 7*c) - 6*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(4*b*x + 2*a
 + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) - 4*cos(a + c)*sin(2
*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(
2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(2*b*x + 4*c)
 - (cos(2*a) + cos(2*c))*sin(a + c) + 4*cos(2*b*x + 2*a + 2*c)*sin(a + c))/
(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x +
a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 - 8*b*cos(2*b*x + a + 3*c)*cos(a +
 c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)
^2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 - 8*b*sin(2*
b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(4*b*cos(6*b*x + a + 7*c) -
6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(8*b
*x + a + 9*c) - 8*(6*b*cos(4*b*x + a + 5*c) - 4*b*cos(2*b*x + a + 3*c) + b*
cos(a + c))*cos(6*b*x + a + 7*c) - 12*(4*b*cos(2*b*x + a + 3*c) - b*cos(a +
 c))*cos(4*b*x + a + 5*c) - 2*(4*b*sin(6*b*x + a + 7*c) - 6*b*sin(4*b*x + a
 + 5*c) + 4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(8*b*x + a + 9*c) - 8
*(6*b*sin(4*b*x + a + 5*c) - 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6
*b*x + a + 7*c) - 12*(4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x +
a + 5*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^5,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.200 $\int \csc^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{8b}-\frac{\sin(a-c)\csc^5(bx+c)}{5b}-\frac{\cos(a-c)\cot(bx+c)\csc^3(bx+c)}{4b}-\frac{3\cos(a-c)\cot(\cdots}{8\cdots}$$

```
[Out] -3/8*arctanh(cos(b*x+c))*cos(a-c)/b-3/8*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1/
4*cos(a-c)*cot(b*x+c)*csc(b*x+c)^3/b-1/5*csc(b*x+c)^5*sin(a-c)/b
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3768, 3770}

$$-\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{8b}-\frac{\sin(a-c)\csc^5(bx+c)}{5b}-\frac{\cos(a-c)\cot(bx+c)\csc^3(bx+c)}{4b}-\frac{3\cos(a-c)\cot(\cdots}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(8*b) - (3*Cos[a - c]*Cot[c + b*x]*Cs
c[c + b*x])/(8*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c + b*x]^3)/(4*b) - (Csc[c
 + b*x]^5*Sin[a - c])/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^5(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^5(c+bx)\,dx$$

$$= -\frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} + \frac{1}{4}(3\cos(a-c))\int \csc^3(c+bx)\,dx - \frac{\text{si}}{}$$

$$= -\frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} -$$

$$= -\frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{8b} - \frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\text{co}}{}$$

Mathematica [A] time = 1.16, size = 79, normalized size = 0.84

$$-\frac{2\csc^5(bx+c)(5\cos(a-c)(14\sin(2(bx+c))-3\sin(4(bx+c)))+64\sin(a-c))+480\cos(a-c)\tanh^{-1}\left(\text{cos}\right.}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] -1/640*(480*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Csc[c + b*
x]^5*(64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] - 3*Sin[4*(c + b*x)
])))/b
```

fricas [B] time = 0.49, size = 197, normalized size = 2.10

$$-\frac{15\left(\cos(bx+c)^4\cos(-a+c) - 2\cos(bx+c)^2\cos(-a+c) + \cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)\sin(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*(cos(b*x + c)^4*cos(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(
-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 15*(cos(b*x + c)^4*cos(
-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*log(-1/2*cos(b*x + c)
 + 1/2)*sin(b*x + c) - 10*(3*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c)*co
s(-a + c))*sin(b*x + c) - 16*sin(-a + c))/((b*cos(b*x + c)^4 - 2*b*cos(b*x
+ c)^2 + b)*sin(b*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b*((-1048576/5*tan((b*x+c)/2)^5*ta
n(a/2)^10*tan(c/2)^9-4194304/5*tan((b*x+c)/2)^5*tan(a/2)^10*tan(c/2)^7-6291
456/5*tan((b*x+c)/2)^5*tan(a/2)^10*tan(c/2)^5-4194304/5*tan((b*x+c)/2)^5*ta
n(a/2)^10*tan(c/2)^3-1048576/5*tan((b*x+c)/2)^5*tan(a/2)^10*tan(c/2)+104857
6/5*tan((b*x+c)/2)^5*tan(a/2)^9*tan(c/2)^10+3145728/5*tan((b*x+c)/2)^5*tan(
a/2)^9*tan(c/2)^8+2097152/5*tan((b*x+c)/2)^5*tan(a/2)^9*tan(c/2)^6-2097152/
5*tan((b*x+c)/2)^5*tan(a/2)^9*tan(c/2)^4-3145728/5*tan((b*x+c)/2)^5*tan(a/2
)^9*tan(c/2)^2-1048576/5*tan((b*x+c)/2)^5*tan(a/2)^9-3145728/5*tan((b*x+c)/
2)^5*tan(a/2)^8*tan(c/2)^9-12582912/5*tan((b*x+c)/2)^5*tan(a/2)^8*tan(c/2)^
```

7-18874368/5*tan((b*x+c)/2)^5*tan(a/2)^8*tan(c/2)^5-12582912/5*tan((b*x+c)/
2)^5*tan(a/2)^8*tan(c/2)^3-3145728/5*tan((b*x+c)/2)^5*tan(a/2)^8*tan(c/2)+4
194304/5*tan((b*x+c)/2)^5*tan(a/2)^7*tan(c/2)^10+12582912/5*tan((b*x+c)/2)^
5*tan(a/2)^7*tan(c/2)^8+8388608/5*tan((b*x+c)/2)^5*tan(a/2)^7*tan(c/2)^6-83
88608/5*tan((b*x+c)/2)^5*tan(a/2)^7*tan(c/2)^4-12582912/5*tan((b*x+c)/2)^5*
tan(a/2)^7*tan(c/2)^2-4194304/5*tan((b*x+c)/2)^5*tan(a/2)^7-2097152/5*tan((
b*x+c)/2)^5*tan(a/2)^6*tan(c/2)^9-8388608/5*tan((b*x+c)/2)^5*tan(a/2)^6*tan
(c/2)^7-12582912/5*tan((b*x+c)/2)^5*tan(a/2)^6*tan(c/2)^5-8388608/5*tan((b*
x+c)/2)^5*tan(a/2)^6*tan(c/2)^3-2097152/5*tan((b*x+c)/2)^5*tan(a/2)^6*tan(c
/2)+6291456/5*tan((b*x+c)/2)^5*tan(a/2)^5*tan(c/2)^10+18874368/5*tan((b*x+c
)/2)^5*tan(a/2)^5*tan(c/2)^8+12582912/5*tan((b*x+c)/2)^5*tan(a/2)^5*tan(c/2
)^6-12582912/5*tan((b*x+c)/2)^5*tan(a/2)^5*tan(c/2)^4-18874368/5*tan((b*x+c
)/2)^5*tan(a/2)^5*tan(c/2)^2-6291456/5*tan((b*x+c)/2)^5*tan(a/2)^5+2097152/
5*tan((b*x+c)/2)^5*tan(a/2)^4*tan(c/2)^9+8388608/5*tan((b*x+c)/2)^5*tan(a/2
)^4*tan(c/2)^7+12582912/5*tan((b*x+c)/2)^5*tan(a/2)^4*tan(c/2)^5+8388608/5*
tan((b*x+c)/2)^5*tan(a/2)^4*tan(c/2)^3+2097152/5*tan((b*x+c)/2)^5*tan(a/2)^
4*tan(c/2)+4194304/5*tan((b*x+c)/2)^5*tan(a/2)^3*tan(c/2)^10+12582912/5*tan
((b*x+c)/2)^5*tan(a/2)^3*tan(c/2)^8+8388608/5*tan((b*x+c)/2)^5*tan(a/2)^3*t
an(c/2)^6-8388608/5*tan((b*x+c)/2)^5*tan(a/2)^3*tan(c/2)^4-12582912/5*tan((
b*x+c)/2)^5*tan(a/2)^3*tan(c/2)^2-4194304/5*tan((b*x+c)/2)^5*tan(a/2)^3+314
5728/5*tan((b*x+c)/2)^5*tan(a/2)^2*tan(c/2)^9+12582912/5*tan((b*x+c)/2)^5*t
an(a/2)^2*tan(c/2)^7+18874368/5*tan((b*x+c)/2)^5*tan(a/2)^2*tan(c/2)^5+1258
2912/5*tan((b*x+c)/2)^5*tan(a/2)^2*tan(c/2)^3+3145728/5*tan((b*x+c)/2)^5*ta
n(a/2)^2*tan(c/2)+1048576/5*tan((b*x+c)/2)^5*tan(a/2)*tan(c/2)^10+3145728/5
*tan((b*x+c)/2)^5*tan(a/2)*tan(c/2)^8+2097152/5*tan((b*x+c)/2)^5*tan(a/2)*t
an(c/2)^6-2097152/5*tan((b*x+c)/2)^5*tan(a/2)*tan(c/2)^4-3145728/5*tan((b*x
+c)/2)^5*tan(a/2)*tan(c/2)^2-1048576/5*tan((b*x+c)/2)^5*tan(a/2)+1048576/5*
tan((b*x+c)/2)^5*tan(c/2)^9+4194304/5*tan((b*x+c)/2)^5*tan(c/2)^7+6291456/5
*tan((b*x+c)/2)^5*tan(c/2)^5+4194304/5*tan((b*x+c)/2)^5*tan(c/2)^3+1048576/
5*tan((b*x+c)/2)^5*tan(c/2)+262144*tan((b*x+c)/2)^4*tan(a/2)^10*tan(c/2)^10
+786432*tan((b*x+c)/2)^4*tan(a/2)^10*tan(c/2)^8+524288*tan((b*x+c)/2)^4*tan
(a/2)^10*tan(c/2)^6-524288*tan((b*x+c)/2)^4*tan(a/2)^10*tan(c/2)^4-786432*t
an((b*x+c)/2)^4*tan(a/2)^10*tan(c/2)^2-262144*tan((b*x+c)/2)^4*tan(a/2)^10+
1048576*tan((b*x+c)/2)^4*tan(a/2)^9*tan(c/2)^9+4194304*tan((b*x+c)/2)^4*tan
(a/2)^9*tan(c/2)^7+6291456*tan((b*x+c)/2)^4*tan(a/2)^9*tan(c/2)^5+4194304*t
an((b*x+c)/2)^4*tan(a/2)^9*tan(c/2)^3+1048576*tan((b*x+c)/2)^4*tan(a/2)^9*t
an(c/2)+786432*tan((b*x+c)/2)^4*tan(a/2)^8*tan(c/2)^10+2359296*tan((b*x+c)/
2)^4*tan(a/2)^8*tan(c/2)^8+1572864*tan((b*x+c)/2)^4*tan(a/2)^8*tan(c/2)^6-1
572864*tan((b*x+c)/2)^4*tan(a/2)^8*tan(c/2)^4-2359296*tan((b*x+c)/2)^4*tan(
a/2)^8*tan(c/2)^2-786432*tan((b*x+c)/2)^4*tan(a/2)^8+4194304*tan((b*x+c)/2)
^4*tan(a/2)^7*tan(c/2)^9+16777216*tan((b*x+c)/2)^4*tan(a/2)^7*tan(c/2)^7+25
165824*tan((b*x+c)/2)^4*tan(a/2)^7*tan(c/2)^5+16777216*tan((b*x+c)/2)^4*tan
(a/2)^7*tan(c/2)^3+4194304*tan((b*x+c)/2)^4*tan(a/2)^7*tan(c/2)+524288*tan(
(b*x+c)/2)^4*tan(a/2)^6*tan(c/2)^10+1572864*tan((b*x+c)/2)^4*tan(a/2)^6*tan
(c/2)^8+1048576*tan((b*x+c)/2)^4*tan(a/2)^6*tan(c/2)^6-1048576*tan((b*x+c)/
2)^4*tan(a/2)^6*tan(c/2)^4-1572864*tan((b*x+c)/2)^4*tan(a/2)^6*tan(c/2)^2-5
24288*tan((b*x+c)/2)^4*tan(a/2)^6+6291456*tan((b*x+c)/2)^4*tan(a/2)^5*tan(c
/2)^9+25165824*tan((b*x+c)/2)^4*tan(a/2)^5*tan(c/2)^7+37748736*tan((b*x+c)/
2)^4*tan(a/2)^5*tan(c/2)^5+25165824*tan((b*x+c)/2)^4*tan(a/2)^5*tan(c/2)^3+
6291456*tan((b*x+c)/2)^4*tan(a/2)^5*tan(c/2)-524288*tan((b*x+c)/2)^4*tan(a/
2)^4*tan(c/2)^10-1572864*tan((b*x+c)/2)^4*tan(a/2)^4*tan(c/2)^8-1048576*tan
((b*x+c)/2)^4*tan(a/2)^4*tan(c/2)^6+1048576*tan((b*x+c)/2)^4*tan(a/2)^4*tan
(c/2)^4+1572864*tan((b*x+c)/2)^4*tan(a/2)^4*tan(c/2)^2+524288*tan((b*x+c)/2
)^4*tan(a/2)^4+4194304*tan((b*x+c)/2)^4*tan(a/2)^3*tan(c/2)^9+16777216*tan(
(b*x+c)/2)^4*tan(a/2)^3*tan(c/2)^7+25165824*tan((b*x+c)/2)^4*tan(a/2)^3*tan
(c/2)^5+16777216*tan((b*x+c)/2)^4*tan(a/2)^3*tan(c/2)^3+4194304*tan((b*x+c)
/2)^4*tan(a/2)^3*tan(c/2)-786432*tan((b*x+c)/2)^4*tan(a/2)^2*tan(c/2)^10-23
59296*tan((b*x+c)/2)^4*tan(a/2)^2*tan(c/2)^8-1572864*tan((b*x+c)/2)^4*tan(a
/2)^2*tan(c/2)^6+1572864*tan((b*x+c)/2)^4*tan(a/2)^2*tan(c/2)^4+2359296*tan

((b*x+c)/2)^4*tan(a/2)^2*tan(c/2)^2+786432*tan((b*x+c)/2)^4*tan(a/2)^2+1048
576*tan((b*x+c)/2)^4*tan(a/2)*tan(c/2)^9+4194304*tan((b*x+c)/2)^4*tan(a/2)*
tan(c/2)^7+6291456*tan((b*x+c)/2)^4*tan(a/2)*tan(c/2)^5+4194304*tan((b*x+c)
/2)^4*tan(a/2)*tan(c/2)^3+1048576*tan((b*x+c)/2)^4*tan(a/2)*tan(c/2)-262144
*tan((b*x+c)/2)^4*tan(c/2)^10-786432*tan((b*x+c)/2)^4*tan(c/2)^8-524288*tan
((b*x+c)/2)^4*tan(c/2)^6+524288*tan((b*x+c)/2)^4*tan(c/2)^4+786432*tan((b*x
+c)/2)^4*tan(c/2)^2+262144*tan((b*x+c)/2)^4-1048576*tan((b*x+c)/2)^3*tan(a/
2)^10*tan(c/2)^9-4194304*tan((b*x+c)/2)^3*tan(a/2)^10*tan(c/2)^7-6291456*ta
n((b*x+c)/2)^3*tan(a/2)^10*tan(c/2)^5-4194304*tan((b*x+c)/2)^3*tan(a/2)^10*
tan(c/2)^3-1048576*tan((b*x+c)/2)^3*tan(a/2)^10*tan(c/2)+1048576*tan((b*x+c
)/2)^3*tan(a/2)^9*tan(c/2)^10+3145728*tan((b*x+c)/2)^3*tan(a/2)^9*tan(c/2)^
8+2097152*tan((b*x+c)/2)^3*tan(a/2)^9*tan(c/2)^6-2097152*tan((b*x+c)/2)^3*t
an(a/2)^9*tan(c/2)^4-3145728*tan((b*x+c)/2)^3*tan(a/2)^9*tan(c/2)^2-1048576
*tan((b*x+c)/2)^3*tan(a/2)^9-3145728*tan((b*x+c)/2)^3*tan(a/2)^8*tan(c/2)^9
-12582912*tan((b*x+c)/2)^3*tan(a/2)^8*tan(c/2)^7-18874368*tan((b*x+c)/2)^3*
tan(a/2)^8*tan(c/2)^5-12582912*tan((b*x+c)/2)^3*tan(a/2)^8*tan(c/2)^3-31457
28*tan((b*x+c)/2)^3*tan(a/2)^8*tan(c/2)+4194304*tan((b*x+c)/2)^3*tan(a/2)^7
*tan(c/2)^10+12582912*tan((b*x+c)/2)^3*tan(a/2)^7*tan(c/2)^8+8388608*tan((b
*x+c)/2)^3*tan(a/2)^7*tan(c/2)^6-8388608*tan((b*x+c)/2)^3*tan(a/2)^7*tan(c/
2)^4-12582912*tan((b*x+c)/2)^3*tan(a/2)^7*tan(c/2)^2-4194304*tan((b*x+c)/2)
^3*tan(a/2)^7-2097152*tan((b*x+c)/2)^3*tan(a/2)^6*tan(c/2)^9-8388608*tan((b
*x+c)/2)^3*tan(a/2)^6*tan(c/2)^7-12582912*tan((b*x+c)/2)^3*tan(a/2)^6*tan(c
/2)^5-8388608*tan((b*x+c)/2)^3*tan(a/2)^6*tan(c/2)^3-2097152*tan((b*x+c)/2)
^3*tan(a/2)^6*tan(c/2)+6291456*tan((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^10+1887
4368*tan((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^8+12582912*tan((b*x+c)/2)^3*tan(a
/2)^5*tan(c/2)^6-12582912*tan((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^4-18874368*t
an((b*x+c)/2)^3*tan(a/2)^5*tan(c/2)^2-6291456*tan((b*x+c)/2)^3*tan(a/2)^5+2
097152*tan((b*x+c)/2)^3*tan(a/2)^4*tan(c/2)^9+8388608*tan((b*x+c)/2)^3*tan(
a/2)^4*tan(c/2)^7+12582912*tan((b*x+c)/2)^3*tan(a/2)^4*tan(c/2)^5+8388608*t
an((b*x+c)/2)^3*tan(a/2)^4*tan(c/2)^3+2097152*tan((b*x+c)/2)^3*tan(a/2)^4*t
an(c/2)+4194304*tan((b*x+c)/2)^3*tan(a/2)^3*tan(c/2)^10+12582912*tan((b*x+c
)/2)^3*tan(a/2)^3*tan(c/2)^8+8388608*tan((b*x+c)/2)^3*tan(a/2)^3*tan(c/2)^6
-8388608*tan((b*x+c)/2)^3*tan(a/2)^3*tan(c/2)^4-12582912*tan((b*x+c)/2)^3*t
an(a/2)^3*tan(c/2)^2-4194304*tan((b*x+c)/2)^3*tan(a/2)^3+3145728*tan((b*x+c
)/2)^3*tan(a/2)^2*tan(c/2)^9+12582912*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)^
7+18874368*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)^5+12582912*tan((b*x+c)/2)^3
*tan(a/2)^2*tan(c/2)^3+3145728*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)+1048576
*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)^10+3145728*tan((b*x+c)/2)^3*tan(a/2)*ta
n(c/2)^8+2097152*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)^6-2097152*tan((b*x+c)/2
)^3*tan(a/2)*tan(c/2)^4-3145728*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)^2-104857
6*tan((b*x+c)/2)^3*tan(a/2)+1048576*tan((b*x+c)/2)^3*tan(c/2)^9+4194304*tan
((b*x+c)/2)^3*tan(c/2)^7+6291456*tan((b*x+c)/2)^3*tan(c/2)^5+4194304*tan((b
*x+c)/2)^3*tan(c/2)^3+1048576*tan((b*x+c)/2)^3*tan(c/2)+2097152*tan((b*x+c)
/2)^2*tan(a/2)^10*tan(c/2)^10+6291456*tan((b*x+c)/2)^2*tan(a/2)^10*tan(c/2)
^8+4194304*tan((b*x+c)/2)^2*tan(a/2)^10*tan(c/2)^6-4194304*tan((b*x+c)/2)^2
*tan(a/2)^10*tan(c/2)^4-6291456*tan((b*x+c)/2)^2*tan(a/2)^10*tan(c/2)^2-209
7152*tan((b*x+c)/2)^2*tan(a/2)^10+8388608*tan((b*x+c)/2)^2*tan(a/2)^9*tan(c
/2)^9+33554432*tan((b*x+c)/2)^2*tan(a/2)^9*tan(c/2)^7+50331648*tan((b*x+c)/
2)^2*tan(a/2)^9*tan(c/2)^5+33554432*tan((b*x+c)/2)^2*tan(a/2)^9*tan(c/2)^3+
8388608*tan((b*x+c)/2)^2*tan(a/2)^9*tan(c/2)+6291456*tan((b*x+c)/2)^2*tan(a
/2)^8*tan(c/2)^10+18874368*tan((b*x+c)/2)^2*tan(a/2)^8*tan(c/2)^8+12582912*
tan((b*x+c)/2)^2*tan(a/2)^8*tan(c/2)^6-12582912*tan((b*x+c)/2)^2*tan(a/2)^8
*tan(c/2)^4-18874368*tan((b*x+c)/2)^2*tan(a/2)^8*tan(c/2)^2-6291456*tan((b*
x+c)/2)^2*tan(a/2)^8+33554432*tan((b*x+c)/2)^2*tan(a/2)^7*tan(c/2)^9+134217
728*tan((b*x+c)/2)^2*tan(a/2)^7*tan(c/2)^7+201326592*tan((b*x+c)/2)^2*tan(a
/2)^7*tan(c/2)^5+134217728*tan((b*x+c)/2)^2*tan(a/2)^7*tan(c/2)^3+33554432*
tan((b*x+c)/2)^2*tan(a/2)^7*tan(c/2)+4194304*tan((b*x+c)/2)^2*tan(a/2)^6*ta
n(c/2)^10+12582912*tan((b*x+c)/2)^2*tan(a/2)^6*tan(c/2)^8+8388608*tan((b*x+
c)/2)^2*tan(a/2)^6*tan(c/2)^6-8388608*tan((b*x+c)/2)^2*tan(a/2)^6*tan(c/2)^

```
4-12582912*tan((b*x+c)/2)^2*tan(a/2)^6*tan(c/2)^2-4194304*tan((b*x+c)/2)^2*
tan(a/2)^6+50331648*tan((b*x+c)/2)^2*tan(a/2)^5*tan(c/2)^9+201326592*tan((b
*x+c)/2)^2*tan(a/2)^5*tan(c/2)^7+301989888*tan((b*x+c)/2)^2*tan(a/2)^5*tan(
c/2)^5+201326592*tan((b*x+c)/2)^2*tan(a/2)^5*tan(c/2)^3+50331648*tan((b*x+c
)/2)^2*tan(a/2)^5*tan(c/2)-4194304*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2)^10-
12582912*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2)^8-8388608*tan((b*x+c)/2)^2*ta
n(a/2)^4*tan(c/2)^6+8388608*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2)^4+12582912
*tan((b*x+c)/2)^2*tan(a/2)^4*tan(c/2)^2+4194304*tan((b*x+c)/2)^2*tan(a/2)^4
+33554432*tan((b*x+c)/2)^2*tan(a/2)^3*tan(c/2)^9+134217728*tan((b*x+c)/2)^2
*tan(a/2)^3*tan(c/2)^7+201326592*tan((b*x+c)/2)^2*tan(a/2)^3*tan(c/2)^5+134
217728*tan((b*x+c)/2)^2*tan(a/2)^3*tan(c/2)^3+33554432*tan((b*x+c)/2)^2*tan
(a/2)^3*tan(c/2)-6291456*tan((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)^10-18874368*t
an((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)^8-12582912*tan((b*x+c)/2)^2*tan(a/2)^2*
tan(c/2)^6+12582912*tan((b*x+c)/2)^2*tan(a/2)^2*tan(c/2)^4+18874368*tan((b*
x+c)/2)^2*tan(a/2)^2*tan(c/2)^2+6291456*tan((b*x+c)/2)^2*tan(a/2)^2+8388608
*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)^9+33554432*tan((b*x+c)/2)^2*tan(a/2)*ta
n(c/2)^7+50331648*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)^5+33554432*tan((b*x+c)
/2)^2*tan(a/2)*tan(c/2)^3+8388608*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)-209715
2*tan((b*x+c)/2)^2*tan(c/2)^10-6291456*tan((b*x+c)/2)^2*tan(c/2)^8-4194304*
tan((b*x+c)/2)^2*tan(c/2)^6+4194304*tan((b*x+c)/2)^2*tan(c/2)^4+6291456*tan
((b*x+c)/2)^2*tan(c/2)^2+2097152*tan((b*x+c)/2)^2-2097152*tan((b*x+c)/2)*ta
n(a/2)^10*tan(c/2)^9-8388608*tan((b*x+c)/2)*tan(a/2)^10*tan(c/2)^7-12582912
*tan((b*x+c)/2)*tan(a/2)^10*tan(c/2)^5-8388608*tan((b*x+c)/2)*tan(a/2)^10*t
an(c/2)^3-2097152*tan((b*x+c)/2)*tan(a/2)^10*tan(c/2)+2097152*tan((b*x+c)/2
)*tan(a/2)^9*tan(c/2)^10+6291456*tan((b*x+c)/2)*tan(a/2)^9*tan(c/2)^8+41943
04*tan((b*x+c)/2)*tan(a/2)^9*tan(c/2)^6-4194304*tan((b*x+c)/2)*tan(a/2)^9*t
an(c/2)^4-6291456*tan((b*x+c)/2)*tan(a/2)^9*tan(c/2)^2-2097152*tan((b*x+c)/
2)*tan(a/2)^9-6291456*tan((b*x+c)/2)*tan(a/2)^8*tan(c/2)^9-25165824*tan((b*
x+c)/2)*tan(a/2)^8*tan(c/2)^7-37748736*tan((b*x+c)/2)*tan(a/2)^8*tan(c/2)^5
-25165824*tan((b*x+c)/2)*tan(a/2)^8*tan(c/2)^3-6291456*tan((b*x+c)/2)*tan(a
/2)^8*tan(c/2)+8388608*tan((b*x+c)/2)*tan(a/2)^7*tan(c/2)^10+25165824*tan((
b*x+c)/2)*tan(a/2)^7*tan(c/2)^8+16777216*tan((b*x+c)/2)*tan(a/2)^7*tan(c/2)
^6-16777216*tan((b*x+c)/2)*tan(a/2)^7*tan(c/2)^4-25165824*tan((b*x+c)/2)*ta
n(a/2)^7*tan(c/2)^2-8388608*tan((b*x+c)/2)*tan(a/2)^7-4194304*tan((b*x+c)/2
)*tan(a/2)^6*tan(c/2)^9-16777216*tan((b*x+c)/2)*tan(a/2)^6*tan(c/2)^7-25165
824*tan((b*x+c)/2)*tan(a/2)^6*tan(c/2)^5-16777216*tan((b*x+c)/2)*tan(a/2)^6
*tan(c/2)^3-4194304*tan((b*x+c)/2)*tan(a/2)^6*tan(c/2)+12582912*tan((b*x+c)
/2)*tan(a/2)^5*tan(c/2)^10+37748736*tan((b*x+c)/2)*tan(a/2)^5*tan(c/2)^8+25
165824*tan((b*x+c)/2)*tan(a/2)^5*tan(c/2)^6-25165824*tan((b*x+c)/2)*tan(a/2
)^5*tan(c/2)^4-37748736*tan((b*x+c)/2)*tan(a/2)^5*tan(c/2)^2-12582912*tan((
b*x+c)/2)*tan(a/2)^5+4194304*tan((b*x+c)/2)*tan(a/2)^4*tan(c/2)^9+16777216*
tan((b*x+c)/2)*tan(a/2)^4*tan(c/2)^7+25165824*tan((b*x+c)/2)*tan(a/2)^4*tan
(c/2)^5+16777216*tan((b*x+c)/2)*tan(a/2)^4*tan(c/2)^3+4194304*tan((b*x+c)/2
)*tan(a/2)^4*tan(c/2)+8388608*tan((b*x+c)/2)*tan(a/2)^3*tan(c/2)^10+2516582
4*tan((b*x+c)/2)*tan(a/2)^3*tan(c/2)^8+16777216*tan((b*x+c)/2)*tan(a/2)^3*t
an(c/2)^6-16777216*tan((b*x+c)/2)*tan(a/2)^3*tan(c/2)^4-25165824*tan((b*x+c
)/2)*tan(a/2)^3*tan(c/2)^2-8388608*tan((b*x+c)/2)*tan(a/2)^3+6291456*tan((b
*x+c)/2)*tan(a/2)^2*tan(c/2)^9+25165824*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)^
7+37748736*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)^5+25165824*tan((b*x+c)/2)*tan
(a/2)^2*tan(c/2)^3+6291456*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)+2097152*tan((
b*x+c)/2)*tan(a/2)*tan(c/2)^10+6291456*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^8+4
194304*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^6-4194304*tan((b*x+c)/2)*tan(a/2)*t
an(c/2)^4-6291456*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^2-2097152*tan((b*x+c)/2)
*tan(a/2)+2097152*tan((b*x+c)/2)*tan(c/2)^9+8388608*tan((b*x+c)/2)*tan(c/2)
^7+12582912*tan((b*x+c)/2)*tan(c/2)^5+8388608*tan((b*x+c)/2)*tan(c/2)^3+209
7152*tan((b*x+c)/2)*tan(c/2))/(33554432*tan(a/2)^10*tan(c/2)^10+167772160*t
an(a/2)^10*tan(c/2)^8+335544320*tan(a/2)^10*tan(c/2)^6+335544320*tan(a/2)^1
0*tan(c/2)^4+167772160*tan(a/2)^10*tan(c/2)^2+33554432*tan(a/2)^10+16777216
0*tan(a/2)^8*tan(c/2)^10+838860800*tan(a/2)^8*tan(c/2)^8+1677721600*tan(a/2
```

```
)^8*tan(c/2)^6+1677721600*tan(a/2)^8*tan(c/2)^4+838860800*tan(a/2)^8*tan(c/
2)^2+167772160*tan(a/2)^8+335544320*tan(a/2)^6*tan(c/2)^10+1677721600*tan(a
/2)^6*tan(c/2)^8+3355443200*tan(a/2)^6*tan(c/2)^6+3355443200*tan(a/2)^6*tan
(c/2)^4+1677721600*tan(a/2)^6*tan(c/2)^2+335544320*tan(a/2)^6+335544320*tan
(a/2)^4*tan(c/2)^10+1677721600*tan(a/2)^4*tan(c/2)^8+3355443200*tan(a/2)^4*
tan(c/2)^6+3355443200*tan(a/2)^4*tan(c/2)^4+1677721600*tan(a/2)^4*tan(c/2)^
2+335544320*tan(a/2)^4+167772160*tan(a/2)^2*tan(c/2)^10+838860800*tan(a/2)^
2*tan(c/2)^8+1677721600*tan(a/2)^2*tan(c/2)^6+1677721600*tan(a/2)^2*tan(c/2
)^4+838860800*tan(a/2)^2*tan(c/2)^2+167772160*tan(a/2)^2+33554432*tan(c/2)^
10+167772160*tan(c/2)^8+335544320*tan(c/2)^6+335544320*tan(c/2)^4+167772160
*tan(c/2)^2+33554432)+(-274*tan((b*x+c)/2)^5*tan(a/2)^2*tan(c/2)^2+274*tan(
(b*x+c)/2)^5*tan(a/2)^2-1096*tan((b*x+c)/2)^5*tan(a/2)*tan(c/2)+274*tan((b*
x+c)/2)^5*tan(c/2)^2-274*tan((b*x+c)/2)^5-40*tan((b*x+c)/2)^4*tan(a/2)^2*ta
n(c/2)+40*tan((b*x+c)/2)^4*tan(a/2)*tan(c/2)^2-40*tan((b*x+c)/2)^4*tan(a/2)
+40*tan((b*x+c)/2)^4*tan(c/2)-40*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)^2+40*
tan((b*x+c)/2)^3*tan(a/2)^2-160*tan((b*x+c)/2)^3*tan(a/2)*tan(c/2)+40*tan((
b*x+c)/2)^3*tan(c/2)^2-40*tan((b*x+c)/2)^3-20*tan((b*x+c)/2)^2*tan(a/2)^2*t
an(c/2)+20*tan((b*x+c)/2)^2*tan(a/2)*tan(c/2)^2-20*tan((b*x+c)/2)^2*tan(a/2
)+20*tan((b*x+c)/2)^2*tan(c/2)-5*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)^2+5*tan
((b*x+c)/2)*tan(a/2)^2-20*tan((b*x+c)/2)*tan(a/2)*tan(c/2)+5*tan((b*x+c)/2)
*tan(c/2)^2-5*tan((b*x+c)/2)-4*tan(a/2)^2*tan(c/2)+4*tan(a/2)*tan(c/2)^2-4*
tan(a/2)+4*tan(c/2))/(640*tan(a/2)^2*tan(c/2)^2+640*tan(a/2)^2+640*tan(c/2)
^2+640)/tan((b*x+c)/2)^5+(3*tan(a/2)^2*tan(c/2)^2-3*tan(a/2)^2+12*tan(a/2)*
tan(c/2)-3*tan(c/2)^2+3)/(16*tan(a/2)^2*tan(c/2)^2+16*tan(a/2)^2+16*tan(c/2
)^2+16)*ln(abs(tan((b*x+c)/2))))
```

maple [B] time = 14.96, size = 97954, normalized size = 1042.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^6*sin(b*x+a),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.57, size = 3879, normalized size = 41.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/80*(2*(15*cos(9*b*x + 2*a + 8*c) + 15*cos(9*b*x + 10*c) - 70*cos(7*b*x +
2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b
*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
2*a) - 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*cos(8*b*x + a + 8*
c) - 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a +
2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) - 10*cos
(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c) + cos(
a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) + 70*cos(7*b*x + 8*c)
 + 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a +
 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8*
b*x + a + 8*c) - 140*(10*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5
*cos(2*b*x + a + 2*c) - cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x
+ a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) - cos(a))*cos
(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*
cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b
*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*cos(4*b*x + a + 4*c) - 5*cos(2*b*
x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*c
) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x +
```

```
2*a + 2*c) + 14*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*co
s(4*b*x + a + 4*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a
+ 2*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 150*(cos(
b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 30*cos(b*x + 2*a)*cos(a
) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*cos(-a + c) + 2
5*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6*b*x + a + 6*c)^2*cos(-a +
c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25*cos(2*b*x + a + 2*c)^2*cos
(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(10*
b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a + 8*c)^2 + 100*cos(-a + c)
*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 25*cos(-
a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
- 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos(6*b*x + a + 6*c)*cos(-a +
c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a +
c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c) - 10*(10*cos(6*b*x + a + 6*
c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a + c) + 5*cos(2*b*x + a + 2*
c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x + a + 8*c) - 20*(10*cos(4*b*
x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(
-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x + a + 2*c)*cos(-a + c) - co
s(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c)
- 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos(-a + c)*sin(6*b*x + a + 6*
c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*
c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c) - 10*(10*cos(-a + c)*sin(6*
b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a + 4*c) + 5*cos(-a + c)*sin(2*
b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x + a + 8*c) - 20*(10*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 20*(5*cos(-a + c)*sin(2*b*x + a + 2*c) - co
s(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
+ cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + 15*(cos(10*b*x +
a + 10*c)^2*cos(-a + c) + 25*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6
*b*x + a + 6*c)^2*cos(-a + c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25
*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a
 + c) + cos(-a + c)*sin(10*b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a
 + 8*c)^2 + 100*cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*
b*x + a + 4*c)^2 + 25*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*s
in(2*b*x + a + 2*c)*sin(a) - 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos
(6*b*x + a + 6*c)*cos(-a + c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos
(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c)
- 10*(10*cos(6*b*x + a + 6*c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a
+ c) + 5*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x +
 a + 8*c) - 20*(10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c
)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x
+ a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)
^2 + sin(a)^2)*cos(-a + c) - 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos
(-a + c)*sin(6*b*x + a + 6*c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos
(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c)
- 10*(10*cos(-a + c)*sin(6*b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a +
4*c) + 5*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x +
 a + 8*c) - 20*(10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b
*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x + a + 6*c) - 20*(5*cos(-a + c
)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2) + 2*(15*sin(9*b*x + 2*a + 8*c) + 15*sin(9*b*x + 10*c) - 70*sin(7*b*
x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) + 128*sin
(5*b*x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*
x + 2*a) - 15*sin(b*x + 2*c))*sin(10*b*x + a + 10*c) - 30*(5*sin(8*b*x + a
+ 8*c) - 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) - 5*sin(2*b*x +
a + 2*c) + sin(a))*sin(9*b*x + 2*a + 8*c) - 30*(5*sin(8*b*x + a + 8*c) - 10
*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) - 5*sin(2*b*x + a + 2*c) +
sin(a))*sin(9*b*x + 10*c) + 10*(70*sin(7*b*x + 2*a + 6*c) + 70*sin(7*b*x +
```

```
8*c) + 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2
*a + 2*c) - 70*sin(3*b*x + 4*c) + 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*si
n(8*b*x + a + 8*c) - 140*(10*sin(6*b*x + a + 6*c) - 10*sin(4*b*x + a + 4*c)
 + 5*sin(2*b*x + a + 2*c) - sin(a))*sin(7*b*x + 2*a + 6*c) - 140*(10*sin(6*
b*x + a + 6*c) - 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) - sin(a))
*sin(7*b*x + 8*c) - 20*(128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) -
 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*b*x + 4*c) + 15*sin(b*x + 2*a) + 15*s
in(b*x + 2*c))*sin(6*b*x + a + 6*c) + 256*(10*sin(4*b*x + a + 4*c) - 5*sin(
2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c) - 256*(10*sin(4*b*x + a +
 4*c) - 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 100*(14*sin(3*b
*x + 2*a + 2*c) + 14*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c)
)*sin(4*b*x + a + 4*c) + 140*(5*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*x +
2*a + 2*c) + 140*(5*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*x + 4*c) - 150*(
sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c) + 30*sin(b*x + 2*a)*s
in(a) + 30*sin(b*x + 2*c)*sin(a))/(b*cos(10*b*x + a + 10*c)^2 + 25*b*cos(8*
b*x + a + 8*c)^2 + 100*b*cos(6*b*x + a + 6*c)^2 + 100*b*cos(4*b*x + a + 4*c
)^2 + 25*b*cos(2*b*x + a + 2*c)^2 - 10*b*cos(2*b*x + a + 2*c)*cos(a) + b*si
n(10*b*x + a + 10*c)^2 + 25*b*sin(8*b*x + a + 8*c)^2 + 100*b*sin(6*b*x + a
+ 6*c)^2 + 100*b*sin(4*b*x + a + 4*c)^2 + 25*b*sin(2*b*x + a + 2*c)^2 - 10*
b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b - 2*(5*b*cos(8*b*x
+ a + 8*c) - 10*b*cos(6*b*x + a + 6*c) + 10*b*cos(4*b*x + a + 4*c) - 5*b*co
s(2*b*x + a + 2*c) + b*cos(a))*cos(10*b*x + a + 10*c) - 10*(10*b*cos(6*b*x
+ a + 6*c) - 10*b*cos(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) - b*cos(a
))*cos(8*b*x + a + 8*c) - 20*(10*b*cos(4*b*x + a + 4*c) - 5*b*cos(2*b*x + a
 + 2*c) + b*cos(a))*cos(6*b*x + a + 6*c) - 20*(5*b*cos(2*b*x + a + 2*c) - b
*cos(a))*cos(4*b*x + a + 4*c) - 2*(5*b*sin(8*b*x + a + 8*c) - 10*b*sin(6*b*
x + a + 6*c) + 10*b*sin(4*b*x + a + 4*c) - 5*b*sin(2*b*x + a + 2*c) + b*sin
(a))*sin(10*b*x + a + 10*c) - 10*(10*b*sin(6*b*x + a + 6*c) - 10*b*sin(4*b*
x + a + 4*c) + 5*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(8*b*x + a + 8*c) -
20*(10*b*sin(4*b*x + a + 4*c) - 5*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(6*
b*x + a + 6*c) - 20*(5*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(4*b*x + a + 4
*c))
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^6,x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.201 $\int \sin^2(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=410

$$-\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(-i(2a+cn)-}{2b+dn}$$

```
[Out] -I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I
*exp(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],exp(2*I*(d*x+c)
))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-d*n
+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, b
/d-1/2*n],[1+b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+
2*b)+I*2^(-1-n)*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, -1/2*n
],[1-1/2*n],exp(2*I*(d*x+c)))/d/((1-exp(2*I*(d*x+c)))^n)/n
```

Rubi [A] time = 0.97, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {4553, 2282, 1980, 2032, 365, 364, 2285, 2253, 2252, 2251}

$$-\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(-i(2a+cn)-}{2b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(I/
E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2,
-n, (2 - (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*
b)/d - n)/2, -n, (2 + (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(I/E^(I*(c + d*x)) - I*E^(I*
(c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^((2*I)*(c + d*x))])/(d
*(1 - E^((2*I)*(c + d*x)))^n*n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = 2^{-2-n}\int \left(2\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - e^{2ia+2ibx}\left(ie\right.\right.$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\left(ie^{-i(c+dx)}-i\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{i(-1+x^2)}{x}\right)^n}{x}dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(\right.\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(\frac{i}{x}-ix\right)^n}{x}dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(\right.\right.$$

$$= -\left(\left(2^{-2-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(2a-cn)+i(2b-dn)x}\left(1\right.\right.$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}\right.}{2b+dn}$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}\right.}{2b+dn}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
[Out] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n, x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sin(dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(dx+c)^n \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

maple [F] time = 7.18, size = 0, normalized size = 0.00

$$\int \left(\sin^2(bx+a)\right)\left(\sin^n(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(dx+c)^n \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+b\,x)^2 \sin(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^n,x)
```

```
[Out] int(sin(a + b*x)^2*sin(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

3.202 $\int \sin^2(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

```
[Out] -1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2638}

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -Cos[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Cos[c + d*x]/(2*d) + Cos[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{4}\sin(2a-c+(2b-d)x)+\frac{1}{2}\sin(c+dx)-\frac{1}{4}\sin(2a+c+(2b+d)x)\right)dx\\
&= \frac{1}{4}\int \sin(2a-c+(2b-d)x)\,dx-\frac{1}{4}\int \sin(2a+c+(2b+d)x)\,dx+\frac{1}{2}\int \sin(c+\\
&= -\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(c+dx)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 80, normalized size = 1.18

$$-\frac{\cos(2a+2bx-c-dx)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}+\frac{1}{2}\left(\frac{\sin(c)\sin(dx)}{d}-\frac{\cos(c)\cos(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -1/4*Cos[2*a - c + 2*b*x - d*x]/(2*b - d) + Cos[2*a + c + (2*b + d)*x]/(4*(
2*b + d)) + (-((Cos[c]*Cos[d*x])/d) + (Sin[c]*Sin[d*x])/d)/2
```

fricas [A] time = 0.46, size = 69, normalized size = 1.01

$$-\frac{2\,bd\cos(bx+a)\sin(bx+a)\sin(dx+c)+\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\cos(dx+c)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*sin(b*x + a)*sin(d*x + c) + (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*cos(d*x + c))/(4*b^2*d - d^3)
```

giac [A] time = 0.20, size = 61, normalized size = 0.90

$$\frac{\cos\left(2\,bx + dx + 2\,a + c\right)}{4\,(2\,b + d)} - \frac{\cos\left(2\,bx - dx + 2\,a - c\right)}{4\,(2\,b - d)} - \frac{\cos\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*cos(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*cos(2*b*x - d*x + 2*a - c)/(
2*b - d) - 1/2*cos(d*x + c)/d
```

maple [A] time = 0.15, size = 63, normalized size = 0.93

$$-\frac{\cos\left(2a - c + (2b - d)\,x\right)}{4\,(2b - d)} - \frac{\cos\left(dx + c\right)}{2d} + \frac{\cos\left(2a + c + (2b + d)\,x\right)}{8b + 4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c),x)
```

```
[Out] -1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x)
/(2*b+d)
```

maxima [B] time = 0.35, size = 371, normalized size = 5.46

$$-\frac{\left(2\,bd\cos(c) - d^2\cos(c)\right)\cos\left((2\,b + d)x + 2\,a + 2\,c\right) + \left(2\,bd\cos(c) - d^2\cos(c)\right)\cos\left((2\,b + d)x + 2\,a\right) - \left(2\,bd\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*cos(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a + 2*c) + (2*b*d*cos
(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a) - (2*b*d*cos(c) + d^2*cos(c))*cos(
-(2*b - d)*x - 2*a + 2*c) - (2*b*d*cos(c) + d^2*cos(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*cos(d*x + 2*c) - 2*(4*b^2*cos(c) - d^2
*cos(c))*cos(d*x) + (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^
2*sin(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*sin(-
(2*b - d)*x - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x + 2*c) + 2*(4*b^
2*sin(c) - d^2*sin(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

mupad [B] time = 0.75, size = 105, normalized size = 1.54

$$-\frac{d^2\,\cos\left(2\,a + c + 2\,b\,x + d\,x\right) - b\,\left(2\,d\,\cos\left(2\,a + c + 2\,b\,x + d\,x\right) - 2\,d\,\cos\left(2\,a - c + 2\,b\,x - d\,x\right)\right) + d^2\,\cos\left(2\,a\right.}{16\,b^2\,d - 4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x),x)
```

```
[Out] - (d^2*cos(2*a + c + 2*b*x + d*x) - b*(2*d*cos(2*a + c + 2*b*x + d*x) - 2*d
*cos(2*a - c + 2*b*x - d*x)) + d^2*cos(2*a - c + 2*b*x - d*x))/(16*b^2*d -
4*d^3) - cos(c + d*x)/(2*d)
```

sympy [A] time = 6.51, size = 408, normalized size = 6.00

$$\begin{cases} x\sin^2(a)\sin(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{4} - \frac{x\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a-\frac{dx}{2}\right)}{4} - \frac{\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{d} - \frac{\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{4} + \frac{x\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a+\frac{dx}{2}\right)}{4} - \frac{\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{d} + \frac{\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos}{2d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\sin(c) \\ -\frac{2b^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2b^2\cos^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\sin(c+dx)\cos(a+bx)}{4b^2d-d^3} + \frac{d^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*s
in(c + d*x)/4 - x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/2 - x*sin(c +
d*x)*cos(a - d*x/2)**2/4 - sin(a - d*x/2)**2*cos(c + d*x)/d - sin(a - d*x/2
)*sin(c + d*x)*cos(a - d*x/2)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2*sin
(c + d*x)/4 + x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/2 - x*sin(c + d*
x)*cos(a + d*x/2)**2/4 - sin(a + d*x/2)**2*cos(c + d*x)/d + sin(a + d*x/2)*
sin(c + d*x)*cos(a + d*x/2)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x*c
os(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c), Eq(d, 0)), (-2*
b**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b**2*cos(a + b*x)**
2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*sin(c + d*x)*cos(a +
b*x)/(4*b**2*d - d**3) + d**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3
), True))
```

3.203 $\int \sin^2(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sin(2*d*x+
2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4569, 2637}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)-\frac{1}{4}\cos(2c+2dx)+\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx-\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+}{16}\end{aligned}$$

Mathematica [A] time = 0.76, size = 106, normalized size = 1.20

$$\frac{\left(2d^3-2b^2d\right)\sin(2(a+bx))+bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(d(\sin(2(a+x(b+d)+c))+4x(b+}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(-2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

fricas [A] time = 0.47, size = 118, normalized size = 1.34

$$-\frac{\left(2\,bd^{2}\cos\left(bx+a\right)^{2}+b^{3}-2\,bd^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^{3}d-bd^{3}\right)x-\left(2\,b^{2}d\cos\left(bx+a\right)\cos\left(dx+c\right)^{2}-(\cdots\right.}{4\left(b^{3}d-bd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) -
(b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - (2*b^2*d - d^3)*
cos(b*x + a))*sin(b*x + a))/(b^3*d - b*d^3)
```

giac [A] time = 0.18, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x+\frac{\sin\left(2\,bx+2\,dx+2\,a+2\,c\right)}{16\left(b+d\right)}+\frac{\sin\left(2\,bx-2\,dx+2\,a-2\,c\right)}{16\left(b-d\right)}-\frac{\sin\left(2\,bx+2\,a\right)}{8\,b}-\frac{\sin\left(2\,dx+2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b - 1/8*sin(2*d*x + 2*c)/d
```

maple [A] time = 1.08, size = 83, normalized size = 0.94

$$\frac{x}{4}-\frac{\sin\left(2bx+2a\right)}{8b}-\frac{\sin\left(2dx+2c\right)}{8d}+\frac{\sin\left(\left(2b-2d\right)x+2a-2c\right)}{16b-16d}+\frac{\sin\left(\left(2b+2d\right)x+2a+2c\right)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^2,x)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

maxima [B] time = 0.38, size = 620, normalized size = 7.05

$$\frac{8\left(\left(b\cos\left(2\,c\right)^{2}+b\sin\left(2\,c\right)^{2}\right)d^{3}-\left(b^{3}\cos\left(2\,c\right)^{2}+b^{3}\sin\left(2\,c\right)^{2}\right)d\right)x+\left(b^{2}d\sin\left(2\,c\right)-bd^{2}\sin\left(2\,c\right)\right)\cos\left(2\left(b+d\right)x\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) + 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) - 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) + 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

mupad [B] time = 0.89, size = 177, normalized size = 2.01

$$\frac{2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^2,x)
```

```
[Out] (2*d^3*sin(2*a + 2*b*x) - 2*b^3*sin(2*c + 2*d*x) + b*d^2*sin(2*a - 2*c + 2*
b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c +
 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) - 2*b^2*d*sin(2*a +
2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2 - d
^2))
```

sympy [A] time = 22.70, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 - 5*s
in(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*
cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 -
5*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) - sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*sin(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) + 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

3.204 $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-\frac{3\,\mathrm{c}}{}$$

```
[Out] 1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-3
/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1/1
6*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4569, 2638}

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-\frac{3\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) + (
3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{16}\sin(2a-3c+(2b-3d)x)+\frac{3}{16}\sin(2a-c+(2b-d)x)+\frac{3}{8}\sin(c+dx)\right.\\ &= -\left(\frac{1}{16}\int \sin(2a-3c+(2b-3d)x)\,dx\right)+\frac{1}{16}\int \sin(2a+3c+(2b+3d)x)\,dx-\\ &= \frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3}{2}\end{aligned}$$

Mathematica [A] time = 1.57, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{3\cos(2a+2bx-3c-3dx)}{2b-3d}-\frac{9\cos(2a+2bx-c-dx)}{2b-d}+\frac{9\cos(2a+2bx+c+dx)}{2b+d}-\frac{3\cos(2a+2bx+3c+}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-18*Cos[c]*Cos[d*x])/d + (2*Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*a - 3*c + 2
*b*x - 3*d*x])/(2*b - 3*d) - (9*Cos[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*
```

```
Cos[2*a + c + 2*b*x + d*x])/(2*b + d) - (3*Cos[2*a + 3*c + 2*b*x + 3*d*x])/
(2*b + 3*d) + (18*Sin[c]*Sin[d*x])/d - (2*Sin[3*c]*Sin[3*d*x])/d)/48
```

fricas [A] time = 0.61, size = 192, normalized size = 1.33

$$\frac{\left(8 b^4 - 38 b^2 d^2 + 9 d^4 + 9 \left(4 b^2 d^2 - d^4\right) \cos(bx+a)^2\right) \cos(dx+c)^3 + 6 \left(\left(4 b^3 d - b d^3\right) \cos(bx+a) \cos(dx+c)\right.}{3 (16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*((8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(
d*x + c)^3 + 6*((4*b^3*d - b*d^3)*cos(b*x + a)*cos(d*x + c)^2 - (4*b^3*d -
7*b*d^3)*cos(b*x + a))*sin(b*x + a)*sin(d*x + c) - 3*(8*b^4 - 26*b^2*d^2 +
9*d^4 + 3*(4*b^2*d^2 - 3*d^4)*cos(b*x + a)^2)*cos(d*x + c))/(16*b^4*d - 40*
b^2*d^3 + 9*d^5)
```

giac [A] time = 2.53, size = 129, normalized size = 0.90

$$-\frac{\cos(2 bx + 3 dx + 2 a + 3 c)}{16 (2 b + 3 d)} + \frac{3 \cos(2 bx + dx + 2 a + c)}{16 (2 b + d)} - \frac{3 \cos(2 bx - dx + 2 a - c)}{16 (2 b - d)} + \frac{\cos(2 bx - 3 dx + 2 a}{16 (2 b - 3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*cos(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*cos(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*cos(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*cos(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*cos(3*d*x + 3*c)/d - 3/8*cos(d*
x + c)/d
```

maple [A] time = 0.28, size = 133, normalized size = 0.92

$$\frac{\cos(2a - 3c + (2b - 3d) x)}{32b - 48d} - \frac{3 \cos(2a - c + (2b - d) x)}{16 (2b - d)} - \frac{3 \cos(dx + c)}{8d} + \frac{\cos(3dx + 3c)}{24d} + \frac{3 \cos(2a + c + (2b +}{16 (2b + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^3,x)
```

```
[Out] 1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-3
/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1/1
6*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [B] time = 0.42, size = 1362, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4
*cos(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) - 12*b^2*d^
2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*cos((2*b + 3*d)*x + 2*a) -
9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c)
 - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*cos((2*b + d)*x + 2*a - 2*c) + 9*(8*
b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 1
8*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
```

```
*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(
-(2*b - 3*d)*x - 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2
*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos
(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x) - 2*(16*b^4*cos(3*
c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*co
s(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x + 4*c) + 18*(16*b^4*
cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x - 2*c) + 3*(8*b^3*
d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b
*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*sin(3
*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*sin((2*b + d
)*x + 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(
3*c) + 9*d^4*sin(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*sin(3*c) +
 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c)
 - 9*d^4*sin(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*d*sin(3*c) + 12
*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x -
 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c)
- 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(3*c) - 40*b^2*d
^2*sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x) - 2*(16*b^4*sin(3*c) - 40*b^2*d^2*
sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*sin(3*c) - 40*b^2*
d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x + 4*c) - 18*(16*b^4*sin(3*c) - 40*b^
2*d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

mupad [B] time = 1.87, size = 469, normalized size = 3.26

$$e^{a\,2\mathrm{i}-c\,3\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,3\mathrm{i}}\left(\frac{3\,d\,\left(2\,b+3\,d\right)}{384\,b^2\,d-864\,d^3}+\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(8\,b^2-18\,d^2\right)}{384\,b^2\,d-864\,d^3}-\frac{3\,d\,e^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-3\,d\right)}{384\,b^2\,d-864\,d^3}\right)+e^{a\,2\mathrm{i}+c\,3\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,3\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^3,x)
```

```
[Out] exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d*(2*b + 3*d))/(384*b^2*d - 864*d^3)
 + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d^3) - (3*d*exp
(- a*4i - b*x*4i)*(2*b - 3*d))/(384*b^2*d - 864*d^3)) + exp(a*2i + c*3i + b
*x*2i + d*x*3i)*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d
^3) - (3*d*(2*b - 3*d))/(384*b^2*d - 864*d^3) + (3*d*exp(- a*4i - b*x*4i)*(
2*b + 3*d))/(384*b^2*d - 864*d^3)) - exp(a*2i - c*1i + b*x*2i - d*x*1i)*((3
*(2*b + d))/(32*(4*b^2 - d^2)) - (3*exp(- a*4i - b*x*4i)*(2*b - d))/(32*(4*
b^2 - d^2)) + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
 - exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*exp(- a*4i - b*x*4i)*(2*b + d))/(
32*(4*b^2 - d^2)) - (3*(2*b - d))/(32*(4*b^2 - d^2)) + (exp(- a*2i - b*x*2i
)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
```

sympy [A] time = 113.79, size = 1999, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x/2)
**2*sin(c + d*x)**3/16 - 3*x*sin(a - 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x)*
*2/16 - 3*x*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*x)/
8 + x*sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(c + d*x)**3/8 - x*sin(c + d*x)*
*3*cos(a - 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c + d*
x)**2/16 - sin(a - 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/d - 5*sin(a - 3
```

```
*d*x/2)**2*cos(c + d*x)**3/(48*d) - sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a
- 3*d*x/2)/(24*d) + 5*sin(a - 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c
+ d*x)**2/(4*d) - 9*cos(a - 3*d*x/2)**2*cos(c + d*x)**3/(16*d), Eq(b, -3*d/
2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a - d*x/2)**2*sin(
c + d*x)*cos(c + d*x)**2/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**2*cos(a - d*
x/2)*cos(c + d*x)/8 - 3*x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)**3/8 -
 3*x*sin(c + d*x)**3*cos(a - d*x/2)**2/16 - 3*x*sin(c + d*x)*cos(a - d*x/2)
**2*cos(c + d*x)**2/16 - sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/d -
 31*sin(a - d*x/2)**2*cos(c + d*x)**3/(48*d) - 3*sin(a - d*x/2)*sin(c + d*x
)**3*cos(a - d*x/2)/(8*d) - sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(
c + d*x)**2/(4*d) - cos(a - d*x/2)**2*cos(c + d*x)**3/(48*d), Eq(b, -d/2)),
 (3*x*sin(a + d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a + d*x/2)**2*sin(c +
d*x)*cos(c + d*x)**2/16 + 3*x*sin(a + d*x/2)*sin(c + d*x)**2*cos(a + d*x/2)
*cos(c + d*x)/8 + 3*x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)**3/8 - 3*x
*sin(c + d*x)**3*cos(a + d*x/2)**2/16 - 3*x*sin(c + d*x)*cos(a + d*x/2)**2*
cos(c + d*x)**2/16 - sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/d - 31*
sin(a + d*x/2)**2*cos(c + d*x)**3/(48*d) + 3*sin(a + d*x/2)*sin(c + d*x)**3
*cos(a + d*x/2)/(8*d) + sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*cos(c +
d*x)**2/(4*d) - cos(a + d*x/2)**2*cos(c + d*x)**3/(48*d), Eq(b, d/2)), (x*s
in(a + 3*d*x/2)**2*sin(c + d*x)**3/16 - 3*x*sin(a + 3*d*x/2)**2*sin(c + d*x
)*cos(c + d*x)**2/16 + 3*x*sin(a + 3*d*x/2)*sin(c + d*x)**2*cos(a + 3*d*x/2
)*cos(c + d*x)/8 - x*sin(a + 3*d*x/2)*cos(a + 3*d*x/2)*cos(c + d*x)**3/8 -
x*sin(c + d*x)**3*cos(a + 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a + 3*d*x/2
)**2*cos(c + d*x)**2/16 - sin(a + 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/
d - 5*sin(a + 3*d*x/2)**2*cos(c + d*x)**3/(48*d) + sin(a + 3*d*x/2)*sin(c +
 d*x)**3*cos(a + 3*d*x/2)/(24*d) - 5*sin(a + 3*d*x/2)*sin(c + d*x)*cos(a +
3*d*x/2)*cos(c + d*x)**2/(4*d) - 9*cos(a + 3*d*x/2)**2*cos(c + d*x)**3/(16*
d), Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*
x)*cos(a + b*x)/(2*b))*sin(c)**3, Eq(d, 0)), (-24*b**4*sin(a + b*x)**2*sin(
c + d*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 16*b**4*si
n(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b*
*4*sin(c + d*x)**2*cos(a + b*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3
+ 27*d**5) - 16*b**4*cos(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b**2*
d**3 + 27*d**5) - 24*b**3*d*sin(a + b*x)*sin(c + d*x)**3*cos(a + b*x)/(48*b
**4*d - 120*b**2*d**3 + 27*d**5) + 78*b**2*d**2*sin(a + b*x)**2*sin(c + d*x
)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2*d**2*sin(
a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b**2
*d**2*sin(c + d*x)**2*cos(a + b*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d*
*3 + 27*d**5) + 40*b**2*d**2*cos(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 1
20*b**2*d**3 + 27*d**5) + 42*b*d**3*sin(a + b*x)*sin(c + d*x)**3*cos(a + b*
x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sin(a + b*x)*sin(c + d
*x)*cos(a + b*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 27
*d**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d*
*3 + 27*d**5) - 18*d**4*sin(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b*
*2*d**3 + 27*d**5), True))
```

3.205 $\int \sin^3(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=600

$$\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b}{3b-dn}$$

```
[Out] 2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*e
xp(I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],exp(2*I*(d
*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+I*(
-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n,
 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x)
)^n)/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(
d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-d*n-b)/
d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*exp(-I*(c*
n+3*a)-I*(d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hyp
ergeom([-n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2
*I*c+2*I*d*x))^n)/(d*n+3*b)
```

Rubi [A] time = 1.72, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4553, 2285, 2253, 2252, 2251}

$$\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b}{3b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 +
(3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I
/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*
d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d
*n)/(2*d), 1 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F
1[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 -
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = 2^{-3-n}\int \left(3ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - i$$

$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3ibx}\left(ie^{-i}\right.$$

$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ibx-in(c+dx}\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(i\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(1\right.$$

$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-\right.}{3b-dn}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
[Out] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sin(dx+c)^n\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(dx+c)^n\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

maple [F] time = 6.42, size = 0, normalized size = 0.00

$$\int \left(\sin^{3}\left(bx+a\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)^{n}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\sin\left(a+b\,x\right)}^3\,{\sin\left(c+d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^n,x)
```

```
[Out] int(sin(a + b*x)^3*sin(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

3.206 $\int \sin^3(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] 3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4569, 2637}

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] (3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*b
 - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)*x
]/(8*(3*b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{3}{8}\cos(a-c+(b-d)x)-\frac{1}{8}\cos(3a-c+(3b-d)x)-\frac{3}{8}\cos(a+c+(b+ \right.\\ &= -\left(\frac{1}{8}\int \cos(3a-c+(3b-d)x)\,dx\right)+\frac{1}{8}\int \cos(3a+c+(3b+d)x)\,dx+\frac{3}{8}\int\\ &= \frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\end{aligned}$$

Mathematica [A] time = 0.54, size = 91, normalized size = 0.94

$$\frac{1}{8}\left(\frac{3\sin(a+bx-c-dx)}{b-d}-\frac{\sin(3a+3bx-c-dx)}{3b-d}+\frac{\sin(3a+3bx+c+dx)}{3b+d}-\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] ((3*Sin[a - c + b*x - d*x])/(b - d) - Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
+ Sin[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

fricas [A] time = 0.56, size = 115, normalized size = 1.19

$$\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)+3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-bd^2\right)\cos\left(b\right.\right.}{9\,b^4-10\,b^2d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a) +
 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*sin(d*x +
c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

giac [A] time = 1.46, size = 89, normalized size = 0.92

$$\frac{\sin\left(3\,bx+dx+3\,a+c\right)}{8\,(3\,b+d)}-\frac{\sin\left(3\,bx-dx+3\,a-c\right)}{8\,(3\,b-d)}-\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,(b+d)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*sin(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c)/
(b - d)
```

maple [A] time = 0.99, size = 90, normalized size = 0.93

$$\frac{3\sin\left(a-c+(b-d)\,x\right)}{8\,(b-d)}-\frac{\sin\left(3a-c+(3b-d)\,x\right)}{8\,(3b-d)}-\frac{3\sin\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\sin\left(3a+c+(3b+d)\,x\right)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c),x)
```

```
[Out] 3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

maxima [B] time = 0.40, size = 789, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*b
 + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*
sin(c))*cos((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin
(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) - (3*b^3*sin(c) + b^2*d*sin
(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*sin(c
) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2*c) +
3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x
 + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c))*cos(-
(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^
3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(
c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) - b^2*d*cos(c
) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) - (3*b^3*cos(c) + b
^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x - 3*a + 2*c) -
(3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*
x - 3*a) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*si
n((b + d)*x + a + 2*c) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*c
```

```
os(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*cos(c) + 9*b^2*d*c
os(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9
*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2
)*d^2)
```

mupad [B] time = 1.72, size = 494, normalized size = 5.09

$$e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}+\frac{e^{-a\,6i-b\,x\,6i}\,\left(-3\,b^3+b^2\,d+3\,b\,d^2-d^3\right)}{b^4\,144i-b^2\,d^2\,160i+d^4\,16i}-\frac{e^{-a\,2i-b\,x\,2i}\,\left(-27\,b^3\right.}{b^4\,144i-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x),x)
```

```
[Out] exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(b^4*14
4i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*b
^3 - d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*2i - b*x*2i)*(3*b
*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (e
xp(- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i + d^4*
16i - b^2*d^2*160i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d
 - 3*b^3 - d^3)/(b^4*144i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)
*(3*b*d^2 - b^2*d - 3*b^3 + d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (ex
p(- a*2i - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i + d^4*1
6i - b^2*d^2*160i) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3
*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i))
```

sympy [A] time = 31.74, size = 933, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*s
in(c + d*x)/8 - 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*sin(a
 - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 - 3*x*cos(a - d*x)**3*cos(c + d*x)/8
 + sin(a - d*x)**3*cos(c + d*x)/(8*d) + 3*sin(a - d*x)**2*sin(c + d*x)*cos(
a - d*x)/(4*d) + 3*sin(c + d*x)*cos(a - d*x)**3/(8*d), Eq(b, -d)), (x*sin(a
 - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/3)*cos(c +
d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x*cos(a - d*
x/3)**3*cos(c + d*x)/8 - 9*sin(a - d*x/3)**3*cos(c + d*x)/(8*d) - 3*sin(a -
 d*x/3)**2*sin(c + d*x)*cos(a - d*x/3)/(4*d) - sin(c + d*x)*cos(a - d*x/3)*
*3/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*sin(c + d*x)/8 + 3*x*sin(a + d
*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 - 3*x*sin(a + d*x/3)*sin(c + d*x)*co
s(a + d*x/3)**2/8 - x*cos(a + d*x/3)**3*cos(c + d*x)/8 - 9*sin(a + d*x/3)**
3*cos(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/(4*d
) + sin(c + d*x)*cos(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*sin(a + d*x)**3
*sin(c + d*x)/8 + 3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d*x)/8 + 3*x*sin
(a + d*x)*sin(c + d*x)*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**3*cos(c + d*x)
/8 - 5*sin(a + d*x)**3*cos(c + d*x)/(8*d) - 3*sin(a + d*x)*cos(a + d*x)**2*
cos(c + d*x)/(4*d) + 3*sin(c + d*x)*cos(a + d*x)**3/(8*d), Eq(b, d)), (-9*b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4
) - 6*b**3*sin(c + d*x)*cos(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + 7*
b**2*d*sin(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 6*b**2
*d*sin(a + b*x)*cos(a + b*x)**2*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4)
 + 3*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10*b**2*d**
2 + d**4) - d**3*sin(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4
), True))
```

3.207 $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-3\text{c}$$

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{4569, 2638\}$

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-3\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) +
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)-\frac{3}{16}\sin(a-2c+(b-2d)x)+\frac{1}{16}\sin(3a-\right.\\ &= \frac{1}{16}\int \sin(3a-2c+(3b-2d)x)\,dx+\frac{1}{16}\int \sin(3a+2c+(3b+2d)x)\,dx-\frac{1}{8}\int\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(}{16(3b-}\end{aligned}$$

Mathematica [A] time = 1.60, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(\frac{9\cos(a+bx-2c-2dx)}{b-2d}-\frac{3\cos(3a+3bx-2c-2dx)}{3b-2d}+\frac{9\cos(a+bx+2c+2dx)}{b+2d}-\frac{3\cos(3a+3bx+2c+}{3b+2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b + (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) - (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (9*
```

```
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) - (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

fricas [A] time = 0.47, size = 189, normalized size = 1.37

$$\frac{\left(9\,b^4-38\,b^2d^2+8\,d^4\right)\cos\left(bx+a\right)^3+6\left(7\,b^3d-4\,bd^3-\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\sin}{3\left(9\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((9*b^4 - 38*b^2*d^2 + 8*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (
b^3*d - 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9
*((b^4 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(
d*x + c)^2 - 3*(9*b^4 - 26*b^2*d^2 + 8*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d
^2 + 16*b*d^4)
```

giac [A] time = 0.18, size = 124, normalized size = 0.90

$$-\frac{\cos\left(3\,bx+2\,dx+3\,a+2\,c\right)}{16\,(3\,b+2\,d)}-\frac{\cos\left(3\,bx-2\,dx+3\,a-2\,c\right)}{16\,(3\,b-2\,d)}+\frac{\cos\left(3\,bx+3\,a\right)}{24\,b}+\frac{3\,\cos\left(bx+2\,dx+a+2\,c\right)}{16\,(b+2\,d)}+\frac{3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) - 1/16*cos(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b + 3/16*cos(b*x + 2*d*x +
a + 2*c)/(b + 2*d) + 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*
x + a)/b
```

maple [A] time = 0.29, size = 127, normalized size = 0.92

$$-\frac{3\cos\left(bx+a\right)}{8b}+\frac{\cos\left(3bx+3a\right)}{24b}+\frac{3\cos\left(a-2c+\left(b-2d\right)x\right)}{16\left(b-2d\right)}-\frac{\cos\left(3a-2c+\left(3b-2d\right)x\right)}{16\left(3b-2d\right)}+\frac{3\cos\left(a+2c+\left(b+\right.\right.}{16\left(b+2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^2,x)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

maxima [B] time = 0.43, size = 1360, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3
*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos
(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) +
3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*
c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c)
- 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*
b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*c
os((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d
^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) +
```

```
18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
 - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) -
8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) - 2*(9*b^4*cos(2*c) - 40*b^2*d^2*co
s(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*cos(2*c) - 40*b
^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) + 18*(9*b^4*cos(2
*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) + 18*(9*b^4
*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(
3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))
*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12
*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*s
in(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(
3*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d
^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*
c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*
d)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c
) + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*s
in(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c
) + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*si
n(2*c))*sin(-(b - 2*d)*x - a) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 1
6*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin
(2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) + 18*(9*b^4*sin(2*c) - 40*b
^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) - 18*(9*b^4*sin(2*c)
- 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c
)^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos
(2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

mupad [B] time = 2.13, size = 437, normalized size = 3.17

$$\frac{81\,b^4\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)-162\,b^4\,\cos\left(a+b\,x\right)-288\,d^4\,\cos\left(a+b\,x\right)-9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^2,x)
```

```
[Out] (81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) - 16
2*b^4*cos(a + b*x) - 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x - 2
*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) + 18*b^4*cos(3*a + 3*b*x) + 32
*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*d^3*
cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x) + 6
*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x - 2*d*
x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) + 720*b^2*d^2*cos(a + b*x) + 36*
b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c + 3*b*x +
 2*d*x) - 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x - 2*d*x)
 + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c + b*x - 2*d*
x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5 - 40*b^3*d
^2))
```

sympy [A] time = 113.35, size = 2030, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq(b
, 0)), (3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**3*co
s(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*cos(c
+ d*x)/8 + 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 - 3*x*si
```

```
n(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(c + d*x)*cos(a
- 2*d*x)**3*cos(c + d*x)/8 - 13*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c + d*x)
/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) - 7*sin(a
- 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) - 17*sin(c + d*x
)**2*cos(a - 2*d*x)**3/(96*d) + 49*cos(a - 2*d*x)**3*cos(c + d*x)**2/(96*d)
, Eq(b, -2*d)), (x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 - x*sin(a - 2*d*x
/3)**3*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*cos(a - 2*
d*x/3)*cos(c + d*x)/8 - 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a - 2*d*x/
3)**2/16 + 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)**2/16 + x*
sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - 15*sin(a - 2*d*x/3)**3*si
n(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(a - 2*d*x/3)*cos
(c + d*x)**2/(2*d) + 9*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d*x/3)**2*co
s(c + d*x)/(8*d) + 21*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d) + 11*cos(a
 - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (x*sin(a + 2*d*x/3)*
*3*sin(c + d*x)**2/16 - x*sin(a + 2*d*x/3)**3*cos(c + d*x)**2/16 + 3*x*sin(
a + 2*d*x/3)**2*sin(c + d*x)*cos(a + 2*d*x/3)*cos(c + d*x)/8 - 3*x*sin(a +
2*d*x/3)*sin(c + d*x)**2*cos(a + 2*d*x/3)**2/16 + 3*x*sin(a + 2*d*x/3)*cos(
a + 2*d*x/3)**2*cos(c + d*x)**2/16 - x*sin(c + d*x)*cos(a + 2*d*x/3)**3*cos
(c + d*x)/8 - 15*sin(a + 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) - 3*s
in(a + 2*d*x/3)**2*cos(a + 2*d*x/3)*cos(c + d*x)**2/(2*d) + 9*sin(a + 2*d*x
/3)*sin(c + d*x)*cos(a + 2*d*x/3)**2*cos(c + d*x)/(8*d) - 21*sin(c + d*x)**
2*cos(a + 2*d*x/3)**3/(32*d) - 11*cos(a + 2*d*x/3)**3*cos(c + d*x)**2/(32*d
), Eq(b, 2*d/3)), (3*x*sin(a + 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a + 2
*d*x)**3*cos(c + d*x)**2/16 + 3*x*sin(a + 2*d*x)**2*sin(c + d*x)*cos(a + 2*
d*x)*cos(c + d*x)/8 + 3*x*sin(a + 2*d*x)*sin(c + d*x)**2*cos(a + 2*d*x)**2/
16 - 3*x*sin(a + 2*d*x)*cos(a + 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c +
d*x)*cos(a + 2*d*x)**3*cos(c + d*x)/8 - 13*sin(a + 2*d*x)**3*sin(c + d*x)*c
os(c + d*x)/(16*d) - sin(a + 2*d*x)**2*cos(a + 2*d*x)*cos(c + d*x)**2/(2*d)
 - 7*sin(a + 2*d*x)*sin(c + d*x)*cos(a + 2*d*x)**2*cos(c + d*x)/(8*d) + 17*
sin(c + d*x)**2*cos(a + 2*d*x)**3/(96*d) - 49*cos(a + 2*d*x)**3*cos(c + d*x
)**2/(96*d), Eq(b, 2*d)), (-27*b**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(a +
 b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 18*b**4*sin(c + d*x)**2*cos(a
 + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 42*b**3*d*sin(a + b*x)**
3*sin(c + d*x)*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 36*b**3
*d*sin(a + b*x)*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(27*b**5 - 120*b*
*3*d**2 + 48*b*d**4) + 78*b**2*d**2*sin(a + b*x)**2*sin(c + d*x)**2*cos(a +
 b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 42*b**2*d**2*sin(a + b*x)**2*
cos(a + b*x)*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40*b**
2*d**2*sin(c + d*x)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4
) + 40*b**2*d**2*cos(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 +
 48*b*d**4) - 24*b*d**3*sin(a + b*x)**3*sin(c + d*x)*cos(c + d*x)/(27*b**5
- 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(
a + b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*co
s(a + b*x)*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d**4*
sin(c + d*x)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*
d**4*cos(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4),
 True))
```

3.208 $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a}{}$$

```
[Out] -3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*a
-3*c+3*(b-d)*x)/(b-d)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*sin(a+c+(b+d)*
x)/(b+d)-1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+d
)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4569, 2637}

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d)) -
Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(a-c)+\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx-\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx-\frac{3}{32}\int\\ &= -\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}\end{aligned}$$

Mathematica [A] time = 1.66, size = 177, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{9\sin(a+bx-3c-3dx)}{b-3d}+\frac{27\sin(a+bx-c-dx)}{b-d}+\frac{\sin(3(a+bx-c-dx))}{b-d}-\frac{9\sin(3a+3bx-c-dx)}{3b-d}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
 - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) - (9*Sin[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Sin[a + c + (b + d)*x])/(b + d) - Sin[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

fricas [A] time = 0.59, size = 291, normalized size = 1.49

$$-\frac{\left(\left(63\,b^4d-88\,b^2d^3+9\,d^5-\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-3\left(21\,b^4d-70\,b^2d^3+9\,d^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(((63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(
b*x + a)^2)*cos(d*x + c)^3 - 3*(21*b^4*d - 70*b^2*d^3 + 9*d^5 - (3*b^4*d -
28*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c))*sin(b*x + a) - ((9*b^5 -
88*b^3*d^2 + 63*b*d^4)*cos(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos
(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^2
 - 3*(9*b^5 - 70*b^3*d^2 + 21*b*d^4)*cos(b*x + a))*sin(d*x + c))/(9*b^6 - 9
1*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

giac [A] time = 0.18, size = 181, normalized size = 0.93

$$-\frac{\sin\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\left(b+d\right)}+\frac{3\,\sin\left(3\,bx+dx+3\,a+c\right)}{32\left(3\,b+d\right)}-\frac{3\,\sin\left(3\,bx-dx+3\,a-c\right)}{32\left(3\,b-d\right)}+\frac{\sin\left(3\,bx-3\,dx+3\,a-\right.}{96\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
 c)/(3*b + d) - 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x
- 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) -
9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) -
 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

maple [A] time = 2.02, size = 184, normalized size = 0.94

$$-\frac{3\sin\left(a-3c+\left(b-3d\right)x\right)}{32\left(b-3d\right)}+\frac{9\sin\left(a-c+\left(b-d\right)x\right)}{32\left(b-d\right)}-\frac{9\sin\left(a+c+\left(b+d\right)x\right)}{32\left(b+d\right)}+\frac{3\sin\left(a+3c+\left(b+3d\right)x\right)}{32\left(b+3d\right)}+\frac{\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^3,x)
```

```
[Out] -3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)-9/32*sin(a+c
+(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*
x+3*a-3*c)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b
+d)-1/96/(b+d)*sin((3*b+3*d)*x+3*a+3*c)
```

maxima [B] time = 0.57, size = 2612, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
```

```
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
 - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) - (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
*cos(-3*(b - d)*x - 3*a) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
 3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
 + 4*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) + (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) + (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) + 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
```

```
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

mupad [B] time = 4.31, size = 997, normalized size = 5.11

$$e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{b^4\,576i-b^2\,d^2\,640i+d^4\,64i}+\frac{e^{-a\,6i-b\,x\,6i}\left(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3\right)}{b^4\,576i-b^2\,d^2\,640i+d^4\,64i}-\frac{e^{-a\,2i-b\,x\,2i}\left(-8\right.}{b^4\,576i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^3,x)
```

```
[Out] exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(b^
4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*d
 - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*2i - b*x*
2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*64
0i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^4*576
i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2
 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*
6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*64i - b^2
*d^2*640i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(
b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^
2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i - c*
3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192i + d^4*1728
i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))
/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 -
27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(-
 a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i + d^4*1728
i - b^2*d^2*1920i)) + exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d
- b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6
i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i)
- (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i +
d^4*1728i - b^2*d^2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 27*b^2*d - 9*
b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.209 $\int \cos^n(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=277

$$-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+}{b-dn}$$

```
[Out] -2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(
d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x+c
)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n+b)*x
+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b
)/d],[1+1/2*(-d*n-b)/d],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+
b)
```

Rubi [A] time = 0.59, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4555, 2285, 2253, 2251}

$$-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+}{b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c +
 d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/d
 - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
- (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
```

```
b, c, d, q}, x] && IGtQ[p, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)dx$$
$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c}\right.$$
$$= \left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+dx)}\left(1+e\right.$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(1\right.\right.$$
$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e\right.}{b-dn}$$

Mathematica [A] time = 1.02, size = 202, normalized size = 0.73

$$-\frac{2^{-n-1}e^{i(c-bx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}\left(e^{idx}(\cos(a)-i\sin(a))(b-dn)\,{}_2F_1\left(1,\frac{1}{2}\left(-\frac{b}{d}+n+2\right);-\frac{b+d(n-2)}{2d};-e^{2i}\right.\right.}{(b-dn)(b+dn)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*E^(I*(c - b*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(
1 + n)*(E^(I*d*x)*(b - d*n)*Hypergeometric2F1[1, (2 - b/d + n)/2, -1/2*(b +
 d*(-2 + n))/d, -E^((2*I)*(c + d*x))]*(Cos[a] - I*Sin[a]) + E^(I*(2*b + d)*
x)*(b + d*n)*Hypergeometric2F1[1, (b + d*(2 + n))/(2*d), (2 + b/d - n)/2, -
E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a])))/((b - d*n)*(b + d*n)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(dx+c\right)^n\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^n*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)^n\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

maple [F] time = 3.26, size = 0, normalized size = 0.00

$$\int \left(\cos^n\left(dx+c\right)\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a),x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos{(dx+c)}^n \sin{(bx+a)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^n\,\sin\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x),x)
[Out] int(cos(c + d*x)^n*sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a),x)
[Out] Timed out
```

3.210 $\int \cos^3(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] -Cos[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) - (3*Cos[a - c + (b - d)*x])/(8*(
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{8}\sin(a-3c+(b-3d)x)+\frac{3}{8}\sin(a-c+(b-d)x)+\frac{3}{8}\sin(a+c+(b+a\right.\\ &= \frac{1}{8}\int \sin(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \sin(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \sin(\\ &= -\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}\end{aligned}$$

Mathematica [A] time = 0.50, size = 87, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\cos(a+bx-3c-3dx)}{b-3d}-\frac{3\cos(a+bx-c-dx)}{b-d}-\frac{\cos(a+bx+3c+3dx)}{b+3d}-\frac{3\cos(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a - 3*c + b*x - 3*d*x]/(b - 3*d)) - (3*Cos[a - c + b*x - d*x])/(b -
d) - Cos[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Cos[a + c + (b + d)*x])/(b +
 d))/8
```

fricas [A] time = 0.42, size = 106, normalized size = 1.16

$$\frac{6\,bd^2\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3+3\left(2\,d^3-\left(b^2d-d^3\right)\cos\left(dx+c\right)^2\right)\sin\left(bx\right.}{b^4-10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*b*d^2*cos(b*x + a)*cos(d*x + c) - (b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c
)^3 + 3*(2*d^3 - (b^2*d - d^3)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c))/(
b^4 - 10*b^2*d^2 + 9*d^4)
```

giac [A] time = 1.92, size = 84, normalized size = 0.92

$$-\frac{\cos\left(bx+3\,dx+a+3\,c\right)}{8\,(b+3\,d)}-\frac{3\,\cos\left(bx+dx+a+c\right)}{8\,(b+d)}-\frac{3\,\cos\left(bx-dx+a-c\right)}{8\,(b-d)}-\frac{\cos\left(bx-3\,dx+a-3\,c\right)}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*cos(b*x + d*x + a + c)/(b +
 d) - 3/8*cos(b*x - d*x + a - c)/(b - d) - 1/8*cos(b*x - 3*d*x + a - 3*c)/(
b - 3*d)
```

maple [A] time = 0.17, size = 84, normalized size = 0.92

$$-\frac{\cos\left(a-3c+(b-3d)\,x\right)}{8\,(b-3d)}-\frac{3\cos\left(a-c+(b-d)\,x\right)}{8\,(b-d)}-\frac{3\cos\left(a+c+(b+d)\,x\right)}{8\,(b+d)}-\frac{\cos\left(a+3c+(b+3d)\,x\right)}{8\,(b+3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a),x)
```

```
[Out] -1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [B] time = 0.39, size = 912, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*
cos((b + 3*d)*x + a + 6*c) + (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3
*c) + 3*d^3*cos(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*cos(3*c) - b^2*d*cos(3*
c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*c
os(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x
 + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*c
os(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b
*d^2*cos(3*c) - 9*d^3*cos(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*cos(3*c) +
 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*cos(-(b - 3*d)*x - a +
 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))
*cos(-(b - 3*d)*x - a) + (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
+ 3*d^3*sin(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(
3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*sin(3
*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*
sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a + 4*c) - 3*(b^3*sin(3*c) + b^
```

```
2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
 - 3*d^3*sin(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

mupad [B] time = 1.61, size = 297, normalized size = 3.26

$$-e^{a\,1i-c\,3i+b\,x\,1i-d\,x\,3i}\left(\frac{b+3\,d}{16\,b^2-144\,d^2}+\frac{e^{-a\,2i-b\,x\,2i}\,(b-3\,d)}{16\,b^2-144\,d^2}\right)-e^{a\,1i+c\,3i+b\,x\,1i+d\,x\,3i}\left(\frac{b-3\,d}{16\,b^2-144\,d^2}+\frac{e^{-a\,2i-b\,x\,2i}\,(b}{16\,b^2-14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x),x)
```

```
[Out] - exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(16*b^2 - 144*d^2) + (exp(-
 a*2i - b*x*2i)*(b - 3*d))/(16*b^2 - 144*d^2)) - exp(a*1i + c*3i + b*x*1i +
 d*x*3i)*((b - 3*d)/(16*b^2 - 144*d^2) + (exp(- a*2i - b*x*2i)*(b + 3*d))/(
16*b^2 - 144*d^2)) - exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/(16*b^
2 - 16*d^2) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(16*b^2 - 16*d^2)) - exp(a
*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(16*b^2 - 16*d^2) + (exp(- a*2i
- b*x*2i)*(3*b + 3*d))/(16*b^2 - 16*d^2))
```

sympy [A] time = 32.16, size = 918, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x)*s
in(c + d*x)**2*cos(c + d*x)/8 + x*sin(a - 3*d*x)*cos(c + d*x)**3/8 - x*sin(
c + d*x)**3*cos(a - 3*d*x)/8 + 3*x*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)
**2/8 - sin(a - 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a - 3*d*x)*sin(c + d*x)
*cos(c + d*x)**2/(4*d) + 3*cos(a - 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -3*d
)), (3*x*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*sin(a - d*x)*cos
(c + d*x)**3/8 + 3*x*sin(c + d*x)**3*cos(a - d*x)/8 + 3*x*sin(c + d*x)*cos(
a - d*x)*cos(c + d*x)**2/8 + 3*sin(a - d*x)*sin(c + d*x)**3/(8*d) + 3*sin(a
 - d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - cos(a - d*x)*cos(c + d*x)**3/(
8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*si
n(a + d*x)*cos(c + d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a + d*x)/8 - 3*x*sin
(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/8 + 3*sin(a + d*x)*sin(c + d*x)**3/(
8*d) + 3*sin(a + d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + cos(a + d*x)*cos
(c + d*x)**3/(8*d), Eq(b, d)), (-3*x*sin(a + 3*d*x)*sin(c + d*x)**2*cos(c +
 d*x)/8 + x*sin(a + 3*d*x)*cos(c + d*x)**3/8 + x*sin(c + d*x)**3*cos(a + 3*
d*x)/8 - 3*x*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/8 - sin(a + 3*d*x)
*sin(c + d*x)**3/(24*d) - sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d)
 - 3*cos(a + 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, 3*d)), (-b**3*cos(a + b*x)
*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sin(a + b*x)*sin
(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sin(c +
 d*x)**2*cos(a + b*x)*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 7*b*d**
2*cos(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*d**3*sin(
a + b*x)*sin(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*sin(a + b*
x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

3.211 $\int \cos^2(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=62

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

```
[Out] -1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]/(2*b) - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)+\frac{1}{4}\sin(a-2c+(b-2d)x)+\frac{1}{4}\sin(a+2c+(b+2d)x)\right)dx\\ &= \frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sin(a+\\ &= -\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A] time = 0.78, size = 71, normalized size = 1.15

$$\frac{1}{4}\left(-\frac{\cos(a+bx-2c-2dx)}{b-2d}-\frac{\cos(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}-\frac{2\cos(a)\cos(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b - Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) - Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

fricas [A] time = 0.50, size = 66, normalized size = 1.06

$$-\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-2\,d^2\cos(bx+a)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
 + c) - 2*d^2*cos(b*x + a))/(b^3 - 4*b*d^2)
```

giac [A] time = 5.90, size = 56, normalized size = 0.90

$$-\frac{\cos\left(bx+2\,dx+a+2\,c\right)}{4\,(b+2\,d)}-\frac{\cos\left(bx-2\,dx+a-2\,c\right)}{4\,(b-2\,d)}-\frac{\cos\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) - 1/4*cos(b*x - 2*d*x + a - 2*c)/
(b - 2*d) - 1/2*cos(b*x + a)/b
```

maple [A] time = 0.17, size = 57, normalized size = 0.92

$$-\frac{\cos\left(bx+a\right)}{2b}-\frac{\cos\left(a-2c+(b-2d)\,x\right)}{4\,(b-2d)}-\frac{\cos\left(a+2c+(b+2d)\,x\right)}{4\,(b+2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a),x)
```

```
[Out] -1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

maxima [B] time = 0.35, size = 414, normalized size = 6.68

$$-\frac{\left(b^2\cos\left(2\,c\right)-2\,bd\cos\left(2\,c\right)\right)\cos\left((b+2\,d)x+a+4\,c\right)+\left(b^2\cos\left(2\,c\right)-2\,bd\cos\left(2\,c\right)\right)\cos\left((b+2\,d)x+a\right)+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos
(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) + 2*(b^
2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b
+ 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) + 2*(b^2*sin(2*c)
- 4*d^2*sin(2*c))*sin(b*x + a + 2*c) - 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

mupad [B] time = 0.77, size = 97, normalized size = 1.56

$$\frac{d\,\left(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\cos\left(a+2\right.}{16\,b\,d^2-4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x),x)
```

```
[Out] (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^2*
cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 - 4*
b^3) - cos(a + b*x)/(2*b)
```

<u>**sympy**</u> **[A]** time = 6.54, size = 405, normalized size = 6.53

$$\begin{cases} x\sin(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ -\frac{x\sin(a-2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}-\frac{\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+\frac{\cos(a-2dx)}{2} \\ -\frac{x\sin(a+2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}-\frac{\cos(a+2dx}{2} \\ -\frac{b^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)**2/4 + x*sin(a - 2*d*x)*cos(c + d*x)**2/4
+ x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 - sin(a - 2*d*x)*sin(c + d*x
)*cos(c + d*x)/(4*d) + cos(a - 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, -2*d)),
(-x*sin(a + 2*d*x)*sin(c + d*x)**2/4 + x*sin(a + 2*d*x)*cos(c + d*x)**2/4 -
 x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - sin(a + 2*d*x)*sin(c + d*x)
*cos(c + d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, 2*d)), (-
b**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(a + b*x)*si
n(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(a +
b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d*
*2), True))
```

3.212 $\int \cos(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] -1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x],x]
[Out] -Cos[a - c + (b - d)*x]/(2*(b - d)) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a-c+(b-d)x)+\frac{1}{2}\sin(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \sin(a-c+(b-d)x)\,dx+\frac{1}{2}\int \sin(a+c+(b+d)x)\,dx\\ &= -\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 43, normalized size = 1.00

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x],x]
[Out] -1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

fricas [A] time = 0.46, size = 42, normalized size = 0.98

$$-\frac{b\cos(bx+a)\cos(dx+c)+d\sin(bx+a)\sin(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x + a)*cos(d*x + c) + d*sin(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

giac [A] time = 0.26, size = 40, normalized size = 0.93

$$-\frac{\cos\left(bx+dx+a+c\right)}{2\,(b+d)}-\frac{\cos\left(bx-dx+a-c\right)}{2\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(b*x - d*x + a - c)/(b - d)
```

maple [A] time = 0.18, size = 40, normalized size = 0.93

$$-\frac{\cos\left(a-c+(b-d)\,x\right)}{2\,(b-d)}-\frac{\cos\left(a+c+(b+d)\,x\right)}{2\,(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a),x)
```

```
[Out] -1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

maxima [A] time = 0.31, size = 40, normalized size = 0.93

$$-\frac{\cos\left(bx+dx+a+c\right)}{2\,(b+d)}-\frac{\cos\left(-bx+dx-a+c\right)}{2\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(-b*x + d*x - a + c)/(b - d)
```

mupad [B] time = 0.84, size = 85, normalized size = 1.98

$$-\frac{b\left(\frac{\cos(a-c+b\,x-d\,x)}{2}+\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}-\frac{d\left(\frac{\cos(a-c+b\,x-d\,x)}{2}-\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x),x)
```

```
[Out] - (b*(cos(a - c + b*x - d*x)/2 + cos(a + c + b*x + d*x)/2))/(b^2 - d^2) - (
d*(cos(a - c + b*x - d*x)/2 - cos(a + c + b*x + d*x)/2))/(b^2 - d^2)
```

sympy [A] time = 1.45, size = 155, normalized size = 3.60

$$\begin{cases} x\sin\left(a\right)\cos\left(c\right) & \text{for}\: b=0 \wedge d=0 \\ \frac{x\sin\left(a-dx\right)\cos\left(c+dx\right)}{2}+\frac{x\sin\left(c+dx\right)\cos\left(a-dx\right)}{2}+\frac{\cos\left(a-dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\: b=-d \\ \frac{x\sin\left(a+dx\right)\cos\left(c+dx\right)}{2}-\frac{x\sin\left(c+dx\right)\cos\left(a+dx\right)}{2}-\frac{\cos\left(a+dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\: b=d \\ -\frac{b\cos\left(a+bx\right)\cos\left(c+dx\right)}{b^2-d^2}-\frac{d\sin\left(a+bx\right)\sin\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*cos(c + d
*x)/2 + x*sin(c + d*x)*cos(a - d*x)/2 + cos(a - d*x)*cos(c + d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)*cos(a + d*x)/2 -
cos(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*cos(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(a + b*x)*sin(c + d*x)/(b**2 - d**2), True))
```

3.213 $\int \sec(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=27

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

```
[Out] -cos(a-c)*ln(cos(b*x+c))/b+x*sin(a-c)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4580, 3475, 8}

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]*Sin[a + b*x],x]
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sec(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \tan(c+bx)\,dx+\sin(a-c)\int 1\,dx\\ &= -\frac{\cos(a-c)\log(\cos(c+bx))}{b}+x\sin(a-c)\end{aligned}$$

Mathematica [A] time = 0.15, size = 27, normalized size = 1.00

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]*Sin[a + b*x],x]
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

fricas [A] time = 0.46, size = 31, normalized size = 1.15

$$-\frac{bx\sin(-a+c)+\cos(-a+c)\log(-\cos(bx+c))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

giac [B] time = 0.20, size = 158, normalized size = 5.85

$$\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)\log}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - ta
n(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^
2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

maple [B] time = 1.56, size = 563, normalized size = 20.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)*sin(b*x+a),x)
```

```
[Out] 1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)/(sin(a)*cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)^2*cos(c)*sin(c)-1/b/(cos(a)
^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*
cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)
+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)*cos(c)^2*sin(a)+1/b/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos
(c)+sin(a)*sin(c))*cos(a)*sin(a)*sin(c)^2-1/b/(cos(a)^2*cos(c)^2+cos(a)^2*s
in(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(a)*sin(c))*
ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*
sin(c))*cos(c)*sin(a)^2*sin(c)+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2
)*ln(1+tan(b*x+a)^2)*cos(a)*cos(c)+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(
a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*sin(c)-1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+si
n(a)^2)*cos(a)*sin(c)*arctan(tan(b*x+a))+1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+
sin(a)^2)*cos(c)*sin(a)*arctan(tan(b*x+a))
```

maxima [B] time = 0.34, size = 73, normalized size = 2.70

$$-\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,b\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

mupad [B] time = 0.89, size = 112, normalized size = 4.15

$$x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)-\frac{\ln\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x),x)
```

```
[Out] x*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2) + x*((exp(c*1i - a*1i
)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - (log(exp(a*2i - c*2i) + exp(a*2i + b*x
*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i - c*1i)/2))/b
```

sympy [B] time = 10.12, size = 435, normalized size = 16.11

$$\left(\left(\begin{cases} -x \\ x \\ 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}\right.}{b\tan^2\left(\frac{c}{2}\right)+b} \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan(
c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**
2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan(
c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) - l
og(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
+ b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2
)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(ta
n(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((-log(sin(b*x))
/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b, 0)), (-
b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(b
*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(t
an(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b
*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**2
 + b), True))*sin(a)
```

3.214 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

```
[Out] cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4580, 2606, 8, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx+\sin(a-c)\int \sec(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}+\frac{\cos(a-c)\,\text{Subst}\left(\int 1\,dx,x,\sec(c+bx)\right)}{b}\\ &= \frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.10, size = 88, normalized size = 2.59

$$\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)
/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Si
n[c])]*Sin[a - c])/b
```

fricas [B] time = 0.48, size = 69, normalized size = 2.03

$$-\frac{\cos\left(bx+c\right)\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-\cos\left(bx+c\right)\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-2\,\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-si
n(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((tan(a/2)^2*tan(c/2)-tan(a/2)*tan(c/2)^2+tan(a/2)-tan(c/2))/(-
tan(a/2)^2*tan(c/2)^2-tan(a/2)^2-tan(c/2)^2-1)*ln(abs(tan((b*x+c)/2)-1))+(t
an(a/2)^2*tan(c/2)-tan(a/2)*tan(c/2)^2+tan(a/2)-tan(c/2))/(tan(a/2)^2*tan(c
/2)^2+tan(a/2)^2+tan(c/2)^2+1)*ln(abs(tan((b*x+c)/2)+1))+(tan(a/2)^2*tan(c/
2)^2-tan(a/2)^2+4*tan(a/2)*tan(c/2)-tan(c/2)^2+1)/(-tan(a/2)^2*tan(c/2)^2-t
an(a/2)^2-tan(c/2)^2-1)/(tan((b*x+c)/2)^2-1))
```

maple [B] time = 1.97, size = 888, normalized size = 26.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] -8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)
^2*sin(c)^2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+
1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos
(c)-cos(a)*cos(c)-sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)+8/b/(-4*c
os(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^
2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2
*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)
*cos(c)-sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)+8/b/(-4*cos(a)^2*co
s(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(cos(a)
*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b
*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-si
n(a)*sin(c))*cos(a)*cos(c)+8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*
cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+
sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan
(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))*sin(a)*sin(c)-8/
b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*
sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*
sin(c)^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2
```

```
*a)+2*cos(a)*sin(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-
cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*cos(a)*sin(c)+8/b/(-4*cos(a)^2*
cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos
(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)
*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*si
n(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)
^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*cos(c)
```

maxima [B] time = 0.52, size = 387, normalized size = 11.38

$$\frac{2\left(\cos\left(bx+2a\right)+\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+2\cos\left(bx+2a\right)\cos(a)+2\cos\left(bx+2c\right)\cos(a)+\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x +
 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a + c
) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*sin(
-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a)^2)
*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) +
sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 +
cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*si
n(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c
) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a + 2
*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*si
n(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

mupad [B] time = 5.35, size = 254, normalized size = 7.47

$$\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i) + exp(a*2i
 + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) -
(exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i)
)^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp
(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(
exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*
2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

sympy [B] time = 165.06, size = 5552, normalized size = 163.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((x/cos(c)**2, Eq(b, 0)), (-1/(b*sin(b*x)), Eq(c, -pi/2) | Eq(c, p
i/2)), (-log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c
/2)**6*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan
(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c
/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/
```

```
2)**2 - b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*t
an(c/2)**6/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*t
an(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan
(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b)
 + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*
*5*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**
5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*
tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
 b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/
2)**4*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(
c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/
2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2
)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*
tan(c/2)**4/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*
tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*ta
n(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b
) - 8*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)
**3*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)*
*5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2
*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2
- b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c
/2)**2*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan
(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c
/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/
2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))
*tan(c/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5
*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*t
an(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
b) + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2
)*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5
*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*t
an(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(b*x/2
)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x
/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2
)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log
(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**6*ta
n(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*ta
n(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 +
 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) + tan(c/2)
/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**6*tan(b*x/2)**2/(b*tan(c/2)**
6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)*
*2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log(tan(b*x/2) + tan(
c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**6/(b*tan(c/2)**6*tan(b*x/
2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/
2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*t
an(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*log(tan(b*x/2) + tan(c/2)/(ta
n(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**5*tan(b*x/2)/(b*tan(c/2)**6*tan(b
*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b
*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*
b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) + tan(c/2)/
(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**4*tan(b*x/2)**2/(b*tan(c/2)**6
*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4
*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**
2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) + tan
(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**4/(b*tan(c/2)**6*tan(b*x
/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x
/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*
```

```
tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 8*log(tan(b*x/2) + tan(c/2)/(t
an(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**6*tan(
b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(
b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4
*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) + tan(c/2)
/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2*tan(b*x/2)**2/(b*tan(c/2)**
6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)*
*2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) + ta
n(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2/(b*tan(c/2)**6*tan(b*
x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*
x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b
*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*log(tan(b*x/2) + tan(c/2)/(
tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)/(b*tan(c/2)**6*tan(b*
x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*
x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b
*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log(tan(b*x/2) + tan(c/2)/(ta
n(c/2) + 1) - 1/(tan(c/2) + 1))*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2
- b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2
)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) +
 1) - 1/(tan(c/2) + 1))/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*
tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*ta
n(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b
*x/2)**2 - b) + 4*tan(c/2)**5/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6
- 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4
- b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b
*tan(b*x/2)**2 - b) + 8*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2
 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/
2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 8*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)
**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)
**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2
)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*tan(c/2)/(b*tan(c
/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c
/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(
c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b), True))*sin(a) + P
iecewise((log(tan(b*x/2))/b, Eq(c, pi/2)), (0, Eq(b, 0)), (log(tan(b*x/2))/
b, Eq(c, -pi/2)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2
) - 1))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)
**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**
2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan
(c/2)**3/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan
(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2)
 - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*ta
n(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*
tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(t
an(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(
b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b
*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 -
4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b)
 + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2
)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(ta
n(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan
(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)/(tan(
c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x
```

```
/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/
2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/
(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan
(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x
/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1)
)*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*t
an(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*tan(c/2)**4/
(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) -
 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)**3*tan(b*x/2)/
(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) -
 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)*tan(b*x/2)/(b*
tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*
b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b), True))*cos(a)
```

3.215 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

```
[Out] 1/2*cos(a-c)*sec(b*x+c)^2/b+sin(a-c)*tan(b*x+c)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3767, 8}

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx+\sin(a-c)\int \sec^2(c+bx)\,dx\\ &= \frac{\cos(a-c)\,\text{Subst}\left(\int x\,dx,x,\sec(c+bx)\right)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int 1\,dx,x,-\tan(c\right.}{b}\\ &= \frac{\cos(a-c)\sec^2(c+bx)}{2b}+\frac{\sin(a-c)\tan(c+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.18, size = 34, normalized size = 0.89

$$\frac{\sec(c)\sec^2(bx+c)(\sin(a-c)\sin(2bx+c)+\cos(a))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

fricas [A] time = 0.54, size = 42, normalized size = 1.11

$$-\frac{2\,\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x + c
)^2)
```

giac [B] time = 3.85, size = 174, normalized size = 4.58

$$\frac{\tan\left(bx+c\right)^2\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(bx+c\right)^2\tan\left(\frac{1}{2}\,a\right)^2+4\,\tan\left(bx+c\right)^2\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+4\,\tan\left(bx+c\right)}{2\left(\tan\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^2
 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*tan
(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/2*c
)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1/2*c
))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

maple [B] time = 3.14, size = 150, normalized size = 3.95

$$\frac{\frac{1}{(\cos(a)\sin(c)-\sin(a)\cos(c))(\sin(a)\cos(c)-\cos(a)\sin(c))(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{}{2(\cos(a)\sin}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/(cos(a)*sin(c)-sin(a)*cos(c))/(sin(a)*cos(c)-cos(a)*sin(c))/(-tan(b*
x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))+1/
2*(-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)*sin(c)-sin(a)*cos(c))/(sin(a)*cos(
c)-cos(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a
)*cos(c)+sin(a)*sin(c))^2)
```

maxima [B] time = 0.34, size = 391, normalized size = 10.29

$$\frac{\left(2\,\cos\left(2\,bx+2\,a+2\,c\right)+\cos\left(2\,a\right)-\cos\left(2\,c\right)\right)\cos\left(4\,bx+a+5\,c\right)+2\left(2\,\cos\left(2\,bx+2\,a+2\,c\right)+\cos\left(2\,a\right)-\cos\left(\right.\right.}{b\cos\left(4\,bx+a+5\,c\right)^2+4\,b\cos\left(2\,bx+a+3\,c\right)^2+4\,b\cos\left(2\,bx+a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2*
(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2*sin
(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*b
*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a +
 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.216 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}$$

```
[Out] 1/3*cos(a-c)*sec(b*x+c)^3/b+1/2*arctanh(sin(b*x+c))*sin(a-c)/b+1/2*sec(b*x+
c)*sin(a-c)*tan(b*x+c)/b
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3768, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^3)/(3*b) + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/(2*b
) + (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^3(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^3(c+bx)\,dx$$
$$= \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b} + \frac{\cos(a-c)\,\text{Subst}\left(\int x^2\,dx, x, \sec(c+bx)\right)}{b}$$
$$= \frac{\cos(a-c)\sec^3(c+bx)}{3b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b} + \frac{\sec(c+bx)\sin(a}{2b}$$

Mathematica [A] time = 0.44, size = 64, normalized size = 0.96

$$\frac{\sec^3(bx+c)(3\sin(a-c)\sin(2(bx+c))+4\cos(a-c))+12\sin(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\sin(c)\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*Co
s[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

fricas [A] time = 0.53, size = 94, normalized size = 1.40

$$-\frac{3\cos(bx+c)^3\log(\sin(bx+c)+1)\sin(-a+c) - 3\cos(bx+c)^3\log(-\sin(bx+c)+1)\sin(-a+c)+6\cos}{12\,b\cos(bx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)^
3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a +
 c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((tan(a/2)^2*tan(c/2)-tan(a/2)*tan(c/2)^2+tan(a/2)-tan(c/2))/(-
2*tan(a/2)^2*tan(c/2)^2-2*tan(a/2)^2-2*tan(c/2)^2-2)*ln(abs(tan((b*x+c)/2)-
1))+(tan(a/2)^2*tan(c/2)-tan(a/2)*tan(c/2)^2+tan(a/2)-tan(c/2))/(2*tan(a/2)
^2*tan(c/2)^2+2*tan(a/2)^2+2*tan(c/2)^2+2)*ln(abs(tan((b*x+c)/2)+1))+(-3*ta
n((b*x+c)/2)^5*tan(a/2)^2*tan(c/2)+3*tan((b*x+c)/2)^5*tan(a/2)*tan(c/2)^2-3
*tan((b*x+c)/2)^5*tan(a/2)+3*tan((b*x+c)/2)^5*tan(c/2)+3*tan((b*x+c)/2)^4*t
an(a/2)^2*tan(c/2)^2-3*tan((b*x+c)/2)^4*tan(a/2)^2+12*tan((b*x+c)/2)^4*tan(
a/2)*tan(c/2)-3*tan((b*x+c)/2)^4*tan(c/2)^2+3*tan((b*x+c)/2)^4+3*tan((b*x+c
)/2)*tan(a/2)^2*tan(c/2)-3*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^2+3*tan((b*x+c)
/2)*tan(a/2)-3*tan((b*x+c)/2)*tan(c/2)+tan(a/2)^2*tan(c/2)^2-tan(a/2)^2+4*t
an(a/2)*tan(c/2)-tan(c/2)^2+1)/(-3*tan(a/2)^2*tan(c/2)^2-3*tan(a/2)^2-3*tan
(c/2)^2-3)/(tan((b*x+c)/2)^2-1)^3)
```

maple [B] time = 8.84, size = 14825, normalized size = 221.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^4*sin(b*x+a),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.58, size = 1424, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6*b
*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos(a)
)*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*
c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*b*x +
 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 16*(3*c
os(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x + 2*c))*co
s(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*
(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*sin(-a + c)
+ 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*cos(a)*sin(
-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a + 4*c)^2*sin
(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x + a + 2*c)*si
n(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*cos(2*b*x + a
+ 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c) + 6*(3*cos(2*
b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(
3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c)*sin(-a + c) + s
in(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x + a + 2*c)*sin(-a
+ c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*sin
(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(
b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(
c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c)
 + sin(c)^2)) + 2*(3*sin(5*b*x + 2*a + 4*c) - 3*sin(5*b*x + 6*c) - 8*sin(3*
b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c)
)*sin(6*b*x + a + 6*c) + 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a + 2*c)
 + sin(a))*sin(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x
 + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) + 8*si
n(3*b*x + 4*c) + 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c)
- 16*(3*sin(2*b*x + a + 2*c) + sin(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2
*b*x + a + 2*c) + sin(a))*sin(3*b*x + 4*c) - 18*(sin(b*x + 2*a) - sin(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 6*sin(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*si
n(a))/(b*cos(6*b*x + a + 6*c)^2 + 9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*
x + a + 2*c)^2 + 6*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2
 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*b*sin(2*b*x + a + 2*c)^2 + 6*b*sin(2*b*x
+ a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b + 2*(3*b*cos(4*b*x + a + 4*c) +
 3*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(6*b*x + a + 6*c) + 6*(3*b*cos(2*b
*x + a + 2*c) + b*cos(a))*cos(4*b*x + a + 4*c) + 2*(3*b*sin(4*b*x + a + 4*c
) + 3*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(6*b*x + a + 6*c) + 6*(3*b*sin(
2*b*x + a + 2*c) + b*sin(a))*sin(4*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^4,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.217 $\int \sec^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\sin(a-c)\tan^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

```
[Out] 1/4*cos(a-c)*sec(b*x+c)^4/b+sin(a-c)*tan(b*x+c)/b+1/3*sin(a-c)*tan(b*x+c)^3
/b
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4580, 2606, 30, 3767}

$$\frac{\sin(a-c)\tan^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^4)/(4*b) + (Sin[a - c]*Tan[c + b*x])/b + (Sin[a -
c]*Tan[c + b*x]^3)/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+bx)\sin(a+bx)\,dx&=\cos(a-c)\int \sec^4(c+bx)\tan(c+bx)\,dx+\sin(a-c)\int \sec^4(c+bx)\,dx\\&=\frac{\cos(a-c)\,\text{Subst}\left(\int x^3\,dx,x,\sec(c+bx)\right)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\right.}{b}\\&=\frac{\cos(a-c)\sec^4(c+bx)}{4b}+\frac{\sin(a-c)\tan(c+bx)}{b}+\frac{\sin(a-c)\tan^3(c+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.34, size = 48, normalized size = 0.81

$$\frac{\sec(c)\sec^4(bx+c)(\sin(a-c)(4\sin(2bx+c)+\sin(4bx+3c))+3\cos(a))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Sec[c]*Sec[c + b*x]^4*(3*Cos[a] + Sin[a - c]*(4*Sin[c + 2*b*x] + Sin[3*c +
 4*b*x])))/(12*b)
```

fricas [A] time = 0.48, size = 53, normalized size = 0.90

$$-\frac{4\left(2\cos\left(bx+c\right)^3+\cos\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\cos\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(2*cos(b*x + c)^3 + cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 3*cos
(-a + c))/(b*cos(b*x + c)^4)
```

giac [B] time = 0.23, size = 327, normalized size = 5.54

$$\frac{3\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-3\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}a\right)^2+12\tan\left(bx+c\right)^4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+8\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*tan(b*x + c)^4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(b*x + c)^4*tan(1/2
*a)^2 + 12*tan(b*x + c)^4*tan(1/2*a)*tan(1/2*c) + 8*tan(b*x + c)^3*tan(1/2*
a)^2*tan(1/2*c) - 3*tan(b*x + c)^4*tan(1/2*c)^2 - 8*tan(b*x + c)^3*tan(1/2*
a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(b*x +
c)^4 + 8*tan(b*x + c)^3*tan(1/2*a) - 6*tan(b*x + c)^2*tan(1/2*a)^2 - 8*tan(
b*x + c)^3*tan(1/2*c) + 24*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 24*tan(b*
x + c)*tan(1/2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 24*tan(b*x
 + c)*tan(1/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2 + 24*tan(b*x + c)*tan(1/2*
a) - 24*tan(b*x + c)*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2
 + tan(1/2*c)^2 + 1)*b)
```

maple [B] time = 13.27, size = 333, normalized size = 5.64

$$\frac{\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\left(\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(a)\right)\left(\sin^2(c)\right)+\left(\cos^2(c)\right)\left(\sin^2(a)\right)+\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)}{4(\cos(a)\sin(c)-\sin(a)\cos(c))^4(\tan(bx+a)\cos(a)\sin(c)-\tan(bx+a)\sin(a)\cos(c)-\sin(a)\sin(c)-\cos(a)\cos(c))^4}-\frac{}{2(\cos(a)\sin(c)-\sin(a)\cos(c))^4(\tan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^5*sin(b*x+a),x)
```

```
[Out] 1/b*(1/4*(cos(a)*cos(c)+sin(a)*sin(c))*(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2
+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))^4/(tan(
b*x+a)*cos(a)*sin(c)-tan(b*x+a)*sin(a)*cos(c)-sin(a)*sin(c)-cos(a)*cos(c))^
4-1/2*(-3*cos(a)*cos(c)-3*sin(a)*sin(c))/(cos(a)*sin(c)-sin(a)*cos(c))^4/(t
an(b*x+a)*cos(a)*sin(c)-tan(b*x+a)*sin(a)*cos(c)-sin(a)*sin(c)-cos(a)*cos(c
))^2+1/(cos(a)*sin(c)-sin(a)*cos(c))^4/(tan(b*x+a)*cos(a)*sin(c)-tan(b*x+a)
```

```
*sin(a)*cos(c)-sin(a)*sin(c)-cos(a)*cos(c))-1/3*(-cos(a)^2*sin(c)^2-3*sin(a
)^2*sin(c)^2-4*cos(a)*cos(c)*sin(a)*sin(c)-3*cos(a)^2*cos(c)^2-cos(c)^2*sin
(a)^2)/(cos(a)*sin(c)-sin(a)*cos(c))^4/(tan(b*x+a)*cos(a)*sin(c)-tan(b*x+a)
*sin(a)*cos(c)-sin(a)*sin(c)-cos(a)*cos(c))^3)
```

maxima [B] time = 0.35, size = 1074, normalized size = 18.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/3*((6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4
*c) + cos(2*a) - cos(2*c))*cos(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*
c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c))*c
os(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*cos(4*b*x + 2
*a + 4*c) + 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(
2*b*x + 4*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 4*(4*cos(2*b*x +
 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) - 4*(4*cos(2*b*x +
a + 3*c) + cos(a + c))*cos(2*b*x + 4*c) + (cos(2*a) - cos(2*c))*cos(a + c)
+ 4*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (6*sin(4*b*x + 2*a + 4*c) + 4*sin(2
*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(8*b*x + a
 + 9*c) + 4*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*
b*x + 4*c) + sin(2*a) - sin(2*c))*sin(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a
 + 3*c) + sin(a + c))*sin(4*b*x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c)
+ 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(
4*b*x + a + 5*c) + 4*(4*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2
*b*x + a + 3*c) - 4*(4*sin(2*b*x + a + 3*c) + sin(a + c))*sin(2*b*x + 4*c)
+ (sin(2*a) - sin(2*c))*sin(a + c) + 4*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(
b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x + a
 + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x + a + 3*c)*cos(a +
c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)^
2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 + 8*b*sin(2*b
*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(4*b*cos(6*b*x + a + 7*c) + 6
*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(8*b*
x + a + 9*c) + 8*(6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*c
os(a + c))*cos(6*b*x + a + 7*c) + 12*(4*b*cos(2*b*x + a + 3*c) + b*cos(a +
c))*cos(4*b*x + a + 5*c) + 2*(4*b*sin(6*b*x + a + 7*c) + 6*b*sin(4*b*x + a
+ 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(8*b*x + a + 9*c) + 8*
(6*b*sin(4*b*x + a + 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6*
b*x + a + 7*c) + 12*(4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a
 + 5*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^5,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.218 $\int \sec^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{8b}+\frac{\cos(a-c)\sec^5(bx+c)}{5b}+\frac{\sin(a-c)\tan(bx+c)\sec^3(bx+c)}{4b}+\frac{3\sin(a-c)\tan$$

```
[Out] 1/5*cos(a-c)*sec(b*x+c)^5/b+3/8*arctanh(sin(b*x+c))*sin(a-c)/b+3/8*sec(b*x+
c)*sin(a-c)*tan(b*x+c)/b+1/4*sec(b*x+c)^3*sin(a-c)*tan(b*x+c)/b
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3768, 3770}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{8b}+\frac{\cos(a-c)\sec^5(bx+c)}{5b}+\frac{\sin(a-c)\tan(bx+c)\sec^3(bx+c)}{4b}+\frac{3\sin(a-c)\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^5)/(5*b) + (3*ArcTanh[Sin[c + b*x]]*Sin[a - c])/(8
*b) + (3*Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(8*b) + (Sec[c + b*x]^3*Sin[
a - c]*Tan[c + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^5(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^5(c+bx)\,dx$$
$$= \frac{\sec^3(c+bx)\sin(a-c)\tan(c+bx)}{4b} + \frac{\cos(a-c)\,\text{Subst}\left(\int x^4\,dx, x, \sec(c+bx)\right)}{b} +$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b} + \frac{\sec^3(c+bx)\sin$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{8b} + \frac{3\sec(c+bx)\sin(a}{8b}$$

Mathematica [A] time = 0.99, size = 78, normalized size = 0.83

$$\frac{2\sec^5(bx+c)(5\sin(a-c)(14\sin(2(bx+c))+3\sin(4(bx+c)))+64\cos(a-c))+480\sin(a-c)\tanh^{-1}\left(\cos(c)\text{t}\right.}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + 2*Sec[c + b*x]^5*(6
4*Cos[a - c] + 5*Sin[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))/(6
40*b)
```

fricas [A] time = 0.50, size = 107, normalized size = 1.14

$$-\frac{15\cos(bx+c)^5\log(\sin(bx+c)+1)\sin(-a+c) - 15\cos(bx+c)^5\log(-\sin(bx+c)+1)\sin(-a+c) + 10\left(3\right.}{80\,b\cos(bx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*cos(b*x + c)^5*log(sin(b*x + c) + 1)*sin(-a + c) - 15*cos(b*x + c
)^5*log(-sin(b*x + c) + 1)*sin(-a + c) + 10*(3*cos(b*x + c)^3 + 2*cos(b*x +
 c))*sin(b*x + c)*sin(-a + c) - 16*cos(-a + c))/(b*cos(b*x + c)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((3*tan(a/2)^2*tan(c/2)-3*tan(a/2)*tan(c/2)^2+3*tan(a/2)-3*tan(
c/2))/(-8*tan(a/2)^2*tan(c/2)^2-8*tan(a/2)^2-8*tan(c/2)^2-8)*ln(abs(tan((b*
x+c)/2)-1))+(3*tan(a/2)^2*tan(c/2)-3*tan(a/2)*tan(c/2)^2+3*tan(a/2)-3*tan(c
/2))/(8*tan(a/2)^2*tan(c/2)^2+8*tan(a/2)^2+8*tan(c/2)^2+8)*ln(abs(tan((b*x+
c)/2)+1))+(-25*tan((b*x+c)/2)^9*tan(a/2)^2*tan(c/2)+25*tan((b*x+c)/2)^9*tan
(a/2)*tan(c/2)^2-25*tan((b*x+c)/2)^9*tan(a/2)+25*tan((b*x+c)/2)^9*tan(c/2)+
20*tan((b*x+c)/2)^8*tan(a/2)^2*tan(c/2)^2-20*tan((b*x+c)/2)^8*tan(a/2)^2+80
*tan((b*x+c)/2)^8*tan(a/2)*tan(c/2)-20*tan((b*x+c)/2)^8*tan(c/2)^2+20*tan((
b*x+c)/2)^8+10*tan((b*x+c)/2)^7*tan(a/2)^2*tan(c/2)-10*tan((b*x+c)/2)^7*tan
(a/2)*tan(c/2)^2+10*tan((b*x+c)/2)^7*tan(a/2)-10*tan((b*x+c)/2)^7*tan(c/2)+
40*tan((b*x+c)/2)^4*tan(a/2)^2*tan(c/2)^2-40*tan((b*x+c)/2)^4*tan(a/2)^2+16
0*tan((b*x+c)/2)^4*tan(a/2)*tan(c/2)-40*tan((b*x+c)/2)^4*tan(c/2)^2+40*tan(
```

```
(b*x+c)/2)^4-10*tan((b*x+c)/2)^3*tan(a/2)^2*tan(c/2)+10*tan((b*x+c)/2)^3*ta
n(a/2)*tan(c/2)^2-10*tan((b*x+c)/2)^3*tan(a/2)+10*tan((b*x+c)/2)^3*tan(c/2)
+25*tan((b*x+c)/2)*tan(a/2)^2*tan(c/2)-25*tan((b*x+c)/2)*tan(a/2)*tan(c/2)^
2+25*tan((b*x+c)/2)*tan(a/2)-25*tan((b*x+c)/2)*tan(c/2)+4*tan(a/2)^2*tan(c/
2)^2-4*tan(a/2)^2+16*tan(a/2)*tan(c/2)-4*tan(c/2)^2+4)/(-20*tan(a/2)^2*tan(
c/2)^2-20*tan(a/2)^2-20*tan(c/2)^2-20)/(tan((b*x+c)/2)^2-1)^5)
```

maple [B] time = 14.87, size = 97703, normalized size = 1039.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^6*sin(b*x+a),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.69, size = 3096, normalized size = 32.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/80*(2*(15*cos(9*b*x + 2*a + 8*c) - 15*cos(9*b*x + 10*c) + 70*cos(7*b*x +
 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*
b*x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
 2*a) + 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) + 30*(5*cos(8*b*x + a + 8
*c) + 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a +
 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) + 10*co
s(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos
(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) - 70*cos(7*b*x + 8*c
) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a
+ 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8
*b*x + a + 8*c) + 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) +
5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x
 + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*co
s(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b*x + 6*c) + 70
*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) - 15*cos(
b*x + 2*c))*cos(6*b*x + a + 6*c) - 256*(10*cos(4*b*x + a + 4*c) + 5*cos(2*b
*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*
c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x
+ 2*a + 2*c) - 14*cos(3*b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*c
os(4*b*x + a + 4*c) - 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a
 + 2*c) + 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 150*(cos
(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 30*cos(b*x + 2*a)*cos(
a) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*sin(-a + c) +
25*cos(8*b*x + a + 8*c)^2*sin(-a + c) + 100*cos(6*b*x + a + 6*c)^2*sin(-a +
 c) + 100*cos(4*b*x + a + 4*c)^2*sin(-a + c) + 25*cos(2*b*x + a + 2*c)^2*si
n(-a + c) + 10*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(10*b*x + a + 1
0*c)^2*sin(-a + c) + 25*sin(8*b*x + a + 8*c)^2*sin(-a + c) + 100*sin(6*b*x
+ a + 6*c)^2*sin(-a + c) + 100*sin(4*b*x + a + 4*c)^2*sin(-a + c) + 25*sin(
2*b*x + a + 2*c)^2*sin(-a + c) + 10*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c)
 + 2*(5*cos(8*b*x + a + 8*c)*sin(-a + c) + 10*cos(6*b*x + a + 6*c)*sin(-a +
 c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a +
 c) + cos(a)*sin(-a + c))*cos(10*b*x + a + 10*c) + 10*(10*cos(6*b*x + a + 6
*c)*sin(-a + c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2
*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(8*b*x + a + 8*c) + 20*(10*cos(4*b
*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin
(-a + c))*cos(6*b*x + a + 6*c) + 20*(5*cos(2*b*x + a + 2*c)*sin(-a + c) + c
os(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(5*sin(8*b*x + a + 8*c)*sin(-a
```

```
+ c) + 10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a + 4*c)*sin(-a
 + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(10*b*x
 + a + 10*c) + 10*(10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a +
 4*c)*sin(-a + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c)
)*sin(8*b*x + a + 8*c) + 20*(10*sin(4*b*x + a + 4*c)*sin(-a + c) + 5*sin(2*
b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 20*
(5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4
*c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 -
 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin
(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2
*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + 2*(15*sin(9*b*x + 2*a + 8*c)
 - 15*sin(9*b*x + 10*c) + 70*sin(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) -
 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2
*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(10*b
*x + a + 10*c) + 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*
sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 2*a + 8
*c) - 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x +
 a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 10*c) + 10*(70*sin
(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) - 12
8*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*s
in(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(8*b*x + a + 8*c) + 140*(10*sin(6*b*x
 + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*si
n(7*b*x + 2*a + 6*c) - 140*(10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*
c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(7*b*x + 8*c) - 20*(128*sin(5*b*x
+ 2*a + 4*c) + 128*sin(5*b*x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*
b*x + 4*c) + 15*sin(b*x + 2*a) - 15*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) -
256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x +
 2*a + 4*c) - 256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a
))*sin(5*b*x + 6*c) - 100*(14*sin(3*b*x + 2*a + 2*c) - 14*sin(3*b*x + 4*c)
+ 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c) - 140*(5*sin(2*
b*x + a + 2*c) + sin(a))*sin(3*b*x + 2*a + 2*c) + 140*(5*sin(2*b*x + a + 2*
c) + sin(a))*sin(3*b*x + 4*c) - 150*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2
*b*x + a + 2*c) - 30*sin(b*x + 2*a)*sin(a) + 30*sin(b*x + 2*c)*sin(a))/(b*c
os(10*b*x + a + 10*c)^2 + 25*b*cos(8*b*x + a + 8*c)^2 + 100*b*cos(6*b*x + a
 + 6*c)^2 + 100*b*cos(4*b*x + a + 4*c)^2 + 25*b*cos(2*b*x + a + 2*c)^2 + 10
*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(10*b*x + a + 10*c)^2 + 25*b*sin(8*b*
x + a + 8*c)^2 + 100*b*sin(6*b*x + a + 6*c)^2 + 100*b*sin(4*b*x + a + 4*c)^
2 + 25*b*sin(2*b*x + a + 2*c)^2 + 10*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a
)^2 + sin(a)^2)*b + 2*(5*b*cos(8*b*x + a + 8*c) + 10*b*cos(6*b*x + a + 6*c)
 + 10*b*cos(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(10*
b*x + a + 10*c) + 10*(10*b*cos(6*b*x + a + 6*c) + 10*b*cos(4*b*x + a + 4*c)
 + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(8*b*x + a + 8*c) + 20*(10*b*cos
(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(6*b*x + a + 6*
c) + 20*(5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(4*b*x + a + 4*c) + 2*(5*b
*sin(8*b*x + a + 8*c) + 10*b*sin(6*b*x + a + 6*c) + 10*b*sin(4*b*x + a + 4*
c) + 5*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(10*b*x + a + 10*c) + 10*(10*b
*sin(6*b*x + a + 6*c) + 10*b*sin(4*b*x + a + 4*c) + 5*b*sin(2*b*x + a + 2*c
) + b*sin(a))*sin(8*b*x + a + 8*c) + 20*(10*b*sin(4*b*x + a + 4*c) + 5*b*si
n(2*b*x + a + 2*c) + b*sin(a))*sin(6*b*x + a + 6*c) + 20*(5*b*sin(2*b*x + a
 + 2*c) + b*sin(a))*sin(4*b*x + a + 4*c))
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^6,x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.219 $\int \cos^n(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=386

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(-i(2a+cn)-}{2b+dn}$$

```
[Out] -I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+ex
p(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],-exp(2*I*(d*x+c)))
/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-d*n+2
*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, b/d-1/
2*n],[1+b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+2*b)
+I*2^(-1-n)*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, -1/2*n],[1-1/
2*n],-exp(2*I*(d*x+c)))/d/((1+exp(2*I*(d*x+c)))^n)/n
```

Rubi [A] time = 0.70, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4555, 2282, 2032, 364, 2285, 2253, 2251}

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(-i(2a+cn)-}{2b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(E^
((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2, -
n, (2 - (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b
)/d - n)/2, -n, (2 + (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(E^((-I)*(c + d*x)) + E^(I*(
c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, -E^((2*I)*(c + d*x))])/(d
*(1 + E^((2*I)*(c + d*x)))^n*n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = 2^{-2-n}\int \left(2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{2ia+2ibx}\left(\right.\right.$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\left(e^{-i(c+dx)}+\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{x}+x\right)^n}{x}\,dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e\right.\right.$$

$$= -\left(\left(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(2a-cn)+i(2b-dn)x}\left(\right.\right.$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+d}\right.}{2b+dn}$$

Mathematica [A] time = 1.83, size = 242, normalized size = 0.63

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}e^{i(c+dx)-2i(a+bx)}\left(e^{2i(a+bx)}(2b+dn)\left(dne^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{d}+\frac{n}{2}+1;\frac{b}{d}-\frac{n}{2}+1;-\right.\right.\right.}{d^3n^3-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-2*I)*(a + b*x) + I*(c + d*x))*((1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x)))^(1 + n)*(d*n*(-2*b + d*n)*Hypergeometric2F1[1, 1 - b/d
 + n/2, 1 - b/d - n/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(2*b + d
```

```
*n)*(d*E^((2*I)*(a + b*x))*n*Hypergeometric2F1[1, 1 + b/d + n/2, 1 + b/d -
n/2, -E^((2*I)*(c + d*x))] + 2*(2*b - d*n)*Hypergeometric2F1[1, (2 + n)/2,
1 - n/2, -E^((2*I)*(c + d*x))])))/(-4*b^2*d*n + d^3*n^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^2-1\right)\cos\left(dx+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)^n\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

maple [F] time = 5.43, size = 0, normalized size = 0.00

$$\int \left(\cos^n\left(dx+c\right)\right)\left(\sin^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
[Out] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)^n\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^n\,{\sin\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x)^2,x)
[Out] int(cos(c + d*x)^n*sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**2,x)
[Out] Timed out
```

3.220 $\int \cos(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=68

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

```
[Out] -1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2637}

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] -Sin[2*a - c + (2*b - d)*x]/(4*(2*b - d)) + Sin[c + d*x]/(2*d) - Sin[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(a+bx)\,dx &= \int \left(-\frac{1}{4}\cos(2a-c+(2b-d)x)+\frac{1}{2}\cos(c+dx)-\frac{1}{4}\cos(2a+c+(2b+d)x)\right)\\ &= -\left(\frac{1}{4}\int \cos(2a-c+(2b-d)x)\,dx\right)-\frac{1}{4}\int \cos(2a+c+(2b+d)x)\,dx+\frac{1}{2}\int\\ &= -\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(c+dx)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A] time = 0.79, size = 76, normalized size = 1.12

$$\frac{1}{4}\left(-\frac{\sin(2a+2bx-c-dx)}{2b-d}-\frac{\sin(2a+2bx+c+dx)}{2b+d}+\frac{2\sin(c)\cos(dx)}{d}+\frac{2\cos(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d - Sin[2*a - c + 2*b*x - d*x]
/(2*b - d) - Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

fricas [A] time = 0.45, size = 70, normalized size = 1.03

$$-\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(bx+a)-\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\sin(dx+c)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*sin(d*x + c))/(4*b^2*d - d^3)
```

giac [A] time = 0.20, size = 61, normalized size = 0.90

$$-\frac{\sin\left(2\,bx+dx+2\,a+c\right)}{4\,(2\,b+d)}-\frac{\sin\left(2\,bx-dx+2\,a-c\right)}{4\,(2\,b-d)}+\frac{\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*sin(2*b*x - d*x + 2*a - c)/
(2*b - d) + 1/2*sin(d*x + c)/d
```

maple [A] time = 0.63, size = 63, normalized size = 0.93

$$-\frac{\sin\left(2a-c+(2b-d)\,x\right)}{4\,(2b-d)}+\frac{\sin\left(dx+c\right)}{2d}-\frac{\sin\left(2a+c+(2b+d)\,x\right)}{4\,(2b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^2,x)
```

```
[Out] -1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x)
/(2*b+d)
```

maxima [B] time = 0.35, size = 371, normalized size = 5.46

$$-\frac{\left(2\,bd\sin(c)-d^2\sin(c)\right)\cos\left((2\,b+d)x+2\,a+2\,c\right)-\left(2\,bd\sin(c)-d^2\sin(c)\right)\cos\left((2\,b+d)x+2\,a\right)-\left(2\,bd\sin(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*sin
(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*cos(
-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) + 2*(4*b^2*sin(c) - d^2
*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c) + d^
2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c))*sin(-
(2*b - d)*x - 2*a) + 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) + 2*(4*b^
2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

mupad [B] time = 0.81, size = 105, normalized size = 1.54

$$\frac{\sin\left(c+d\,x\right)}{2\,d}-\frac{b\,\left(2\,d\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+2\,d\,\sin\left(2\,a-c+2\,b\,x-d\,x\right)\right)-d^2\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+d^2}{16\,b^2\,d-4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x)^2,x)
```

```
[Out] sin(c + d*x)/(2*d) - (b*(2*d*sin(2*a + c + 2*b*x + d*x) + 2*d*sin(2*a - c +
 2*b*x - d*x)) - d^2*sin(2*a + c + 2*b*x + d*x) + d^2*sin(2*a - c + 2*b*x -
 d*x))/(16*b^2*d - 4*d^3)
```

sympy [A] time = 6.70, size = 408, normalized size = 6.00

$$\begin{cases} x\sin^2(a)\cos(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4}+\frac{x\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2}-\frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4}+\frac{\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{d}-\frac{\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4}-\frac{x\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2}-\frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4}+\frac{\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{d}+\frac{\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2d} \\ \left(\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos(c) \\ \frac{2b^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3}+\frac{2b^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3}-\frac{2bd\sin(a+bx)\cos(a+bx)\cos(c+dx)}{4b^2d-d^3}-\frac{d^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*c
os(c + d*x)/4 + x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 - x*cos(a -
d*x/2)**2*cos(c + d*x)/4 + sin(a - d*x/2)**2*sin(c + d*x)/d - sin(a - d*x/2
)*cos(a - d*x/2)*cos(c + d*x)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2*cos
(c + d*x)/4 - x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 - x*cos(a + d*
x/2)**2*cos(c + d*x)/4 + sin(a + d*x/2)**2*sin(c + d*x)/d + sin(a + d*x/2)*
cos(a + d*x/2)*cos(c + d*x)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x*c
os(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(d, 0)), (2*b
**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*sin(c + d*x)*co
s(a + b*x)**2/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*cos(a + b*x)*cos(c + d
*x)/(4*b**2*d - d**3) - d**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3)
, True))
```

3.221 $\int \cos^2(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b-1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x+
2*c)/d-1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2637}

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)-\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx)-\frac{1}{8}\text{c}\right.\\ &= \frac{x}{4}-\frac{1}{8}\int\cos(2(a-c)+2(b-d)x)\,dx-\frac{1}{8}\int\cos(2(a+c)+2(b+d)x)\,dx-\frac{1}{4}\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}-\frac{\sin(2(a+c)}{16(b}\end{aligned}$$

Mathematica [A] time = 0.73, size = 108, normalized size = 1.23

$$\frac{\left(2d^3-2b^2d\right)\sin(2(a+bx))-bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(-d\sin(2(a+x(b+d)+c))+2(b+d)}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] - b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sin[2*(c + d*x)] - d*Sin[2*(a + c
 + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

fricas [A]   time = 0.44, size = 108, normalized size = 1.23

$$\frac{\left(2\,bd^{2}\cos\left(bx+a\right)^{2}+b^{3}-2\,bd^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(b^{3}d-bd^{3}\right)x-\left(2\,b^{2}d\cos\left(bx+a\right)\cos\left(dx+c\right)^{2}-\right.}{4\left(b^{3}d-bd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) + (
b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))
*sin(b*x + a))/(b^3*d - b*d^3)
```

giac [A]   time = 0.20, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x-\frac{\sin\left(2\,bx+2\,dx+2\,a+2\,c\right)}{16\,(b+d)}-\frac{\sin\left(2\,bx-2\,dx+2\,a-2\,c\right)}{16\,(b-d)}-\frac{\sin\left(2\,bx+2\,a\right)}{8\,b}+\frac{\sin\left(2\,dx+2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x - 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) - 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

maple [A]   time = 1.23, size = 83, normalized size = 0.94

$$\frac{x}{4}-\frac{\sin\left(2bx+2a\right)}{8b}+\frac{\sin\left(2dx+2c\right)}{8d}-\frac{\sin\left(\left(2b-2d\right)x+2a-2c\right)}{16\left(b-d\right)}-\frac{\sin\left(\left(2b+2d\right)x+2a+2c\right)}{16\left(b+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d-1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)-1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

maxima [B]   time = 0.38, size = 620, normalized size = 7.05

$$\frac{8\left(\left(b\cos\left(2\,c\right)^{2}+b\sin\left(2\,c\right)^{2}\right)d^{3}-\left(b^{3}\cos\left(2\,c\right)^{2}+b^{3}\sin\left(2\,c\right)^{2}\right)d\right)x-\left(b^{2}d\sin\left(2\,c\right)-bd^{2}\sin\left(2\,c\right)\right)\cos\left(2\,(b+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) +
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) + (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) - (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) - (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

mupad [B] time = 0.93, size = 177, normalized size = 2.01

$$-\frac{b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x)^2,x)
```

```
[Out] -(b*d^2*sin(2*a - 2*c + 2*b*x - 2*d*x) - 2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin
(2*a + 2*b*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
+ 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a +
 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) + 4*b*d^3*x - 4*b^3*d*x)/(16*b*d*(b^2 -
d^2))
```

sympy [A] time = 22.98, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (x*sin(a - d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a - d*x)**2*cos(c + d
*x)**2/8 + x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + 3*x*si
n(c + d*x)**2*cos(a - d*x)**2/8 + x*cos(a - d*x)**2*cos(c + d*x)**2/8 + sin
(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*co
s(c + d*x)**2/(2*d) + 3*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (x*sin(a + d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + 3*x
*sin(c + d*x)**2*cos(a + d*x)**2/8 + x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) - 5*sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

3.222 $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=144

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+$$

```
[Out] -1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-1/
16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2637}

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) -
(3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3*d)
*x]/(16*(2*b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin^2(a+bx)\,dx &= \int\left(-\frac{1}{16}\cos(2a-3c+(2b-3d)x)-\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos(c+\right.\\ &= -\left(\frac{1}{16}\int\cos(2a-3c+(2b-3d)x)\,dx\right)-\frac{1}{16}\int\cos(2a+3c+(2b+3d)x)\,d\\ &= -\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\mathrm{si}}{}\end{aligned}$$

Mathematica [A] time = 1.76, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(-\frac{3\sin(2a+2bx-3c-3dx)}{2b-3d}-\frac{9\sin(2a+2bx-c-dx)}{2b-d}-\frac{9\sin(2a+2bx+c+dx)}{2b+d}-\frac{3\sin(2a+2bx+3c}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d - (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
```

```
*d) - (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) - (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) - (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

fricas [A] time = 0.43, size = 174, normalized size = 1.21

$$\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+a\right)^2-1\right.}{3\left(16\,b^4d-40\,b^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)*
cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 - 16*b^4 + 40*b^2*d^2
 - 18*d^4 - (8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^
2)*cos(d*x + c)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

giac [A] time = 0.28, size = 129, normalized size = 0.90

$$-\frac{\sin\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\,(2\,b+3\,d)}-\frac{3\,\sin\left(2\,bx+dx+2\,a+c\right)}{16\,(2\,b+d)}-\frac{3\,\sin\left(2\,bx-dx+2\,a-c\right)}{16\,(2\,b-d)}-\frac{\sin\left(2\,bx-3\,dx+2\,a-3\,c\right.}{16\,(2\,b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/16*sin(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/16*sin(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*
x + c)/d
```

maple [A] time = 1.19, size = 133, normalized size = 0.92

$$-\frac{\sin\left(2a-3c+(2b-3d)\,x\right)}{16\,(2b-3d)}-\frac{3\sin\left(2a-c+(2b-d)\,x\right)}{16\,(2b-d)}+\frac{3\sin\left(dx+c\right)}{8d}+\frac{\sin\left(3dx+3c\right)}{24d}-\frac{3\sin\left(2a+c+(2b+d)\right.}{16\,(2b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^2,x)
```

```
[Out] -1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-1/
16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [B] time = 0.43, size = 1362, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*
sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2
*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) + 9
*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c
))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
- 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*b
^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*co
s(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18
*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*
d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-
```

```
(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*
b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(
3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) - 2*(16*b^4*sin(3*c
) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) - 18*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) + 18*(16*b^4*s
in(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*d
*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2
*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*
d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3*
c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d)
*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3
*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x -
 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
- 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12*
b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) -
 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*cos(3*c) - 40*b^2*d^
2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) + 2*(16*b^4*cos(3*c) - 40*b^2*d^2*c
os(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) + 18*(16*b^4*cos(3*c) - 40*b^2
*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^2
)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^4
*sin(3*c)^2)*d)
```

mupad [B] time = 1.97, size = 495, normalized size = 3.44

$$-\mathrm{e}^{a\,2\mathrm{i}-c\,1\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}+\frac{3\,d\,\left(2\,b+d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}-\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}\right)+\mathrm{e}^{a\,2\mathrm{i}+c\,1\mathrm{i}+b\,x\,2\mathrm{i}+d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x)^2,x)
```

```
[Out] exp(a*2i + c*1i + b*x*2i + d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))
/(b^2*d*128i - d^3*32i) - (3*d*(2*b - d))/(b^2*d*128i - d^3*32i) + (3*d*exp
(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i + b*
x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*3
2i) + (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) - (3*d*exp(- a*4i - b*x*4i)*(2
*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d
*(2*b + 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d
^2))/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d))/(b^2*
d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((exp(- a*2i - b*x
*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b - 3*d))/(b^2*d*3
84i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*384i - d^3*
864i))
```

sympy [A] time = 115.10, size = 1999, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x
/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a - 3*d*x/2)**2*cos(c + d*x)
**3/16 - x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 + 3*x*sin(a
- 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 + 3*x*sin(c + d*
x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a - 3*d*x/2)**2*cos(c + d
*x)**3/16 + 5*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*d*x/2)
```

```
**2*sin(c + d*x)*cos(c + d*x)**2/d + 5*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos
(a - 3*d*x/2)*cos(c + d*x)/(4*d) - sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(c
+ d*x)**3/(24*d) + 9*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d), Eq(b, -3*d
/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*x*sin(a -
d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a - d*x/2)*sin(c + d*x)**3*cos(a - d
*x/2)/8 + 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/8
- 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 - 3*x*cos(a - d*x/2
)**2*cos(c + d*x)**3/16 + 31*sin(a - d*x/2)**2*sin(c + d*x)**3/(48*d) + sin
(a - d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - sin(a - d*x/2)*sin(c + d*x)
**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) - 3*sin(a - d*x/2)*cos(a - d*x/2)*cos
(c + d*x)**3/(8*d) + sin(c + d*x)**3*cos(a - d*x/2)**2/(48*d), Eq(b, -d/2))
, (3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*x*sin(a + d*x/
2)**2*cos(c + d*x)**3/16 - 3*x*sin(a + d*x/2)*sin(c + d*x)**3*cos(a + d*x/2
)/8 - 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*cos(c + d*x)**2/8 - 3*
x*sin(c + d*x)**2*cos(a + d*x/2)**2*cos(c + d*x)/16 - 3*x*cos(a + d*x/2)**2
*cos(c + d*x)**3/16 + 31*sin(a + d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a +
 d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + sin(a + d*x/2)*sin(c + d*x)**2*
cos(a + d*x/2)*cos(c + d*x)/(4*d) + 3*sin(a + d*x/2)*cos(a + d*x/2)*cos(c +
 d*x)**3/(8*d) + sin(c + d*x)**3*cos(a + d*x/2)**2/(48*d), Eq(b, d/2)), (-3
*x*sin(a + 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a + 3*d*x/2)
**2*cos(c + d*x)**3/16 + x*sin(a + 3*d*x/2)*sin(c + d*x)**3*cos(a + 3*d*x/2
)/8 - 3*x*sin(a + 3*d*x/2)*sin(c + d*x)*cos(a + 3*d*x/2)*cos(c + d*x)**2/8
+ 3*x*sin(c + d*x)**2*cos(a + 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a + 3*d*x
/2)**2*cos(c + d*x)**3/16 + 5*sin(a + 3*d*x/2)**2*sin(c + d*x)**3/(48*d) +
sin(a + 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - 5*sin(a + 3*d*x/2)*sin
(c + d*x)**2*cos(a + 3*d*x/2)*cos(c + d*x)/(4*d) + sin(a + 3*d*x/2)*cos(a +
 3*d*x/2)*cos(c + d*x)**3/(24*d) + 9*sin(c + d*x)**3*cos(a + 3*d*x/2)**2/(1
6*d), Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a +
b*x)*cos(a + b*x)/(2*b))*cos(c)**3, Eq(d, 0)), (16*b**4*sin(a + b*x)**2*sin
(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**4*sin(a + b*x)**
2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 16*b
**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) +
 24*b**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**4*d - 120*b**2
*d**3 + 27*d**5) - 24*b**3*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**3/(48*
b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sin(a + b*x)**2*sin(c + d*
x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sin(a + b*x)**2*
sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**
2*d**2*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5
) - 42*b**2*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**4*d -
120*b**2*d**3 + 27*d**5) + 36*b*d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b
*x)*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b*d**3*sin(a +
b*x)*cos(a + b*x)*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 1
8*d**4*sin(a + b*x)**2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5
) + 27*d**4*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b
**2*d**3 + 27*d**5), True))
```

3.223 $\int \cos^n(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=568

$$\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(}{3b-dn}$$

```
[Out] 2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(
I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],-exp(2*I*(d*x
+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+I*(-d
*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*
(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)
/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c
))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-d*n-b)/d],-exp
(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*exp(-I*(c*n+3*a)
-I*(d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-
n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I
*d*x))^n)/(d*n+3*b)
```

Rubi [A] time = 1.18, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4555, 2285, 2253, 2251}

$$\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(}{3b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 + (
3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E
^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d
), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*
n)/(2*d), 1 - (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1
[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 +
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = 2^{-3-n}\int \left(3ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - ie^{-3ia-}\right.$$

$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3ibx}\left(e^{-i(c+dx)}\right.$$

$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ibx-in(c+dx)}\left(1\right.\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(1+e^{2}\right.$$

$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(}\right.}{3b-dn}$$

Mathematica [A] time = 23.94, size = 329, normalized size = 0.58

$$2^{-n-3}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}e^{i(-3a+c+d(n+1)x)}\left(-\frac{3e^{2ia-ix(b+dn)}\,{}_2F_1\left(1,\frac{1}{2}\left(-\frac{b}{d}+n+2\right);-\frac{b+d(n-2)}{2d};-e^{2i(c+dx)}\right)}{b+dn}+e^{i(}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

```
[Out] 2^(-3 - n)*E^(I*(-3*a + c + d*(1 + n)*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x)))^(1 + n)*(Hypergeometric2F1[1, (2 - (3*b)/d + n)/2, 1 - (3*b)/(2*
d) - n/2, -E^((2*I)*(c + d*x))]/(E^(I*(3*b + d*n)*x)*(3*b + d*n)) - (3*E^((
2*I)*a - I*(b + d*n)*x)*Hypergeometric2F1[1, (2 - b/d + n)/2, -1/2*(b + d*(
-2 + n))/d, -E^((2*I)*(c + d*x))])/(b + d*n) + E^(I*(4*a + b*x - d*n*x))*((
E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (2 + (3*b)/d + n)/2, 1 + (3*b)/(2*
d) - n/2, -E^((2*I)*(c + d*x))])/(3*b - d*n) - (3*Hypergeometric2F1[1, (b +
 d*(2 + n))/(2*d), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/(b - d*n)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos(bx+a)^2-1\right)\cos(dx+c)^n\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(dx+c)^n\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

maple [F] time = 5.52, size = 0, normalized size = 0.00

$$\int (\cos^n (dx + c)) \left(\sin^3 (bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos (dx + c)^n \sin (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c + d\,x)^n \sin (a + b\,x)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x)^3,x)
```

```
[Out] int(cos(c + d*x)^n*sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.224 $\int \cos(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] -3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d)*
x]/(8*(3*b + d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{8}\sin(a-c+(b-d)x)-\frac{1}{8}\sin(3a-c+(3b-d)x)+\frac{3}{8}\sin(a+c+(b+d)x)\right.\\ &= -\left(\frac{1}{8}\int \sin(3a-c+(3b-d)x)\,dx\right)-\frac{1}{8}\int \sin(3a+c+(3b+d)x)\,dx+\frac{3}{8}\int \sin\\ &= -\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\end{aligned}$$

Mathematica [A] time = 0.53, size = 90, normalized size = 0.93

$$\frac{1}{8}\left(-\frac{3\cos(a+bx-c-dx)}{b-d}+\frac{\cos(3a+3bx-c-dx)}{3b-d}+\frac{\cos(3a+3bx+c+dx)}{3b+d}-\frac{3\cos(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] ((-3*Cos[a - c + b*x - d*x])/(b - d) + Cos[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Cos[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cos[a + c + (b + d)*x])/(b + d
))/8
```

fricas [A] time = 0.44, size = 116, normalized size = 1.20

$$-\frac{\left(7\,b^{2}d-d^{3}-\left(b^{2}d-d^{3}\right)\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-3\left(\left(b^{3}-bd^{2}\right)\cos\left(bx+a\right)^{3}-\left(3\,b^{3}-bd^{2}\right)\mathrm{co}\right.}{9\,b^{4}-10\,b^{2}d^{2}+d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*sin(b*x + a)*sin(d*x + c)
- 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*cos(d*x +
 c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

giac [A] time = 0.25, size = 89, normalized size = 0.92

$$\frac{\cos\left(3\,bx+dx+3\,a+c\right)}{8\left(3\,b+d\right)}+\frac{\cos\left(3\,bx-dx+3\,a-c\right)}{8\left(3\,b-d\right)}-\frac{3\,\cos\left(bx+dx+a+c\right)}{8\left(b+d\right)}-\frac{3\,\cos\left(bx-dx+a-c\right)}{8\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*cos(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*cos(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*cos(b*x + d*x + a + c)/(b + d) - 3/8*cos(b*x - d*x + a - c)/
(b - d)
```

maple [A] time = 0.27, size = 90, normalized size = 0.93

$$-\frac{3\cos\left(a-c+\left(b-d\right)x\right)}{8\left(b-d\right)}+\frac{\cos\left(3a-c+\left(3b-d\right)x\right)}{24b-8d}-\frac{3\cos\left(a+c+\left(b+d\right)x\right)}{8\left(b+d\right)}+\frac{\cos\left(3a+c+\left(3b+d\right)x\right)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^3,x)
```

```
[Out] -3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

maxima [B] time = 0.38, size = 785, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*cos((3*b
+ d)*x + 3*a + 2*c) + (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*c
os(c))*cos((3*b + d)*x + 3*a) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(
c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*cos(c) + b^2*d*cos(
c) - 3*b*d^2*cos(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*cos(c)
 - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x + a + 2*c) - 3
*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x
+ a) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*cos(-(
b - d)*x - a + 2*c) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3
*cos(c))*cos(-(b - d)*x - a) + (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c
) + d^3*sin(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c)
 - 3*b*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^
2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x - 3*a + 2*c) - (
3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x
 - 3*a) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*sin
((b + d)*x + a + 2*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d
^3*sin(c))*sin((b + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
```

```
n(c) - d^3*sin(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*si
n(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9*
b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2)
*d^2)
```

mupad [B] time = 1.61, size = 471, normalized size = 4.86

$$-e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}+\frac{e^{-a\,6i-b\,x\,6i}\,\left(-3\,b^3+b^2\,d+3\,b\,d^2-d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{e^{-a\,2i-b\,x\,2i}\,\left(-27\,b^3-2\right.}{144\,b^4-160}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(144*
b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*b^
3 - d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2i)*(3*b*d^2
 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*
4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160
*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d - 3*b^3 -
 d^3)/(144*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 - b
^2*d - 3*b^3 + d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2)
- (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 1
6*d^4 - 160*b^2*d^2))
```

sympy [A] time = 32.62, size = 933, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*c
os(c + d*x)/8 + 3*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/8 + 3*x*sin(a
 - d*x)*cos(a - d*x)**2*cos(c + d*x)/8 + 3*x*sin(c + d*x)*cos(a - d*x)**3/8
 - sin(a - d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a - d*x)**2*cos(a - d*x)*cos(
c + d*x)/(4*d) + 3*cos(a - d*x)**3*cos(c + d*x)/(8*d), Eq(b, -d)), (x*sin(a
 - d*x/3)**3*cos(c + d*x)/8 + 3*x*sin(a - d*x/3)**2*sin(c + d*x)*cos(a - d*
x/3)/8 - 3*x*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/8 - x*sin(c + d*
x)*cos(a - d*x/3)**3/8 + 9*sin(a - d*x/3)**3*sin(c + d*x)/(8*d) - 3*sin(a -
 d*x/3)**2*cos(a - d*x/3)*cos(c + d*x)/(4*d) - cos(a - d*x/3)**3*cos(c + d*
x)/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*cos(c + d*x)/8 - 3*x*sin(a + d
*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/8 - 3*x*sin(a + d*x/3)*cos(a + d*x/3)*
*2*cos(c + d*x)/8 + x*sin(c + d*x)*cos(a + d*x/3)**3/8 + 9*sin(a + d*x/3)**
3*sin(c + d*x)/(8*d) + 3*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/(4*d
) + cos(a + d*x/3)**3*cos(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sin(a + d*x)**3
*cos(c + d*x)/8 - 3*x*sin(a + d*x)**2*sin(c + d*x)*cos(a + d*x)/8 + 3*x*sin
(a + d*x)*cos(a + d*x)**2*cos(c + d*x)/8 - 3*x*sin(c + d*x)*cos(a + d*x)**3
/8 + 5*sin(a + d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x)*sin(c + d*x)*cos
(a + d*x)**2/(4*d) + 3*cos(a + d*x)**3*cos(c + d*x)/(8*d), Eq(b, d)), (-9*b
**3*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4
) - 6*b**3*cos(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 7*
b**2*d*sin(a + b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 6*b**2
*d*sin(a + b*x)*sin(c + d*x)*cos(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4)
 + 3*b*d**2*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2*d**
2 + d**4) + d**3*sin(a + b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4
), True))
```

3.225 $\int \cos^2(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}$$

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) -
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)+\frac{3}{16}\sin(a-2c+(b-2d)x)-\frac{1}{16}\sin(3\right.\\ &= -\left(\frac{1}{16}\int \sin(3a-2c+(3b-2d)x)\,dx\right)-\frac{1}{16}\int \sin(3a+2c+(3b+2d)x)\,dx\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c}{16(3\end{aligned}$$

Mathematica [A] time = 1.57, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{9\cos(a+bx-2c-2dx)}{b-2d}+\frac{3\cos(3a+3bx-2c-2dx)}{3b-2d}-\frac{9\cos(a+bx+2c+2dx)}{b+2d}+\frac{3\cos(3a+3bx+}{3b+2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b - (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) + (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) - (9*
```

```
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

fricas [A] time = 0.44, size = 179, normalized size = 1.30

$$-\frac{2\left(b^2d^2-4\,d^4\right)\cos\left(bx+a\right)^3+6\left(7\,b^3d-4\,bd^3-\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\sin\left(dx+\right.}{3\left(9\,b^5-40\,b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(b^2*d^2 - 4*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (b^3*d -
4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9*((b^4 -
 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(d*x + c)
^2 - 6*(7*b^2*d^2 - 4*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4)
```

giac [A] time = 0.19, size = 124, normalized size = 0.90

$$\frac{\cos\left(3\,bx+2\,dx+3\,a+2\,c\right)}{16\left(3\,b+2\,d\right)}+\frac{\cos\left(3\,bx-2\,dx+3\,a-2\,c\right)}{16\left(3\,b-2\,d\right)}+\frac{\cos\left(3\,bx+3\,a\right)}{24\,b}-\frac{3\,\cos\left(bx+2\,dx+a+2\,c\right)}{16\left(b+2\,d\right)}-\frac{3\,\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*cos(3*b*x - 2*d*x +
3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b - 3/16*cos(b*x + 2*d*x + a
 + 2*c)/(b + 2*d) - 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*x
 + a)/b
```

maple [A] time = 0.29, size = 127, normalized size = 0.92

$$-\frac{3\cos\left(bx+a\right)}{8b}+\frac{\cos\left(3bx+3a\right)}{24b}-\frac{3\cos\left(a-2c+\left(b-2d\right)x\right)}{16\left(b-2d\right)}+\frac{\cos\left(3a-2c+\left(3b-2d\right)x\right)}{48b-32d}-\frac{3\cos\left(a+2c+\left(b+2d\right.\right.}{16\left(b+2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^3,x)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

maxima [B] time = 0.42, size = 1360, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*
cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos(
2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) + 3
*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c
))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) -
 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*b
^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*co
s((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^
2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) + 1
8*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
```

```
- a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8
*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*cos(2*c) - 40*b^2*d^2*cos
(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) + 2*(9*b^4*cos(2*c) - 40*b^
2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) - 18*(9*b^4*cos(2*
c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) - 18*(9*b^4*
cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(3
*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*
sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*
b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*si
n(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3
*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^
2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*c
) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*d
)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c)
 + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*si
n(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin
(2*c))*sin(-(b - 2*d)*x - a) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 16
*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(
2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) - 18*(9*b^4*sin(2*c) - 40*b^
2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) + 18*(9*b^4*sin(2*c) -
 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c)
^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos(
2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

mupad [B] time = 2.03, size = 438, normalized size = 3.17

$$-\frac{81\,b^4\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)+162\,b^4\,\cos\left(a+b\,x\right)+288\,d^4\,\cos\left(a+b\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x)^3,x)
```

```
[Out] -(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) + 1
62*b^4*cos(a + b*x) + 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x -
2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) - 18*b^4*cos(3*a + 3*b*x) - 3
2*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*d^3
*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x) +
6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x - 2*d
*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) - 720*b^2*d^2*cos(a + b*x) + 36
*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c + 3*b*x
+ 2*d*x) + 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x - 2*d*x
) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c + b*x - 2*d
*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5 - 40*b^3*
d^2))
```

sympy [A] time = 117.13, size = 2020, normalized size = 14.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq(b
, 0)), (-3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**3*c
os(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*cos(c
 + d*x)/8 - 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 + 3*x*s
in(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)*cos(a
```

```
 - 2*d*x)**3*cos(c + d*x)/8 - 3*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c + d*x)
/(16*d) + sin(a - 2*d*x)**2*cos(a - 2*d*x)*cos(c + d*x)**2/(2*d) - sin(a -
2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) + sin(c + d*x)**2*
cos(a - 2*d*x)**3/(96*d) + 31*cos(a - 2*d*x)**3*cos(c + d*x)**2/(96*d), Eq(
b, -2*d)), (-x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a - 2*d*x/3)*
*3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*cos(a - 2*d*x/
3)*cos(c + d*x)/8 + 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a - 2*d*x/3)**
2/16 - 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)**2/16 - x*sin(
c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 - sin(a - 2*d*x/3)**3*sin(c + d
*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*cos(a - 2*d*x/3)*cos(c + d*
x)**2/(2*d) + 15*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d*x/3)**2*cos(c +
d*x)/(8*d) + 27*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d) + 5*cos(a - 2*d*
x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (-x*sin(a + 2*d*x/3)**3*sin
(c + d*x)**2/16 + x*sin(a + 2*d*x/3)**3*cos(c + d*x)**2/16 - 3*x*sin(a + 2*
d*x/3)**2*sin(c + d*x)*cos(a + 2*d*x/3)*cos(c + d*x)/8 + 3*x*sin(a + 2*d*x/
3)*sin(c + d*x)**2*cos(a + 2*d*x/3)**2/16 - 3*x*sin(a + 2*d*x/3)*cos(a + 2*
d*x/3)**2*cos(c + d*x)**2/16 + x*sin(c + d*x)*cos(a + 2*d*x/3)**3*cos(c + d
*x)/8 - sin(a + 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) - 3*sin(a + 2*
d*x/3)**2*cos(a + 2*d*x/3)*cos(c + d*x)**2/(2*d) + 15*sin(a + 2*d*x/3)*sin(
c + d*x)*cos(a + 2*d*x/3)**2*cos(c + d*x)/(8*d) - 27*sin(c + d*x)**2*cos(a
+ 2*d*x/3)**3/(32*d) - 5*cos(a + 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b,
2*d/3)), (-3*x*sin(a + 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a + 2*d*x)**3
*cos(c + d*x)**2/16 - 3*x*sin(a + 2*d*x)**2*sin(c + d*x)*cos(a + 2*d*x)*cos
(c + d*x)/8 - 3*x*sin(a + 2*d*x)*sin(c + d*x)**2*cos(a + 2*d*x)**2/16 + 3*x
*sin(a + 2*d*x)*cos(a + 2*d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(c + d*x)*cos
(a + 2*d*x)**3*cos(c + d*x)/8 - 3*sin(a + 2*d*x)**3*sin(c + d*x)*cos(c + d*
x)/(16*d) - sin(a + 2*d*x)**2*cos(a + 2*d*x)*cos(c + d*x)**2/(2*d) - sin(a
+ 2*d*x)*sin(c + d*x)*cos(a + 2*d*x)**2*cos(c + d*x)/(8*d) - sin(c + d*x)**
2*cos(a + 2*d*x)**3/(96*d) - 31*cos(a + 2*d*x)**3*cos(c + d*x)**2/(96*d), E
q(b, 2*d)), (-27*b**4*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)**2/(27*b**5
 - 120*b**3*d**2 + 48*b*d**4) - 18*b**4*cos(a + b*x)**3*cos(c + d*x)**2/(27
*b**5 - 120*b**3*d**2 + 48*b*d**4) - 42*b**3*d*sin(a + b*x)**3*sin(c + d*x)
*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 36*b**3*d*sin(a + b*x
)*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b
*d**4) + 42*b**2*d**2*sin(a + b*x)**2*sin(c + d*x)**2*cos(a + b*x)/(27*b**5
 - 120*b**3*d**2 + 48*b*d**4) + 78*b**2*d**2*sin(a + b*x)**2*cos(a + b*x)*c
os(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40*b**2*d**2*sin(c +
 d*x)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40*b**2*d*
*2*cos(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) +
24*b*d**3*sin(a + b*x)**3*sin(c + d*x)*cos(c + d*x)/(27*b**5 - 120*b**3*d**
2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(a + b*x)/(27*b
**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*cos(a + b*x)*cos
(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d**4*sin(c + d*x)**
2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d**4*cos(a + b
*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4), True))
```

3.226 $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9c}{}$$

```
[Out] -3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)+1/96*cos(3*a
-3*c+3*(b-d)*x)/(b-d)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cos(a+c+(b+d)*
x)/(b+d)+1/96*cos(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*b+d
)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.12, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x])
/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d)) +
Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}\sin(a-3c+(b-3d)x)+\frac{9}{32}\sin(a-c+(b-d)x)-\frac{1}{32}\sin(3(a-c)\right.\\ &= -\left(\frac{1}{32}\int \sin(3(a-c)+3(b-d)x)\,dx\right)-\frac{1}{32}\int \sin(3(a+c)+3(b+d)x)\,dx\\ &= -\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}+\frac{\cos(3(a-c)+3(b-}{96(b-d)}\end{aligned}$$

Mathematica [A] time = 1.55, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(-\frac{9\cos(a+bx-3c-3dx)}{b-3d}-\frac{27\cos(a+bx-c-dx)}{b-d}+\frac{\cos(3(a+bx-c-dx))}{b-d}+\frac{9\cos(3a+3bx-c-dx}{3b-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ((-9*Cos[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cos[a - c + b*x - d*x])/(b
 - d) + Cos[3*(a - c + b*x - d*x)]/(b - d) + (9*Cos[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Cos[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cos[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Cos[a + c + (b + d)*x])/(b + d) + Cos[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

fricas [A] time = 0.50, size = 264, normalized size = 1.35

$$\frac{\left(\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^3-3\left(9\,b^5-28\,b^3d^2+3\,bd^4\right)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^3+\left(122\,b^2d^3-18\,d^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2
 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^3 + (122*b^2*d^3 - 18*d^5 - 2*(b^2*d
^3 - 9*d^5)*cos(b*x + a)^2 - (63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82
*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c)
 - 6*((b^3*d^2 - 9*b*d^4)*cos(b*x + a)^3 - 3*(7*b^3*d^2 - 3*b*d^4)*cos(b*x
+ a))*cos(d*x + c))/(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

giac [A] time = 0.28, size = 181, normalized size = 0.93

$$\frac{\cos\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\,(b+d)}+\frac{3\,\cos\left(3\,bx+dx+3\,a+c\right)}{32\,(3\,b+d)}+\frac{3\,\cos\left(3\,bx-dx+3\,a-c\right)}{32\,(3\,b-d)}+\frac{\cos\left(3\,bx-3\,dx+3\,a-3\right.}{96\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*cos(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*cos(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*cos(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*cos(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) - 3/32*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9
/32*cos(b*x + d*x + a + c)/(b + d) - 9/32*cos(b*x - d*x + a - c)/(b - d) -
3/32*cos(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

maple [A] time = 0.31, size = 184, normalized size = 0.94

$$-\frac{3\cos\left(a-3c+\left(b-3d\right)x\right)}{32\left(b-3d\right)}-\frac{9\cos\left(a-c+\left(b-d\right)x\right)}{32\left(b-d\right)}-\frac{9\cos\left(a+c+\left(b+d\right)x\right)}{32\left(b+d\right)}-\frac{3\cos\left(a+3c+\left(b+3d\right)x\right)}{32\left(b+3d\right)}+\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^3,x)
```

```
[Out] -3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)-9/32*cos(a+c
+(b+d)*x)/(b+d)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*cos((3*b-3*d)*
x+3*a-3*c)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*b
+d)+1/96/(b+d)*cos((3*b+3*d)*x+3*a+3*c)
```

maxima [B] time = 0.54, size = 2612, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^
3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a + 4*
c) + 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^3*
```

```
cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a - 2*c)
 + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*co
s(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
+ 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*cos
(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a - 2*c) -
 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*c
os(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*(
9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*cos(3
*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a) + (9*b^5*cos(3*
c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4
*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*cos(3*c)
- 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*co
s(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c) - 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 - 9*d^5*cos(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) - 9*b^4*d*
cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) - 9
*d^5*cos(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(
3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5
*cos(3*c))*cos(-(b - d)*x - a + 4*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*cos(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82
*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c)
)*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82*b
^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*
cos(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^
2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b
 - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^2*c
os(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b -
3*d)*x - a) + 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10
*b^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3
*a + 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b
^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3*a
 - 2*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a -
 2*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^
2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a + 6*c
) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^
3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a) + (9*b^5
*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*si
n(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b
*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9
*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3
*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4
*d*sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c)
+ 9*d^5*sin(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*
sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9
*d^5*sin(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c)
 - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin
(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10
*b^3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*
sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^
3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*sin
```

```
(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3*
c)^2 + b^4*sin(3*c)^2)*d^2)
```

mupad [B] time = 4.14, size = 951, normalized size = 4.88

$$-e^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{576\,b^4-640\,b^2\,d^2+64\,d^4}+\frac{e^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}-\frac{e^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-81\right.}{576\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(
576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*d
- 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- a*2i - b*x*2i)*
(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (
exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d
^4 - 640*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2 + 3*b^2*d
 - 9*b^3 - 3*d^3)/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*
(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (ex
p(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4
 - 640*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^
3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)) - exp(a*3i - c*3i + b*x*3i - d*x*3i)*
((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp
(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 19
20*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/
(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b
^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)) - exp(a*3i + c
*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(192*b^4 + 1728*d^4
 - 1920*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^
2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i -
b*x*4i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b
^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.227 $\int \cos(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos(a-c)\log(\sin(bx+c))}{b}-x\sin(a-c)$$

```
[Out] cos(a-c)*ln(sin(b*x+c))/b-x*sin(a-c)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4581, 3475, 8}

$$\frac{\cos(a-c)\log(\sin(bx+c))}{b}-x\sin(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] (Cos[a - c]*Log[Sin[c + b*x]])/b - x*Sin[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\,dx-\sin(a-c)\int 1\,dx\\ &= \frac{\cos(a-c)\log(\sin(c+bx))}{b}-x\sin(a-c)\end{aligned}$$

Mathematica [C] time = 0.18, size = 58, normalized size = 2.15

$$\frac{-2bx\sin(a-c)-2i\cos(a-c)\tan^{-1}(\tan(bx+c))+\cos(a-c)\left(\log\left(\sin^2(bx+c)\right)+2ibx\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[c
 + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

fricas [A] time = 0.48, size = 30, normalized size = 1.11

$$\frac{bx\sin(-a+c)+\cos(-a+c)\log\left(\frac{1}{2}\sin(bx+c)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

giac [B] time = 1.56, size = 482, normalized size = 17.85

$$-\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx+a)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log(}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)
^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/
2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*
c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c
)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2
*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(
1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)
 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan
(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(
1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*a)*tan(1/2*c)
 + 1))/b
```

maple [B] time = 1.94, size = 325, normalized size = 12.04

$$\frac{\ln\left(\tan\left(bx+a\right)\cos(a)\cos(c)+\tan\left(bx+a\right)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c)\right)\cos(a)\cos(c)}{b\left(\left(\cos^{2}(a)\right)\left(\cos^{2}(c)\right)+\left(\cos^{2}(a)\right)\left(\sin^{2}(c)\right)+\left(\cos^{2}(c)\right)\left(\sin^{2}(a)\right)+\left(\sin^{2}(a)\right)\left(\sin^{2}(c)\right)\right)}+\frac{\ln\left(\tan\left(b\right.\right.}{b\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c),x)
```

```
[Out] 1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a
)*cos(c))*cos(a)*cos(c)+1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*s
in(a)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*si
n(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*sin(c)-1/2/b/(cos(c)^2+sin(c)^2)/(
cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*cos(c)-1/2/b/(cos(c)^2+sin(c)^
2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*sin(c)+1/b/(cos(c)^2+sin(c
)^2)/(cos(a)^2+sin(a)^2)*cos(a)*sin(c)*arctan(tan(b*x+a))-1/b/(cos(c)^2+sin
(c)^2)/(cos(a)^2+sin(a)^2)*cos(c)*sin(a)*arctan(tan(b*x+a))
```

maxima [B] time = 0.36, size = 106, normalized size = 3.93

$$\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos(c)+\cos(c)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin(c)+\sin(c\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2))/b
```

mupad [B] time = 0.70, size = 115, normalized size = 4.26

$$-x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)-x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)+\frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x),x)
```

```
[Out] (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i
- c*1i)/2))/b - x*((exp(c*1i - a*1i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - x*(
(exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2)
```

sympy [B] time = 8.03, size = 333, normalized size = 12.33

$$-\left(\begin{cases}0\\ x\\ 0\\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x)
```

```
[Out] -Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x, Eq(c, 0)), (0, Eq(b, 0)), (-b*x*ta
n(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) +
tan(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*t
an(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)
**2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (log(sin(
b*x))/b, Eq(c, 0)), (x/sin(c), Eq(b, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 +
b) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c
/2) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/
2)**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b
) + log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)
**2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

3.228 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4581, 2606, 8, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc^2(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx-\sin(a-c)\int \csc(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}-\frac{\cos(a-c)\,\text{Subst}\left(\int 1\,dx,x,\csc(c+bx)\right)}{b}\\ &= -\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.10, size = 90, normalized size = 2.57

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]
*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]
*Sin[c])]*Sin[a - c])/b
```

fricas [B] time = 0.47, size = 71, normalized size = 2.03

$$-\frac{\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+2\cos}{2\,b\sin\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(b
*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

giac [B] time = 0.47, size = 893, normalized size = 25.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1
/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)
 - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/
2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) -
 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)
^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)
+ 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
 tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^4 - 2*ta
n(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/2*a)*tan(1
/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*a)
*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/
2*a)*tan(1/2*a)^4 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) + 2*tan(
1/2*a)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 - 1
2*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^
3 + 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan(1/2*c)^4 - 2*ta
n(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 + 2*tan(1/2*a)^
3 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan(1/2*a)^2*tan(1/2*
c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1/2*c)^2 - 2*t
an(1/2*c)^3 + tan(1/2*b*x + 1/2*a) - 2*tan(1/2*a) + 2*tan(1/2*c))/((tan(1/2
*b*x + 1/2*a)^2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)
*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*
x + 1/2*a)^2*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 - tan(1/2*b*x +
 1/2*a)^2*tan(1/2*c) + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1
/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*b*x + 1/2*a) - tan(1/2*a) + tan(1/2*c))*(tan(1/2*a)^2*tan(1
/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))))/b
```

maple [B] time = 2.16, size = 1062, normalized size = 30.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^2,x)
```

```
[Out] -1/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x
+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)
+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+
cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*
b*x+1/2*a)*cos(a)^2*cos(c)^2-2/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1
/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(
1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2
*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*si
n(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)*sin(a)*sin(c)-1/b/(-1/
2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2
+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*cos(
a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*
sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a
)*sin(a)^2*sin(c)^2-1/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)
*sin(a)*tan(1/2*b*x+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1
/2*a)*sin(a)*sin(c)+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*cos(a)*cos(c)-1/b/(
-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a
)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*c
os(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)
^2*sin(a)^2+sin(a)^2*sin(c)^2)*sin(a)*sin(c)+4/b/(2*cos(a)^2*cos(c)^2+2*cos
(a)^2*sin(c)^2+2*cos(c)^2*sin(a)^2+2*sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2
-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2
*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*
sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*si
n(c)^2)^(1/2))*cos(a)*sin(c)-4/b/(2*cos(a)^2*cos(c)^2+2*cos(a)^2*sin(c)^2+2
*cos(c)^2*sin(a)^2+2*sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)
^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-
cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(a)
^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*s
in(a)*cos(c)
```

maxima [B] time = 0.38, size = 450, normalized size = 12.86

$$\frac{2\left(\sin\left(bx+2\,a\right)+\sin\left(bx+2\,c\right)\right)\cos\left(2\,bx+a+2\,c\right)-\left(\cos\left(2\,bx+a+2\,c\right)^{2}\sin\left(-a+c\right)-2\,\cos\left(2\,bx+a+2\,c\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
+ a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(
2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c)
+ (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a + 2*c
)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a
 + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)
^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) + cos(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x +
2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
+ 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*b
*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

mupad [B] time = 5.21, size = 252, normalized size = 7.20

$$-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\cdots}\right.}{\sqrt{-e^{a\,2\cdots}}}\right.}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x)^2,x)
```

```
[Out] (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2
i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i
- c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1i)*
(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) -
1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*
2i - c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp
(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

sympy [B] time = 102.51, size = 3266, normalized size = 93.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**2,x)
```

```
[Out] -Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (log(tan(b*x/2))/b, Eq(c,
 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*ta
n(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x
/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)**
3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b
*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(
tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**
3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b
*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c
/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2
)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + tan(b*x/2))*t
an(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
- log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2
)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2)
- b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(
b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2
)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**
4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*ta
n(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan
(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) +
 b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b
*tan(c/2) - b*tan(b*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(b*tan
(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c
/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - 2*log(tan(b*x/2) - 1/tan(c
/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b
*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*
x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*
tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b
*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/
2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
+ b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) -
1/tan(c/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)
**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 + tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x
/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/
2)) - 2*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2
 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) -
2*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(
c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - tan(b*x/2
)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 +
```

```
 b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)), True))*sin(a) + Pie
cewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (-1/(b*sin(b*x)), Eq(c, 0)), (x/sin(c)
**2, Eq(b, 0)), (4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(2*b*t
an(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2
*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) +
 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2)**5*ta
n(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**
2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/
2) + tan(b*x/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*
tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c
/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**
2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 -
2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan
(c/2)*tan(b*x/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)/(
2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**
4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/
2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2
)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan
(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log
(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan
(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 -
2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log(tan(b*x/2) - 1/tan(c/2))
*tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b
*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**
2 - 2*b*tan(c/2)*tan(b*x/2)) + tan(c/2)**6*tan(b*x/2)/(2*b*tan(c/2)**5*tan(
b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*
tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(c/2)**5/
(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)*
*4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x
/2)) - tan(c/2)**4*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4
*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(
c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**
5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/
2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/
2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/
2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(
b*x/2)) + tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/
2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 -
 2*b*tan(c/2)*tan(b*x/2)), True))*cos(a)
```

3.229 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

```
[Out] -1/2*cos(a-c)*csc(b*x+c)^2/b+cot(b*x+c)*sin(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4581, 2606, 30, 3767, 8}

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

```
[Out] -(Cos[a - c]*Csc[c + b*x]^2)/(2*b) + (Cot[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc^3(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx-\sin(a-c)\int \csc^2(c+bx)\,dx\\ &= -\frac{\cos(a-c)\operatorname{Subst}(\int x\,dx,x,\csc(c+bx))}{b}+\frac{\sin(a-c)\operatorname{Subst}(\int 1\,dx,x,\cot(c-\\ &= -\frac{\cos(a-c)\csc^2(c+bx)}{2b}+\frac{\cot(c+bx)\sin(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.21, size = 35, normalized size = 0.92

$$-\frac{\csc(c)\csc^2(bx+c)(\sin(a)-\sin(a-c)\cos(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

```
[Out] -1/2*(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/b
```

fricas [A] time = 0.47, size = 45, normalized size = 1.18

$$\frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)+\cos(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c)
^2 - b)
```

giac [B] time = 0.29, size = 327, normalized size = 8.61

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}a\right)^4}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*ta
n(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)
*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2
*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

maple [A] time = 12.39, size = 55, normalized size = 1.45

$$-\frac{1}{2b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^3,x)
```

```
[Out] -1/2/b/(cos(a)*cos(c)+sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2
```

maxima [B] time = 0.34, size = 395, normalized size = 10.39

$$\frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a)+\cos(}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2*sin
(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**3,x)
```

```
[Out] Timed out
```

3.230 $\int \sin(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{3\cos(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\sin(a+bx)}{b}$$

```
[Out] -3/2*arctanh(sin(b*x+c))*cos(a-c)/b+sec(b*x+c)*sin(a-c)/b+sin(b*x+a)/b+1/2*
cos(a-c)*sec(b*x+c)*tan(b*x+c)/b
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4576, 4579, 2637, 3770, 2606, 8, 2611}

$$\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{3\cos(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + b*x]]*Cos[a - c])/(2*b) + (Sec[c + b*x]*Sin[a - c])/b +
 Sin[a + b*x]/b + (Cos[a - c]*Sec[c + b*x]*Tan[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx - \int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \sec(c+bx)\,dx + \sin(a-$$
$$= -\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \cos(a$$
$$= -\frac{3\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)}{b} + \frac{\sin(a+bx)}{b} + \cdots$$

Mathematica [A] time = 0.37, size = 70, normalized size = 0.97

$$\frac{\sec^2(bx+c)(2\sin(a-bx-2c)+\sin(a+3bx+2c)+5\sin(a+bx)) - 12\cos(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\cdots\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^2*(2*S
in[a - 2*c - b*x] + 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

fricas [B] time = 0.53, size = 376, normalized size = 5.22

$$-\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2+\cos(-2a+2c)^2-1\right)\log\left(-\frac{2\cos(bx+a)^{2}\cdots}{\cdots}\right.}{\sqrt{\cos(-2a+2c)+1}}}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a
 + 2*c) - 2*(cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a
 + 2*c)^2 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) +
cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) -
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x +
 a)^2*cos(-2*a + 2*c) - 3*cos(-2*a + 2*c) + 5)*sin(b*x + a) - 4*(4*cos(b*x
+ a)^3 - 5*cos(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx+a)\tan(bx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c)^3, x)
```

maple [C] time = 1.00, size = 186, normalized size = 2.58

$$-\frac{i e^{i(bx+a)}}{2b}+\frac{i e^{-i(bx+a)}}{2b}-\frac{i\left(3\,e^{i(3bx+5a+2c)}-e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3\,e^{i(bx+3a+2c)}\right)}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(a-c}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^3,x)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+
5*a))-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a
-c)-3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.55, size = 1027, normalized size = 14.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) + 2*s
in(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*
x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a +
2*c) + sin(b*x + a))*cos(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x
 + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*c
os(3*b*x + a + 2*c)^2*cos(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos
(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2
+ 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)
*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 + 2*(2*cos(3*b*x + a + 2*c)*cos(
-a + c) + cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*cos(-a + c)
*sin(3*b*x + a + 2*c) + cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log
((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
+ 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(
c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
 - 2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b
*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) + 2*cos(
2*b*x + 2*a) - 5*cos(2*b*x + 2*c) - 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b
*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a +
 2*c) + cos(b*x + a))*sin(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) - 5*cos(2*b*
x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) + 10*c
os(b*x + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) - 10*cos(2*b
*x + 2*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*co
s(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x +
a)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*
x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c)
+ b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*si
n(b*x + a))*sin(5*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\tan^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c)**3, x)
```

3.231 $\int \sin(a+bx)\tan^2(c+bx)\,dx$

Optimal. Leaf size=44

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

```
[Out] cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4576, 4579, 2638, 3770, 2606, 8}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a
 - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx - \int \cos(a+bx)\tan(c+bx)\,dx$$
$$= \frac{\cos(a-c)\,\text{Subst}(\int 1\,dx, x, \sec(c+bx))}{b} + \sin(a-c)\int \sec(c+bx)\,dx - \int \sin$$
$$= \frac{\cos(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

Mathematica [C] time = 0.10, size = 109, normalized size = 2.48

$$\frac{\cos(a-c)\sec(bx+c)}{b} - \frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\cos}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (Cos[a]*Cos[b*x])/b + (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c
] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/
2] - I*Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

fricas [B] time = 0.47, size = 315, normalized size = 7.16

$$-\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^2 - 4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right) + \frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cos(bx+a)\right.}{}}{4\left(b\sin\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*
c) + (cos(-2*a + 2*c)^2 - 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a
+ 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2
*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a
+ 2*c) + 1) + 4*cos(-2*a + 2*c) + 4)/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(bx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c)^2, x)
```

maple [C] time = 0.79, size = 143, normalized size = 3.25

$$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} + \frac{e^{i(bx+3a)} + e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)} + e^{2ia}\right)} + \frac{\ln\left(e^{i(bx+a)} + ie^{i(a-c)}\right)\sin\left(a-c\right)}{b} - \frac{\ln\left(e^{i(bx+a)} - ie^{i(a-c)}\right)\sin\left(a-c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^2,x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(exp(2*I*(b*x+a+c))+exp(2*I*
a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))
/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.53, size = 520, normalized size = 11.82

$$\frac{(\cos(3\,bx + a + 2\,c) + \cos(bx + a))\cos(4\,bx + 2\,a + 2\,c) + (3\,\cos(2\,bx + 2\,a) + 3\,\cos(2\,bx + 2\,c) + 1)\cos(3\,bx \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + (3*cos(
2*b*x + 2*a) + 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) + 3*cos(2*b*x + 2*c)*cos(b*x + a) + (cos(3*b*x + a + 2*c
)^2*sin(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) + 2*sin(3*b*x + a
+ 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log((cos(b*x
+ 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*
x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x
 + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + (sin(3*
b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 3*(sin(2*b*x + 2*a)
 + sin(2*b*x + 2*c))*sin(3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a)
 + 3*sin(2*b*x + 2*c)*sin(b*x + a) + cos(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

mupad [B] time = 5.31, size = 294, normalized size = 6.68

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2\cdots}\right.}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + b*x)^2,x)
```

```
[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i)
*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*1i
)) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)
*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(
exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp
(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*
exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/
(2*b*(-exp(a*2i - c*2i))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a + bx)\tan^2(bx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c)**2, x)
```

3.232 $\int \sin(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sin(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4576, 2637, 3770}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x],x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan(c+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\,dx-\int \cos(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 3.24

$$-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sin(a)\cos(bx)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Cos[a]*Sin[b*x])/b
```

fricas [B] time = 0.45, size = 188, normalized size = 6.48

$$\frac{\sqrt{2}\,\sqrt{\cos\left(-2\,a+2\,c\right)+1}\,\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\frac{2\,\sqrt{2}\,((\cos(-2\,a+2\,c)+1)\sin(bx+a)+\cos(bx+a)\sin(-}{\sqrt{\cos(-2\,a+2\,c)+1}}}{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\cos(-2\,a+2\,c)+1}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c
) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*
c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) +
 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - 4*sin(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(bx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c), x)
```

maple [C] time = 0.52, size = 99, normalized size = 3.41

$$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}+i e^{i(a-c)}\right)\cos\left(a-c\right)}{b}-\frac{\ln\left(e^{i(bx+a)}-i e^{i(a-c)}\right)\cos\left(a-c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c),x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))/b*cos(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.52, size = 131, normalized size = 4.52

$$-\frac{\cos\left(-a+c\right)\log\left(\frac{\cos(bx+2\,c)^2+\cos(c)^2-2\,\cos(c)\sin(bx+2\,c)+\sin(bx+2\,c)^2+2\,\cos(bx+2\,c)\sin(c)+\sin(c)^2}{\cos(bx+2\,c)^2+\cos(c)^2+2\,\cos(c)\sin(bx+2\,c)+\sin(bx+2\,c)^2-2\,\cos(bx+2\,c)\sin(c)+\sin(c)^2}\right)+2\,\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c
) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^
2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c
)*sin(c) + sin(c)^2)) + 2*sin(b*x + a))/b
```

mupad [B] time = 5.49, size = 227, normalized size = 7.83

$$-\frac{e^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2\,b}+\frac{e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2\,b}+\frac{\ln\left(-e^{a\,1\mathrm{i}}\,e^{b\,x\,1\mathrm{i}}\,\left(e^{a\,2\mathrm{i}}\,e^{-c\,2\mathrm{i}}+1\right)-\frac{e^{a\,2\mathrm{i}}\,e^{-c\,2\mathrm{i}}\,\left(e^{a\,2\mathrm{i}}\,e^{-c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{\sqrt{e^{a\,2\mathrm{i}}\,e^{-c\,2\mathrm{i}}}}\right)\,\left(e^{a\,2\mathrm{i}-c\,2\mathrm{i}}+1\right)}{2\,b\,\sqrt{e^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}-\frac{\ln\left(-e^{a\,1\mathrm{i}}\,e^{b\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + b*x),x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) + (log(- ex
p(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c*2i)*(exp
(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
+ 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*
exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(e
xp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c), x)
```

3.233 $\int \cot(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

```
[Out] -arctanh(cos(b*x+c))*sin(a-c)/b+sin(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4578, 2637, 3770}

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cot(c+bx)\sin(a+bx)\,dx &= \sin(a-c)\int \csc(c+bx)\,dx+\int \cos(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 3.21

$$-\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b - ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/
2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*S
in[a - c])/b + (Cos[a]*Sin[b*x])/b
```

fricas [B] time = 0.45, size = 197, normalized size = 6.79

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)\sin(-2a+2c)}{\sqrt{\cos(-2a+2c)+1}} -}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) - 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) + 4
*sin(b*x + a))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((-tan(a/2)^2*tan(c/2)+tan(a/2)*tan(c/2)^2-tan(a/2)+tan(c/2))/(
-tan(a/2)^2*tan(c/2)^2-tan(a/2)^2-tan(c/2)^2-1)*ln(abs(tan(b*x/2)+tan(c/2))
)+(-tan(a/2)^2*tan(c/2)^2+tan(a/2)*tan(c/2)^3-tan(a/2)*tan(c/2)+tan(c/2)^2)
/(tan(a/2)^2*tan(c/2)^3+tan(a/2)^2*tan(c/2)+tan(c/2)^3+tan(c/2))*ln(abs(tan
(b*x/2)*tan(c/2)-1))+(-tan(b*x/2)*tan(a/2)^2+tan(b*x/2)+2*tan(a/2))/(tan(a/
2)^2+1)/(tan(b*x/2)^2+1))
```

maple [C] time = 0.80, size = 95, normalized size = 3.28

$$-\frac{i e^{i(bx+a)}}{2b}+\frac{i e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)*sin(b*x+a),x)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))+exp(I*(a-
c)))/b*sin(a-c)+ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.35, size = 105, normalized size = 3.62

$$\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(-a+c)-\log\left(\cos(b\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos
(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) + 2*sin(b*x
+ a))/b
```

mupad [B] time = 4.85, size = 233, normalized size = 8.03

$$\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + b*x)*sin(a + b*x),x)
```

```
[Out] (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i + b*x*1i)*1i)/(2*b) - (log(- ex
p(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp
(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
 - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*
2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) - 1))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*
2i))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\cot(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c), x)
```

3.234 $\int \cot^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b-csc(b*x+c)*sin(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4578, 4577, 2638, 3770, 2606, 8}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*Si
n[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot(c+bx)\,dx$$
$$= \cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a-c)\,\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} - \int \sin(a$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b} + \frac{\cos(a+bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

Mathematica [C] time = 0.10, size = 111, normalized size = 2.41

$$-\frac{\sin(a-c)\csc(bx+c)}{b} - \frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\cos(b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Csc[c + b*x]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

fricas [B] time = 0.45, size = 316, normalized size = 6.87

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cos(bx+a)\sin(-2\,a+2\,c)+\left(\cos(-2\,a+2\,c)^2+2\,\cos(-2\,a+2\,c)+\right.\right.}{}}{4\left(b\cos\left(bx+a\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos
(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-
2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
+ a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*
(cos(b*x + a)^2 + 1)*sin(-2*a + 2*c))/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*
cos(-2*a + 2*c) + b)*sin(b*x + a))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((tan(c/2)^2*tan(a/2)^2-tan(c/2)^2+4*tan(c/2)*tan(a/2)-tan(a/2)
^2+1)/(2*tan(c/2)^2*tan(a/2)^2+2*tan(c/2)^2+2*tan(a/2)^2+2)*ln(abs(tan(b*x/
2)+tan(c/2)))+(tan(c/2)^3*tan(a/2)^2-tan(c/2)^3+4*tan(c/2)^2*tan(a/2)-tan(c
/2)*tan(a/2)^2+tan(c/2))/(-2*tan(c/2)^3*tan(a/2)^2-2*tan(c/2)^3-2*tan(c/2)*
tan(a/2)^2-2*tan(c/2))*ln(abs(tan(b*x/2)*tan(c/2)-1))+(-tan(b*x/2)^3*tan(c/
```

```
2)^4*tan(a/2)+tan(b*x/2)^3*tan(c/2)^3*tan(a/2)^2-tan(b*x/2)^3*tan(c/2)^3+6*
tan(b*x/2)^3*tan(c/2)^2*tan(a/2)-tan(b*x/2)^3*tan(c/2)*tan(a/2)^2+tan(b*x/2
)^3*tan(c/2)-tan(b*x/2)^3*tan(a/2)+6*tan(b*x/2)^2*tan(c/2)^3*tan(a/2)-6*tan
(b*x/2)^2*tan(c/2)*tan(a/2)-tan(b*x/2)*tan(c/2)^4*tan(a/2)+3*tan(b*x/2)*tan
(c/2)^3*tan(a/2)^2-3*tan(b*x/2)*tan(c/2)^3-2*tan(b*x/2)*tan(c/2)^2*tan(a/2)
-3*tan(b*x/2)*tan(c/2)*tan(a/2)^2+3*tan(b*x/2)*tan(c/2)-tan(b*x/2)*tan(a/2)
+2*tan(c/2)^3*tan(a/2)-4*tan(c/2)^2*tan(a/2)^2+4*tan(c/2)^2-2*tan(c/2)*tan(
a/2))/(-2*tan(c/2)*tan(a/2)^2-2*tan(c/2))/(tan(b*x/2)^4*tan(c/2)+tan(b*x/2)
^3*tan(c/2)^2-tan(b*x/2)^3+tan(b*x/2)*tan(c/2)^2-tan(b*x/2)-tan(c/2)))
```

maple [C] time = 1.01, size = 143, normalized size = 3.11

$$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(-exp(2*I*(b*x+a+c))+exp(2*I
*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))-exp(I*(a-c)))/
b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.37, size = 612, normalized size = 13.30

$$\frac{(\cos(3\,bx+a+2\,c)-\cos(bx+a))\cos(4\,bx+2\,a+2\,c)-(3\,\cos(2\,bx+2\,a)-3\,\cos(2\,bx+2\,c)+1)\cos(3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - (3*cos(
2*b*x + 2*a) - 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) - 3*cos(2*b*x + 2*c)*cos(b*x + a) - (cos(3*b*x + a + 2*c
)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*x
 + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 2*cos(-a + c)*si
n(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log(cos(b*x)^
2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^
2) + (cos(3*b*x + a + 2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x +
 a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(
b*x + a)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(c) + sin(c)^2) + (sin(3*b*x + a + 2*c) - sin(b*x + a))*sin(4*b
*x + 2*a + 2*c) - 3*(sin(2*b*x + 2*a) - sin(2*b*x + 2*c))*sin(3*b*x + a + 2
*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a) - 3*sin(2*b*x + 2*c)*sin(b*x + a) + c
os(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

mupad [B] time = 5.26, size = 290, normalized size = 6.30

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + b*x)^2*sin(a + b*x),x)
```

```
[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i)
*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*1i
)) - (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2
i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*
(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(
-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*
1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*
b*exp(a*2i - c*2i)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\cot^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**2, x)
```

3.235 $\int \cot^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{3\sin(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+3/2*arctanh(cos(b*x+c))*sin(a-c)/b-1/2*cot(b*x+c)*cs
c(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4578, 4577, 2637, 3770, 2606, 8, 2611}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{3\sin(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (3*ArcTanh[Cos[c + b*x]]*Sin[a - c])/(2*b)
 - (Cot[c + b*x]*Csc[c + b*x]*Sin[a - c])/(2*b) - Sin[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \sin(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot^2(c+bx)\,dx$$
$$= -\frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} + \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \frac{1}{2}$$
$$= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} - \frac{\cos(a-c)}{}$$
$$= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{3\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+}{2b}$$

Mathematica [A] time = 0.37, size = 71, normalized size = 0.96

$$\frac{\csc^2(bx+c)(2\sin(a-bx-2c)+\sin(a+3bx+2c)-5\sin(a+bx))+12\sin(a-c)\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^2*(2*Si
n[a - 2*c - b*x] - 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

fricas [B] time = 0.51, size = 372, normalized size = 5.03

$$\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)^2-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^2\cos(-2a+2c)-\cos(-2a+2c)-1\right)\sin(-2a+2c)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+}{2\cos(b}\right.}{\sqrt{\cos(-2a+2c)+1}}}{8\left(2b\cos(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*co
s(b*x + a)^2*cos(-2*a + 2*c) - cos(-2*a + 2*c) - 1)*sin(-2*a + 2*c))*log(-(
2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2
*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a
 + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2
*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a +
 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x + a)^2*cos(-2*a + 2*c)
 - 3*cos(-2*a + 2*c) - 5)*sin(b*x + a) - 4*(4*cos(b*x + a)^3 - 5*cos(b*x +
a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((tan(b*x/2)*tan(a/2)^2-tan(b*x/2)-2*tan(a/2))/(tan(a/2)^2+1)/(
tan(b*x/2)^2+1)+(2*tan(b*x/2)^3*tan(c/2)^7*tan(a/2)+6*tan(b*x/2)^3*tan(c/2)
^5*tan(a/2)-4*tan(b*x/2)^3*tan(c/2)^4*tan(a/2)^2+4*tan(b*x/2)^3*tan(c/2)^4-
6*tan(b*x/2)^3*tan(c/2)^3*tan(a/2)-2*tan(b*x/2)^3*tan(c/2)*tan(a/2)+tan(b*x
/2)^2*tan(c/2)^8*tan(a/2)+tan(b*x/2)^2*tan(c/2)^7*tan(a/2)^2-tan(b*x/2)^2*t
an(c/2)^7+2*tan(b*x/2)^2*tan(c/2)^6*tan(a/2)-5*tan(b*x/2)^2*tan(c/2)^5*tan(
a/2)^2+5*tan(b*x/2)^2*tan(c/2)^5-22*tan(b*x/2)^2*tan(c/2)^4*tan(a/2)+5*tan(
b*x/2)^2*tan(c/2)^3*tan(a/2)^2-5*tan(b*x/2)^2*tan(c/2)^3+2*tan(b*x/2)^2*tan
(c/2)^2*tan(a/2)-tan(b*x/2)^2*tan(c/2)*tan(a/2)^2+tan(b*x/2)^2*tan(c/2)+tan
(b*x/2)^2*tan(a/2)-2*tan(b*x/2)*tan(c/2)^7*tan(a/2)-4*tan(b*x/2)*tan(c/2)^6
*tan(a/2)^2+4*tan(b*x/2)*tan(c/2)^6-14*tan(b*x/2)*tan(c/2)^5*tan(a/2)+4*tan
(b*x/2)*tan(c/2)^4*tan(a/2)^2-4*tan(b*x/2)*tan(c/2)^4+14*tan(b*x/2)*tan(c/2
)^3*tan(a/2)-4*tan(b*x/2)*tan(c/2)^2*tan(a/2)^2+4*tan(b*x/2)*tan(c/2)^2+2*t
an(b*x/2)*tan(c/2)*tan(a/2)+2*tan(c/2)^6*tan(a/2)+2*tan(c/2)^5*tan(a/2)^2-2
*tan(c/2)^5+12*tan(c/2)^4*tan(a/2)-2*tan(c/2)^3*tan(a/2)^2+2*tan(c/2)^3+2*t
an(c/2)^2*tan(a/2))/(-8*tan(c/2)^2*tan(a/2)^2-8*tan(c/2)^2)/(tan(b*x/2)^2*t
an(c/2)+tan(b*x/2)*tan(c/2)^2-tan(b*x/2)-tan(c/2))^2+(-3*tan(c/2)^2*tan(a/2
)+3*tan(c/2)*tan(a/2)^2-3*tan(c/2)+3*tan(a/2))/(-2*tan(c/2)^2*tan(a/2)^2-2*
tan(c/2)^2-2*tan(a/2)^2-2)*ln(abs(tan(b*x/2)+tan(c/2)))+(-3*tan(c/2)^3*tan(
a/2)+3*tan(c/2)^2*tan(a/2)^2-3*tan(c/2)^2+3*tan(c/2)*tan(a/2))/(2*tan(c/2)^
3*tan(a/2)^2+2*tan(c/2)^3+2*tan(c/2)*tan(a/2)^2+2*tan(c/2))*ln(abs(tan(b*x/
2)*tan(c/2)-1)))
```

maple [C] time = 0.88, size = 184, normalized size = 2.49

$$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}+\frac{i\left(-3\,e^{i(3bx+5a+2c)}-e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+3\,e^{i(bx+3a+2c)}\right)}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+1/2*I/b/(-exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x
+5*a))+3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-
c)-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.40, size = 1254, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) - 2*si
n(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
 + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2
*c) - sin(b*x + a))*cos(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x
+ 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*co
s(3*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(
-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) +
 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)
*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-
a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x +
a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(
cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c)
+ sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c
```

```
)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) + 4*sin(3*b*x + a
+ 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + si
n(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-a + c) - cos(b*x
+ a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x + a + 2*c)*sin(-a +
 c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2*co
s(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(
cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b*x + 2
*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) - 2*cos(2*b*x
+ 2*a) - 5*cos(2*b*x + 2*c) + 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b*x + a
 + 2*c) - cos(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x + a + 2*c)
- cos(b*x + a))*sin(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*
c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) - 10*cos(b*x
 + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) + 10*cos(2*b*x + 2
*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b*
x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
+ 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*co
s(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
+ a))*sin(5*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + b*x)^3*sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a + bx)\cot^3(bx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**3, x)
```

3.236 $\int \sin(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=143

$$-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

```
[Out] 1/2*I/b/exp(I*(b*x+a))+1/2*I*exp(I*(b*x+a))/b-I*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-I*exp(I*(b*x+a))*hypergeom([1,
 1/2*b/d],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4557, 2194, 2251}

$$-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2F
1[1, -b/(2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) - (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c +
d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4557

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan(c+dx)\,dx &= \int \left(\frac{1}{2}e^{-i(a+bx)}-\frac{1}{2}e^{i(a+bx)}-\frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\right)dx\\ &= \frac{1}{2}\int e^{-i(a+bx)}\,dx-\frac{1}{2}\int e^{i(a+bx)}\,dx-\int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx+\int \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\,dx\\ &= \frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\right.}{}\end{aligned}$$

Mathematica [A] time = 1.81, size = 116, normalized size = 0.81

$$-\frac{ie^{-i(a+bx)}\left(2e^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)-e^{2i(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] ((-1/2*I)*(-1 - E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), -E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(b*E^(I*(a + b*x)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(bx+a\right)\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(sin(b*x+a)*tan(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\tan\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + d*x),x)
```

```
[Out] int(sin(a + b*x)*tan(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(c + d*x), x)
```

3.237 $\int \cot(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

```
[Out] -1/2*I/b/exp(I*(b*x+a))-1/2*I*exp(I*(b*x+a))/b+I*hypergeom([1, -1/2*b/d],[1
-1/2*b/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+I*exp(I*(b*x+a))*hypergeom([1,
 1/2*b/d],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4559, 2194, 2251}

$$\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (-I/2)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometric2
F1[1, -b/(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) + (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d
*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[Cot[(c_.) + (d_.)*(x_)]*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-(1/(
E^(I*(a + b*x))*2)) + E^(I*(a + b*x))/2 + 1/(E^(I*(a + b*x))*(1 - E^(2*I*(c
 + d*x)))) - E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\sin(a+bx)\,dx &= \int \left(-\frac{1}{2}e^{-i(a+bx)}+\frac{1}{2}e^{i(a+bx)}+\frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}\,dx\right)+\frac{1}{2}\int e^{i(a+bx)}\,dx+\int \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx-\int \frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\,dx\\ &= -\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}+\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\right.}{b}\end{aligned}$$

Mathematica [A] time = 3.62, size = 260, normalized size = 1.87

$$\frac{-\frac{ie^{-i(a+bx-2c)}\left(be^{2idx}\,{}_2F_1\left(1,1-\frac{b}{2d};2-\frac{b}{2d};e^{2i(c+dx)}\right)-(b-2d)\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)(b-2d)}-\frac{ie^{i(a+bx+2c)}\left(be^{2idx}\,{}_2F_1\left(1,\frac{b}{2d}+1;\frac{b}{2d}+2;e^{2i(c+dx)}\right)-(b+2d)\,{}_2F}{\left(-1+e^{2ic}\right)(b+2d)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a]*Cos[b*x]*Cot[c]) - (I*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b
/(2*d), 2 - b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1,
-1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]))/((b - 2*d)*E^(I*(a - 2*c + b*
x))*(-1 + E^((2*I)*c))) - (I*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyperge
ometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*Hy
pergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b + 2*d)*
(-1 + E^((2*I)*c))) + Cot[c]*Sin[a]*Sin[b*x])/b
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\cot\left(dx+c\right)\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*sin(b*x+a),x)
```

```
[Out] int(cot(d*x+c)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*sin(a + b*x),x)
[Out] int(cot(c + d*x)*sin(a + b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*cot(c + d*x), x)
```

3.238 $\int \cos(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] 1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(b
 - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*x
]/(8*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)+\frac{3}{8}\cos(a+c+(b+\right.\\ &= \frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \cos\\ &= \frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\end{aligned}$$

Mathematica [A] time = 0.52, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\sin(a+bx-3c-3dx)}{b-3d}+\frac{3\sin(a+bx-c-dx)}{b-d}+\frac{\sin(a+bx+3c+3dx)}{b+3d}+\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] (Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sin[a - c + b*x - d*x])/(b - d)
+ Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

fricas [A] time = 0.43, size = 109, normalized size = 1.20

$$-\frac{\left(6\,bd^2\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-3\left(2\,d^3\cos\left(bx+a\right)-\left(b^2d-d^3\right)\cos\left(bx+a\right)\cos\left(d\right.\right.}{b^4-10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -((6*b*d^2*cos(d*x + c) - (b^3 - b*d^2)*cos(d*x + c)^3)*sin(b*x + a) - 3*(2
*d^3*cos(b*x + a) - (b^2*d - d^3)*cos(b*x + a)*cos(d*x + c)^2)*sin(d*x + c)
)/(b^4 - 10*b^2*d^2 + 9*d^4)
```

giac [A] time = 5.95, size = 84, normalized size = 0.92

$$\frac{\sin\left(bx+3\,dx+a+3\,c\right)}{8\,(b+3\,d)}+\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,(b+d)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,(b-d)}+\frac{\sin\left(bx-3\,dx+a-3\,c\right)}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) + 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

maple [A] time = 0.94, size = 84, normalized size = 0.92

$$\frac{\sin\left(a-3c+(b-3d)\,x\right)}{8b-24d}+\frac{3\sin\left(a-c+(b-d)\,x\right)}{8\,(b-d)}+\frac{3\sin\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\sin\left(a+3c+(b+3d)\,x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^3,x)
```

```
[Out] 1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [B] time = 0.40, size = 914, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) - 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) - (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
 6*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) - 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
```

```
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

mupad [B] time = 1.95, size = 313, normalized size = 3.44

$$e^{a\,1i-c\,3i+b\,x\,1i-d\,x\,3i}\left(\frac{b+3\,d}{b^2\,16i-d^2\,144i}-\frac{e^{-a\,2i-b\,x\,2i}\,\left(b-3\,d\right)}{b^2\,16i-d^2\,144i}\right)+e^{a\,1i+c\,3i+b\,x\,1i+d\,x\,3i}\left(\frac{b-3\,d}{b^2\,16i-d^2\,144i}-\frac{e^{-a\,2i-b\,x\,2i}\,(}{b^2\,16i-d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x)^3,x)
```

```
[Out] exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) - (exp(-
 a*2i - b*x*2i)*(b - 3*d))/(b^2*16i - d^2*144i)) + exp(a*1i + c*3i + b*x*1i
 + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) - (exp(- a*2i - b*x*2i)*(b + 3*d
))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/
(b^2*16i - d^2*16i) - (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^2*16i
)) + exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*16i) -
(exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

sympy [A] time = 32.44, size = 918, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 - 3*sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a - 3*d
*x)/(24*d) - sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, -3*d)
), (-3*x*sin(a - d*x)*sin(c + d*x)**3/8 - 3*x*sin(a - d*x)*sin(c + d*x)*cos
(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*cos(
a - d*x)*cos(c + d*x)**3/8 + sin(a - d*x)*cos(c + d*x)**3/(8*d) + 3*sin(c +
 d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)**2/(
4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*si
n(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x
)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 - sin(a + d*x)*cos(c + d*x)**3/(8*
d) + 3*sin(c + d*x)**3*cos(a + d*x)/(8*d) + 3*sin(c + d*x)*cos(a + d*x)*cos
(c + d*x)**2/(4*d), Eq(b, d)), (-x*sin(a + 3*d*x)*sin(c + d*x)**3/8 + 3*x*s
in(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a +
3*d*x)*cos(c + d*x)/8 + x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + 3*sin(a + 3*d*
x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a + 3*d*x)/(24*d) - sin(c +
d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)), (b**3*sin(a + b*x)*
cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sin(c + d*x)*cos(
a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*b*d**2*sin(a +
b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2
*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*d**3*sin(c
 + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*sin(c + d*x
)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

3.239 $\int \cos(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

```
[Out] 1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)/
(b+2*d)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*c
 + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos^2(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a+bx)+\frac{1}{4}\cos(a-2c+(b-2d)x)+\frac{1}{4}\cos(a+2c+(b+2d)x)\right)dx\\ &= \frac{1}{4}\int \cos(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \cos(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \cos(a\\ &= \frac{\sin(a+bx)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A] time = 0.76, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\sin(a+bx-2c-2dx)}{b-2d}+\frac{\sin(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\cos(bx)}{b}+\frac{2\cos(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] ((2*Cos[b*x]*Sin[a])/b + (2*Cos[a]*Sin[b*x])/b + Sin[a - 2*c + b*x - 2*d*x]
/(b - 2*d) + Sin[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

fricas [A] time = 0.45, size = 63, normalized size = 1.02

$$-\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(dx+c)-\left(b^2\cos(dx+c)^2-2\,d^2\right)\sin(bx+a)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*d^
2)*sin(b*x + a))/(b^3 - 4*b*d^2)
```

giac [A] time = 0.20, size = 56, normalized size = 0.90

$$\frac{\sin\left(bx+2\,dx+a+2\,c\right)}{4\,(b+2\,d)}+\frac{\sin\left(bx-2\,dx+a-2\,c\right)}{4\,(b-2\,d)}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*sin(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*sin(b*x - 2*d*x + a - 2*c)/(
b - 2*d) + 1/2*sin(b*x + a)/b
```

maple [A] time = 0.65, size = 57, normalized size = 0.92

$$\frac{\sin\left(bx+a\right)}{2b}+\frac{\sin\left(a-2c+\left(b-2d\right)x\right)}{4b-8d}+\frac{\sin\left(a+2c+\left(b+2d\right)x\right)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^2,x)
```

```
[Out] 1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)/
(b+2*d)
```

maxima [B] time = 0.37, size = 416, normalized size = 6.71

$$-\frac{\left(b^2\sin\left(2\,c\right)-2\,bd\sin\left(2\,c\right)\right)\cos\left((b+2\,d)x+a+4\,c\right)-\left(b^2\sin\left(2\,c\right)-2\,bd\sin\left(2\,c\right)\right)\cos\left((b+2\,d)x+a\right)-\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*sin(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a + 4*c) - (b^2*sin
(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a) - (b^2*sin(2*c) + 2*b*d*sin(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a + 2*c) - 2*(b^
2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a - 2*c) - (b^2*cos(2*c) - 2*b*d*cos
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*cos(2*c) - 2*b*d*cos(2*c))*sin((b
+ 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*cos(2*c)
- 4*d^2*cos(2*c))*sin(b*x + a + 2*c) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

mupad [B] time = 1.02, size = 98, normalized size = 1.58

$$\frac{\sin\left(a+b\,x\right)}{2\,b}-\frac{d\,\left(2\,b\,\sin\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\sin\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\sin\left(a-2\,c+b\,x-2\,d\,x\right)+}{16\,b\,d^2-4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x)^2,x)
```

```
[Out] sin(a + b*x)/(2*b) - (d*(2*b*sin(a - 2*c + b*x - 2*d*x) - 2*b*sin(a + 2*c +
 b*x + 2*d*x)) + b^2*sin(a - 2*c + b*x - 2*d*x) + b^2*sin(a + 2*c + b*x + 2
*d*x))/(16*b*d^2 - 4*b^3)
```

sympy [A] time = 6.68, size = 405, normalized size = 6.53

$$\begin{cases} x\cos(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\cos(a) \\ -\frac{x\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a-2dx)}{4}+\frac{x\cos(a-2dx)\cos^2(c+dx)}{4}-\frac{\sin(a-2dx)\cos^2(c+dx)}{2d}-\frac{\sin(c+dx)\cos(a-2dx}{4d} \\ \frac{x\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a+2dx)}{4}+\frac{x\cos(a+2dx)\cos^2(c+dx)}{4}+\frac{\sin(a+2dx)\cos^2(c+dx)}{2d}-\frac{\sin(c+dx)\cos(a+2dx)}{4d} \\ \frac{b^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(c+dx)\cos(a+bx)\cos(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\sin^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a -
 2*d*x)/4 + x*cos(a - 2*d*x)*cos(c + d*x)**2/4 - sin(a - 2*d*x)*cos(c + d*x
)**2/(2*d) - sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(4*d), Eq(b, -2*d)),
(x*sin(a + 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a + 2
*d*x)/4 + x*cos(a + 2*d*x)*cos(c + d*x)**2/4 + sin(a + 2*d*x)*cos(c + d*x)*
*2/(2*d) - sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/(4*d), Eq(b, 2*d)), (b*
*2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(c + d*x)*cos(
a + b*x)*cos(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*sin(c + d*x)*
*2/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2
), True))
```

3.240 $\int \cos(a+bx)\cos(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x],x]
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\cos(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)+\frac{1}{2}\cos(a+c+(b+d)x)\right)dx\\
&= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\
&= \frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 43, normalized size = 1.00

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x],x]
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

fricas [A] time = 0.45, size = 42, normalized size = 0.98

$$\frac{b\cos(dx+c)\sin(bx+a)-d\cos(bx+a)\sin(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="fricas")
[Out] (b*cos(d*x + c)*sin(b*x + a) - d*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

giac [A] time = 0.82, size = 40, normalized size = 0.93

$$\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}+\frac{\sin\left(bx-dx+a-c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="giac")
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

maple [A] time = 0.66, size = 40, normalized size = 0.93

$$\frac{\sin\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sin\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c),x)
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

maxima [A] time = 0.33, size = 40, normalized size = 0.93

$$\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\sin\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="maxima")
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

mupad [B] time = 1.30, size = 84, normalized size = 1.95

$$\frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2}+\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}-\frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2}-\frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x),x)
[Out] (b*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (d*
(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

sympy [A] time = 1.48, size = 153, normalized size = 3.56

$$\begin{cases} x\cos\left(a\right)\cos\left(c\right) & \text{for}\: b=0\wedge d=0 \\ -\frac{x\sin\left(a-dx\right)\sin\left(c+dx\right)}{2}+\frac{x\cos\left(a-dx\right)\cos\left(c+dx\right)}{2}-\frac{\sin\left(a-dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\: b=-d \\ \frac{x\sin\left(a+dx\right)\sin\left(c+dx\right)}{2}+\frac{x\cos\left(a+dx\right)\cos\left(c+dx\right)}{2}+\frac{\sin\left(c+dx\right)\cos\left(a+dx\right)}{2d} & \text{for}\: b=d \\ \frac{b\sin\left(a+bx\right)\cos\left(c+dx\right)}{b^2-d^2}-\frac{d\sin\left(c+dx\right)\cos\left(a+bx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x)
```

```
[Out] Piecewise((x*cos(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x)*sin(c +
d*x)/2 + x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d), E
q(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 +
 sin(c + d*x)*cos(a + d*x)/(2*d), Eq(b, d)), (b*sin(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(c + d*x)*cos(a + b*x)/(b**2 - d**2), True))
```

3.241 $\int \cos(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

```
[Out] x*cos(a-c)+ln(cos(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4583, 3475, 8}

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec(c+bx)\,dx&=\cos(a-c)\int 1\,dx-\sin(a-c)\int \tan(c+bx)\,dx\\&=x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 26, normalized size = 1.00

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

fricas [A] time = 0.54, size = 31, normalized size = 1.19

$$\frac{bx\cos(-a+c)-\log(-\cos(bx+c))\sin(-a+c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

giac [B] time = 0.26, size = 440, normalized size = 16.92

$$\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^{2}+1\right)(bx+a)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1}-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)\log(}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2
*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*
tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*ta
n(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(
1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1
/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2
+ 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*
tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2*c)^3 - tan
(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^4
+ tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

maple [B] time = 1.57, size = 461, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c),x)
```

```
[Out] 1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)/(sin(a)*cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)^2*sin(c)^2-2/b/(cos(a)^2*co
s(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c
)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(
a)*cos(c)+sin(a)*sin(c))*cos(a)*cos(c)*sin(a)*sin(c)+1/b/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos
(c)+sin(a)*sin(c))*cos(c)^2*sin(a)^2+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+si
n(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*sin(c)-1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^
2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*cos(c)+1/b/(cos(c)^2+sin(c)^2)/(cos(a
)^2+sin(a)^2)*cos(a)*cos(c)*arctan(tan(b*x+a))+1/b/(cos(c)^2+sin(c)^2)/(cos
(a)^2+sin(a)^2)*sin(a)*sin(c)*arctan(tan(b*x+a))
```

maxima [B] time = 0.34, size = 74, normalized size = 2.85

$$\frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)+\text{s}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c
)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

mupad [B] time = 1.04, size = 109, normalized size = 4.19

$$x\left(\frac{e^{-a\,1i+c\,1i}}{2}-\frac{e^{a\,1i-c\,1i}}{2}\right)+x\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)+\frac{\ln\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x),x)
```

```
[Out] x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(a
*1i - c*1i)/2) + (log(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))*((exp(c*1i - a
*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

sympy [B] time = 10.43, size = 435, normalized size = 16.73

$$-\left(\begin{cases}-x \\ x \\ 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)}\right.}{b\tan^2\left(\frac{c}{2}\right)}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x)
```

```
[Out] -Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)*
*2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan
(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) -
log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/
2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(t
an(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((-log(sin(b*x)
)/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b, 0)), (
-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(
b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(
tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(
b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**
2 + b), True))*cos(a)
```

3.242 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4583, 2606, 8, 3770}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec^2(c+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\,dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a-c)\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+bx))}{b}\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.09, size = 89, normalized size = 2.54

$$-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec[c
 + b*x]*Sin[a - c])/b
```

fricas [A] time = 0.46, size = 69, normalized size = 1.97

$$\frac{\cos(bx+c)\cos(-a+c)\log(\sin(bx+c)+1)-\cos(bx+c)\cos(-a+c)\log(-\sin(bx+c)+1)+2\sin(-a+c)}{2b\cos(bx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a +
 c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

giac [B] time = 1.83, size = 1341, normalized size = 38.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan
(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2
*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*a)*
tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1
/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*t
an(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan
(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(
1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2
 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*c) - tan(1
/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) +
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan(1/2*c)^3 +
tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/
2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3
 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 - t
an(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(1/2*a) - tan(1/2
*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x
+ 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1))/(tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2
*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 + ta
n(1/2*a)^3 - tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)
^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c) + 1) - 4*(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 - 4*tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^3 + 2*
tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 - tan(1/2*a)^3*tan(1/2*c)^4
+ 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^4*tan(1/2*c)
- 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*a)^3*tan(1/2
*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan
(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 -
 tan(1/2*a)^3 - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*a)
^2*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 - 6*tan(1/2*a)*tan(1/2*
c)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))/((tan(1/2*b*x + 1/2*a)^2*tan
(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)^2 + 4*tan(1/2*b*
```

```
x + 1/2*a)^2*tan(1/2*a)*tan(1/2*c) - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan
(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)^2 -
 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*a)*
tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)*(tan(1/2*a)^2*tan(
1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)))/b
```

maple [B] time = 2.25, size = 1049, normalized size = 29.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^2,x)
```

```
[Out] 2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+
2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a
)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a
)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos
(a)^2*sin(c)^2-4/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/
2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(
a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2
+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2
*b*x+1/2*a)*cos(a)*cos(c)*sin(a)*sin(c)+2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*
a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-
2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*c
os(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(
c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos(c)^2*sin(a)^2-2/b/(cos(a)*cos(c)*t
an(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)
*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(
c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)*cos(a)*sin(c)+2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(
1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*si
n(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)
^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*sin(a)*cos(c)-2/b/(cos(a)^2*cos(c)^
2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^
2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(
2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*sin(c)-2*sin(a)
*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*s
in(c)^2)^(1/2))*cos(a)*cos(c)-2/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(
c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(
c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*
sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*sin(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(
c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*s
in(c)
```

maxima [B] time = 0.52, size = 391, normalized size = 11.17

$$-\frac{2\left(\sin\left(bx+2a\right)-\sin\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+\left(\cos\left(2bx+a+2c\right)^{2}\cos\left(-a+c\right)+2\cos\left(2bx+a+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*x
 + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos
(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
 + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*
cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)
```

```
^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)
^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(b*x + 2
*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x + 2*
c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a +
2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*s
in(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

mupad [B] time = 6.52, size = 246, normalized size = 7.03

$$\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,(}{\sqrt{e}}\right.}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x)^2,x)
```

```
[Out] (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c
*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i
 - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b
*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i -
c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i
- c*2i) + exp(a*2i + b*x*2i)))
```

sympy [B] time = 160.75, size = 5552, normalized size = 158.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**2,x)
```

```
[Out] Piecewise((x/cos(c)**2, Eq(b, 0)), (-1/(b*sin(b*x)), Eq(c, -pi/2) | Eq(c, p
i/2)), (-log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c
/2)**6*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan
(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c
/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/
2)**2 - b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*t
an(c/2)**6/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*t
an(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan
(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b)
 + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*
*5*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**
5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*
tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
 b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/
2)**4*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(
c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/
2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2
)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*
tan(c/2)**4/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*
tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*ta
n(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b
) - 8*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)
**3*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)*
*5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2
*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2
- b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c
/2)**2*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan
(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c
```

```
/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/
2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))
*tan(c/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5
*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*t
an(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
b) + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2
)*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5
*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*t
an(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 -
b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(b*x/2
)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x
/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2
)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log
(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**6*ta
n(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*ta
n(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 +
 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) + tan(c/2)
/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**6*tan(b*x/2)**2/(b*tan(c/2)**
6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)*
*2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log(tan(b*x/2) + tan(
c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**6/(b*tan(c/2)**6*tan(b*x/
2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/
2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*t
an(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*log(tan(b*x/2) + tan(c/2)/(ta
n(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**5*tan(b*x/2)/(b*tan(c/2)**6*tan(b
*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b
*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*
b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) + tan(c/2)/
(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**4*tan(b*x/2)**2/(b*tan(c/2)**6
*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4
*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**
2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) + tan
(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**4/(b*tan(c/2)**6*tan(b*x
/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x
/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*
tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 8*log(tan(b*x/2) + tan(c/2)/(t
an(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**6*tan(
b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(
b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4
*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) + tan(c/2)
/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2*tan(b*x/2)**2/(b*tan(c/2)**
6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**
4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)*
*2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) + ta
n(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2/(b*tan(c/2)**6*tan(b*
x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*
x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b
*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*log(tan(b*x/2) + tan(c/2)/(
tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)/(b*tan(c/2)**6*tan(b*
x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*
x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b
*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - log(tan(b*x/2) + tan(c/2)/(ta
n(c/2) + 1) - 1/(tan(c/2) + 1))*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2
- b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2
+ b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2
)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) +
 1) - 1/(tan(c/2) + 1))/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*
tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*ta
```

```
n(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b
*x/2)**2 - b) + 4*tan(c/2)**5/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6
- 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4
- b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b
*tan(b*x/2)**2 - b) + 8*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2
 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2
 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/
2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) + 8*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)
**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)
**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2
)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b) - 4*tan(c/2)/(b*tan(c
/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c
/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(
c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) + b*tan(b*x/2)**2 - b), True))*cos(a) - P
iecewise((log(tan(b*x/2))/b, Eq(c, pi/2)), (0, Eq(b, 0)), (log(tan(b*x/2))/
b, Eq(c, -pi/2)), (-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2
) - 1))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)
**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**
2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan
(c/2)**3/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan
(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2)
 - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*ta
n(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*
tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(t
an(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(
b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b
*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 -
4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b)
 + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2
)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(ta
n(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan
(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)/(tan(
c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x
/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/
2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/
(tan(c/2) + 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan
(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x
/2)**2 + b) + 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1)
)*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*t
an(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*tan(c/2)**4/
(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) -
 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)**3*tan(b*x/2)/
(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) -
 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)*tan(b*x/2)/(b*
tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*
b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b), True))*sin(a)
```

3.243 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

```
[Out] -1/2*sec(b*x+c)^2*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4583, 2606, 30, 3767, 8}

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

```
[Out] -(Sec[c + b*x]^2*Sin[a - c])/(2*b) + (Cos[a - c]*Tan[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec^3(c+bx)\,dx &= \cos(a-c)\int \sec^2(c+bx)\,dx-\sin(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx\\ &= -\frac{\cos(a-c)\,\text{Subst}(\int 1\,dx,x,-\tan(c+bx))}{b}-\frac{\sin(a-c)\,\text{Subst}(\int x\,dx,x,\sec(}{b}\\ &= -\frac{\sec^2(c+bx)\sin(a-c)}{2b}+\frac{\cos(a-c)\tan(c+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 35, normalized size = 0.92

$$-\frac{\sec(c)\sec^2(bx+c)(\sin(a)-\cos(a-c)\sin(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
[Out] -1/2*(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/b
```

fricas [A] time = 0.51, size = 40, normalized size = 1.05

$$\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)+\sin(-a+c)}{2\,b\cos(bx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c)
^2)
```

giac [B] time = 5.75, size = 315, normalized size = 8.29

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}a\right)^4}{4\left(2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
[Out] -1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/
2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1
)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*b)
```

maple [A] time = 3.67, size = 56, normalized size = 1.47

$$\frac{1}{2b(\cos(a)\sin(c)-\sin(a)\cos(c))(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^3,x)
[Out] 1/2/b/(cos(a)*sin(c)-sin(a)*cos(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*s
in(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^2
```

maxima [B] time = 0.33, size = 382, normalized size = 10.05

$$-\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2a)+\sin(}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) + (s
in(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos
(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(2*co
s(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (cos(2*a
) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*
x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(
a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*
c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*
b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a
+ 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.244 $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\,\mathrm{si}}{}$$

```
[Out] 1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3
/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1/1
6*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

```
[Out] Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) + (
3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{16}\cos(2a-3c+(2b-3d)x)+\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos(c+dx)\right.\\ &= \frac{1}{16}\int \cos(2a-3c+(2b-3d)x)\,dx+\frac{1}{16}\int \cos(2a+3c+(2b+3d)x)\,dx+\frac{1}{8}\\ &= \frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3}{2}\end{aligned}$$

Mathematica [A] time = 1.57, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{3\sin(2a+2bx-3c-3dx)}{2b-3d}+\frac{9\sin(2a+2bx-c-dx)}{2b-d}+\frac{9\sin(2a+2bx+c+dx)}{2b+d}+\frac{3\sin(2a+2bx+3c+3}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d + (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
```

```
*d) + (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) + (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

fricas [A] time = 0.63, size = 163, normalized size = 1.13

$$-\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+a\right)\right.}{3\left(16\,b^4d-40\,b^2d^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)
*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 + 16*b^4 - 40*b^2*d^
2 + (8*b^4 - 2*b^2*d^2 - 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(d*x + c)^2
)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

giac [A] time = 4.94, size = 129, normalized size = 0.90

$$\frac{\sin\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\,(2\,b+3\,d)}+\frac{3\,\sin\left(2\,bx+dx+2\,a+c\right)}{16\,(2\,b+d)}+\frac{3\,\sin\left(2\,bx-dx+2\,a-c\right)}{16\,(2\,b-d)}+\frac{\sin\left(2\,bx-3\,dx+2\,a-\right.}{16\,(2\,b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*sin(2*b*x + d*x + 2*
a + c)/(2*b + d) + 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*sin(2*b
*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*x
 + c)/d
```

maple [A] time = 1.29, size = 133, normalized size = 0.92

$$\frac{\sin\left(2a-3c+(2b-3d)\,x\right)}{32b-48d}+\frac{3\sin\left(2a-c+(2b-d)\,x\right)}{16\,(2b-d)}+\frac{3\sin\left(dx+c\right)}{8d}+\frac{\sin\left(3dx+3c\right)}{24d}+\frac{3\sin\left(2a+c+(2b+\right.}{16\,(2b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^3,x)
```

```
[Out] 1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3
/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1/1
6*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [B] time = 0.43, size = 1362, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4
*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^
2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) +
9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
 - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*
b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 1
8*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(
```

```
-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2
*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) + 2*(16*b^4*sin(3*
c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*si
n(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) - 18*(16*b^4*
sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*
d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b
*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3
*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d
)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(
3*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
 - 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12
*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c)
- 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) - 2*(16*b^4*cos(3*c) - 40*b^2*d^2*
cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) - 18*(16*b^4*cos(3*c) - 40*b^2*
d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) - 18*(16*b^4*cos(3*c) - 40*b^
2*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

mupad [B] time = 2.44, size = 495, normalized size = 3.44

$$-\mathrm{e}^{a\,2\mathrm{i}-c\,1\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}-\frac{3\,d\,\left(2\,b+d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}+\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}\right)+\mathrm{e}^{a\,2\mathrm{i}+c\,1\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cos(c + d*x)^3,x)
```

```
[Out] exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*d*(2*b - d))/(b^2*d*128i - d^3*32i)
+ (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*32i) - (3*d*exp
(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i + b*
x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*3
2i) - (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) + (3*d*exp(- a*4i - b*x*4i)*(2
*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)*((exp
(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b + 3
*d))/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d))/(b^2*
d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((3*d*(2*b - 3*d))
/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*3
84i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*384i - d^3*
864i))
```

sympy [A] time = 114.29, size = 2009, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cos(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x/
2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*sin(a - 3*d*x/2)**2*cos(c + d*x)*
*3/16 + x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 - 3*x*sin(a -
 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x
)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 + x*cos(a - 3*d*x/2)**2*cos(c + d*
x)**3/16 + 11*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a - 3*d*x/2)
```

```
**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos
(a - 3*d*x/2)*cos(c + d*x)/(4*d) - 5*sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(
c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a - 3*d*x/2)**2/(16*d), Eq(b, -3*
d/2)), (-3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - 3*x*sin(a
- d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**3*cos(a -
 d*x/2)/8 - 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/
8 + 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 + 3*x*cos(a - d*x
/2)**2*cos(c + d*x)**3/16 + 49*sin(a - d*x/2)**2*sin(c + d*x)**3/(48*d) + s
in(a - d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - 7*sin(a - d*x/2)*sin(c +
d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) - 13*sin(a - d*x/2)*cos(a - d*x/2
)*cos(c + d*x)**3/(8*d) - 17*sin(c + d*x)**3*cos(a - d*x/2)**2/(48*d), Eq(b
, -d/2)), (-3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - 3*x*sin
(a + d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a + d*x/2)*sin(c + d*x)**3*cos(
a + d*x/2)/8 + 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*cos(c + d*x)*
*2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x/2)**2*cos(c + d*x)/16 + 3*x*cos(a +
d*x/2)**2*cos(c + d*x)**3/16 + 49*sin(a + d*x/2)**2*sin(c + d*x)**3/(48*d)
+ sin(a + d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 7*sin(a + d*x/2)*sin(c
 + d*x)**2*cos(a + d*x/2)*cos(c + d*x)/(4*d) + 13*sin(a + d*x/2)*cos(a + d*
x/2)*cos(c + d*x)**3/(8*d) - 17*sin(c + d*x)**3*cos(a + d*x/2)**2/(48*d), E
q(b, d/2)), (3*x*sin(a + 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*si
n(a + 3*d*x/2)**2*cos(c + d*x)**3/16 - x*sin(a + 3*d*x/2)*sin(c + d*x)**3*c
os(a + 3*d*x/2)/8 + 3*x*sin(a + 3*d*x/2)*sin(c + d*x)*cos(a + 3*d*x/2)*cos(
c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a + 3*d*x/2)**2*cos(c + d*x)/16 + x
*cos(a + 3*d*x/2)**2*cos(c + d*x)**3/16 + 11*sin(a + 3*d*x/2)**2*sin(c + d*
x)**3/(48*d) + sin(a + 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d - 3*sin(a
 + 3*d*x/2)*sin(c + d*x)**2*cos(a + 3*d*x/2)*cos(c + d*x)/(4*d) + 5*sin(a +
 3*d*x/2)*cos(a + 3*d*x/2)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a
+ 3*d*x/2)**2/(16*d), Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)
**2/2 + sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c)**3, Eq(d, 0)), (16*b**4*sin
(a + b*x)**2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**
4*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 +
 27*d**5) + 16*b**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d
**3 + 27*d**5) + 24*b**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b
**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**3*d*sin(a + b*x)*cos(a + b*x)*cos(
c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sin(a + b*
x)**2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b**2*d**2*
sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) - 40*b**2*d**2*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) - 78*b**2*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)*
*2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 36*b*d**3*sin(a + b*x)*sin(c + d
*x)**2*cos(a + b*x)*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42
*b*d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**
3 + 27*d**5) + 18*d**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) + 27*d**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(4
8*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

3.245 $\int \cos^2(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] 1/4*x+1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x+
2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cos^2(c+dx)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{8}\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx+\frac{1}{4}\\ &= \frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)}{16(b}\end{aligned}$$

Mathematica [A] time = 0.72, size = 105, normalized size = 1.19

$$\frac{2d\left(b^2-d^2\right)\sin(2(a+bx))+bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(d(\sin(2(a+x(b+d)+c))+4x(b+d))}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x)]
+ b*(b - d)*(2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c + (
b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

fricas [A] time = 0.44, size = 105, normalized size = 1.19

$$-\frac{\left(2\,bd^2\cos\left(bx+a\right)^2-b^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-d^3\cos\right.}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 - b^3)*cos(d*x + c)*sin(d*x + c) - (b^3*d - b
*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))*sin(b*x
+ a))/(b^3*d - b*d^3)
```

giac [A] time = 1.99, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x+\frac{\sin\left(2\,bx+2\,dx+2\,a+2\,c\right)}{16\,(b+d)}+\frac{\sin\left(2\,bx-2\,dx+2\,a-2\,c\right)}{16\,(b-d)}+\frac{\sin\left(2\,bx+2\,a\right)}{8\,b}+\frac{\sin\left(2\,dx+2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) + 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

maple [A] time = 1.69, size = 83, normalized size = 0.94

$$\frac{x}{4}+\frac{\sin\left(2bx+2a\right)}{8b}+\frac{\sin\left(2dx+2c\right)}{8d}+\frac{\sin\left(\left(2b-2d\right)x+2a-2c\right)}{16b-16d}+\frac{\sin\left(\left(2b+2d\right)x+2a+2c\right)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^2,x)
```

```
[Out] 1/4*x+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

maxima [B] time = 0.37, size = 620, normalized size = 7.05

$$\frac{8\left(\left(b\cos\left(2\,c\right)^2+b\sin\left(2\,c\right)^2\right)d^3-\left(b^3\cos\left(2\,c\right)^2+b^3\sin\left(2\,c\right)^2\right)d\right)x+\left(b^2d\sin\left(2\,c\right)-bd^2\sin\left(2\,c\right)\right)\cos\left(2\left(b+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) - 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) - 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

mupad [B] time = 1.06, size = 177, normalized size = 2.01

$$\frac{2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cos(c + d*x)^2,x)
```

```
[Out] (2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin(2*a + 2*b*x) + b*d^2*sin(2*a - 2*c + 2*
b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c +
 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a +
2*b*x) - 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2 - d
^2))
```

sympy [A] time = 23.83, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 + 3*s
in(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a - d*x)*cos(a - d*x)*
cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
3*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

3.246 $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin}{}$$

```
[Out] 3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*a-
3*c+3*(b-d)*x)/(b-d)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sin(a+c+(b+d)*x
)/(b+d)+1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+d)
+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] (3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])/
(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c +
 (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d)) + S
in[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/(
32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(a-c)\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx+\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx+\frac{3}{3}\\ &= \frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)}{96(b-d)}\end{aligned}$$

Mathematica [A] time = 1.64, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(\frac{9\sin(a+bx-3c-3dx)}{b-3d}+\frac{27\sin(a+bx-c-dx)}{b-d}+\frac{\sin(3(a+bx-c-dx))}{b-d}+\frac{9\sin(3a+3bx-c-dx)}{3b-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] ((9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
- d) + Sin[3*(a - c + b*x - d*x)]/(b - d) + (9*Sin[3*a - c + 3*b*x - d*x])/
(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*x
 + 3*d*x])/(b + 3*d) + (27*Sin[a + c + (b + d)*x])/(b + d) + Sin[3*(a + c +
 (b + d)*x)]/(b + d))/96
```

fricas [A] time = 0.48, size = 240, normalized size = 1.23

$$\frac{\left(\left(18\,b^5-2\,b^3d^2+\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-6\left(20\,b^3d^2+\left(b^3d^2-9\,bd^4\right)\cos\left(bx+a\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((18*b^5 - 2*b^3*d^2 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^2)*
cos(d*x + c)^3 - 6*(20*b^3*d^2 + (b^3*d^2 - 9*b*d^4)*cos(b*x + a)^2)*cos(d*
x + c))*sin(b*x + a) + (120*b^2*d^3*cos(b*x + a) + 2*(b^2*d^3 - 9*d^5)*cos(
b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^3 + 6*(9*b^4*d -
b^2*d^3)*cos(b*x + a))*cos(d*x + c)^2)*sin(d*x + c))/(9*b^6 - 91*b^4*d^2 +
91*b^2*d^4 - 9*d^6)
```

giac [A] time = 0.17, size = 181, normalized size = 0.93

$$\frac{\sin\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\,(b+d)}+\frac{3\,\sin\left(3\,bx+dx+3\,a+c\right)}{32\,(3\,b+d)}+\frac{3\,\sin\left(3\,bx-dx+3\,a-c\right)}{32\,(3\,b-d)}+\frac{\sin\left(3\,bx-3\,dx+3\,a-3\,c\right)}{96\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9
/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) +
3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

maple [A] time = 2.00, size = 184, normalized size = 0.94

$$\frac{3\sin\left(a-3c+\left(b-3d\right)x\right)}{32\left(b-3d\right)}+\frac{9\sin\left(a-c+\left(b-d\right)x\right)}{32\left(b-d\right)}+\frac{9\sin\left(a+c+\left(b+d\right)x\right)}{32\left(b+d\right)}+\frac{3\sin\left(a+3c+\left(b+3d\right)x\right)}{32\left(b+3d\right)}+\frac{\sin((3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*cos(d*x+c)^3,x)
```

```
[Out] 3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+9/32*sin(a+c+
(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*x
+3*a-3*c)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+
d)+1/96/(b+d)*sin((3*b+3*d)*x+3*a+3*c)
```

maxima [B] time = 0.58, size = 2614, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
```

```
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
 + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) + (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) - (9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) + 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
*cos(-3*(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
 3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
 + 4*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) - (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
```

```
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

mupad [B] time = 4.54, size = 999, normalized size = 5.12

$$-e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{b^4\,576i-b^2\,d^2\,640i+d^4\,64i}-\frac{e^{-a\,6i-b\,x\,6i}\,\left(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3\right)}{b^4\,576i-b^2\,d^2\,640i+d^4\,64i}+\frac{e^{-a\,2i-b\,x\,2i}\,\left(-81\right.}{b^4\,576i\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*cos(c + d*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(
b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2
*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*2i - b*
x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*
640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^4*5
76i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d
^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(-
a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*64i - b
^2*d^2*640i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))
/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*
b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i -
c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192i + d^4*17
28i - b^2*d^2*1920i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3
))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*2i - b*x*2i)*(9*b*d^2
- 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp
(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i + d^4*17
28i - b^2*d^2*1920i)) - exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*
d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*6i - b*x
*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i
) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i
+ d^4*1728i - b^2*d^2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 27*b^2*d -
9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*cos(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.247 $\int \cos(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\cos(a+bx)}{b}$$

```
[Out] cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+3/2*arctanh(sin(b*x+c))*sin(a-c)/b-1/2*s
ec(b*x+c)*sin(a-c)*tan(b*x+c)/b
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4579, 4576, 2638, 3770, 2606, 8, 2611}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (3*ArcTanh[Sin[c + b*x]]*Sin
[a - c])/(2*b) - (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = -\left(\sin(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx\right)+\int \sin(a+bx)\tan^2(c+bx)\,dx$$
$$= -\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}+\cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx+$$
$$= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}+\frac{\cos(a-c)}{}$$
$$= \frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c}{}$$

Mathematica [A] time = 0.37, size = 70, normalized size = 0.97

$$\frac{\sec^2(bx+c)(2\cos(a-bx-2c)+\cos(a+3bx+2c)+5\cos(a+bx))+12\sin(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\sin\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^3,x]
[Out] ((2*Cos[a - 2*c - b*x] + 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Sec[c + b*x
]^2 + 12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c])/(4*b)
```

fricas [B] time = 0.49, size = 366, normalized size = 5.08

$$\frac{16\cos(bx+a)^3\cos(-2a+2c)-4\left(4\cos(bx+a)^2+1\right)\sin(bx+a)\sin(-2a+2c)-4(\cos(-2a+2c)-5)\cos}{8\left(2b\cos(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
[Out] 1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x +
 a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) - 5)*cos(b*x + a) + 3*sqrt(2)*(2*(
cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*cos(b*x + a)^2*cos(-2
*a + 2*c) - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*log(-(2*cos(b*x + a)^2*co
s(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((c
os(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-
2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) -
2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(co
s(-2*a + 2*c) + 1))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*
sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\tan(bx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^3, x)
```

maple [C] time = 0.96, size = 181, normalized size = 2.51

$$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+3\,e^{i(bx+3a+2c)}}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2}-\frac{3\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{2b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^3,x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/2/b/(exp(2*I*(b*x+a+c))+exp(2*
I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))+3
*exp(I*(b*x+3*a+2*c)))-3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)+3/2
*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.54, size = 1027, normalized size = 14.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) + 2*co
s(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) + 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(3
*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a
 + 2*c) + cos(b*x + a))*cos(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) + 5*cos(2*
b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) + 10
*cos(2*b*x + 2*c)*cos(b*x + a) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*
cos(3*b*x + a + 2*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*si
n(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c)
 + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) + 4*sin(3*b*x + a + 2*c)*sin(b*x +
a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) + 2*(2*cos(3*b*x + a + 2*c)*sin
(-a + c) + cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*sin(3*b*x
+ a + 2*c)*sin(-a + c) + sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*lo
g((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos
(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)
) + 2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*
b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) + 2*sin
(2*b*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
+ a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2*
c) + sin(b*x + a))*sin(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x +
 2*c))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) + 10*sin(2*b*
x + 2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos
(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a
)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*x
 + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c) +
 b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*sin
(b*x + a))*sin(5*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c)**3, x)
```

3.248 $\int \cos(a+bx)\tan^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4579, 4576, 2637, 3770, 2606, 8}

$$-\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
+ b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = -(\sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx) + \int \sin(a+bx)\tan(c+bx)\,dx$$
$$= \cos(a-c)\int \sec(c+bx)\,dx - \frac{\sin(a-c)\,\text{Subst}(\int 1\,dx, x, \sec(c+bx))}{b} - \int \cos($$
$$= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [C] time = 0.09, size = 111, normalized size = 2.41

$$-\frac{\sin(a-c)\sec(bx+c)}{b} - \frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\cos(bx)}{b} - \frac{\cos(a)\sin(b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Sec[c + b*x]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

fricas [B] time = 0.51, size = 316, normalized size = 6.87

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + \frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\sin(-2\,a+2\,c)-\left(\cos(-2\,a+2\,c)^2+2\,\cos(-2\,a+2\,c)+1\right.\right.}{}}{4\left(b\sin\left(bx+a\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*cos(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(
b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2
*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x +
 a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*(
cos(b*x + a)^2 - 2)*sin(-2*a + 2*c))/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\tan\left(bx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^2, x)
```

maple [C] time = 0.86, size = 149, normalized size = 3.24

$$\frac{i\mathrm{e}^{i(bx+a)}}{2b} - \frac{i\mathrm{e}^{-i(bx+a)}}{2b} - \frac{i\left(-\mathrm{e}^{i(bx+3a)}+\mathrm{e}^{i(bx+a+2c)}\right)}{b\left(\mathrm{e}^{2i(bx+a+c)}+\mathrm{e}^{2ia}\right)} + \frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(a-c)}\right)\cos\left(a-c\right)}{b} - \frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\mathrm{e}^{i(a-c)}\right)\cos\left(a-c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^2,x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))-I/b/(exp(2*I*(b*x+a+c))+exp(
2*I*a))*(-exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))/b*cos(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.52, size = 526, normalized size = 11.43

$$\frac{(\sin(3bx+a+2c)+\sin(bx+a))\cos(4bx+2a+2c)-3(\sin(2bx+2a)-\sin(2bx+2c))\cos(3bx+a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 3*(sin(
2*b*x + 2*a) - sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*cos(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*
x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 2*cos(-a + c)*s
in(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log((cos(b*x
 + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b
*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*
x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - (cos(3
*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + (3*cos(2*b*x + 2*a
) - 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)*sin(2*b*x
 + 2*a) + 3*cos(b*x + a)*sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a)
 - 3*cos(2*b*x + 2*c)*sin(b*x + a) - sin(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

mupad [B] time = 5.24, size = 285, normalized size = 6.20

$$-\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right.}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right.}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i
 + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x
*2i)*1i)) + (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(
a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2
))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*e
xp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp
(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b
*exp(a*2i - c*2i)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c)**2, x)
```

3.249 $\int \cos(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b-arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4579, 2638, 3770}

$$-\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan(c+bx)\,dx &= -(\sin(a-c)\int \sec(c+bx)\,dx)+\int \sin(a+bx)\,dx\\ &= -\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.05, size = 93, normalized size = 3.10

$$\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sin(a)\sin(bx)}{b}-\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*S
in[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])
]*Sin[a - c])/b + (Sin[a]*Sin[b*x])/b
```

fricas [B] time = 0.47, size = 196, normalized size = 6.53

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\frac{2\,\sqrt{2}\left((\cos(-2\,a+2\,c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2\,a+2\,c)\right)}{\sqrt{\cos(-2\,a+2\,c)+1}}-\cos(-2\,a+2\,c)-3}{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\cos(-2\,a+2\,c)+1}\right)\sin(-2\,a+2\,c)}{\sqrt{\cos(-2\,a+2\,c)+1}} -}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) + 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) - 4
*cos(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c), x)
```

maple [C] time = 0.61, size = 97, normalized size = 3.23

$$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c),x)
```

```
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*sin(a-c)+ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.51, size = 131, normalized size = 4.37

$$-\frac{\log\left(\frac{\cos(bx+2\,c)^2+\cos(c)^2-2\,\cos(c)\sin(bx+2\,c)+\sin(bx+2\,c)^2+2\,\cos(bx+2\,c)\sin(c)+\sin(c)^2}{\cos(bx+2\,c)^2+\cos(c)^2+2\,\cos(c)\sin(bx+2\,c)+\sin(bx+2\,c)^2-2\,\cos(bx+2\,c)\sin(c)+\sin(c)^2}\right)\sin(-a+c)+2\,\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2
 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2))*sin(-a + c) + 2*cos(b*x + a))/b
```

mupad [B] time = 4.73, size = 237, normalized size = 7.90

$$-\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x),x)
```

```
[Out] (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)*ex
p(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp
(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - exp(a*1i + b*x*1i)/(2
*b) - exp(- a*1i - b*x*1i)/(2*b) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*ex
p(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(ex
p(a*2i)*exp(-c*2i)*1i - 1i))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i
))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c), x)
```

3.250 $\int \cos(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4577, 2638, 3770}

$$\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot(c+bx)\,dx &= \cos(a-c)\int \csc(c+bx)\,dx-\int \sin(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 3.24

$$-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Sin[a]*Sin[b*x])/b
```

fricas [B] time = 0.46, size = 190, normalized size = 6.55

$$\frac{\sqrt{2}\,\sqrt{\cos\left(-2\,a+2\,c\right)+1}\,\log\left(-\frac{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\frac{2\,\sqrt{2}\,((\cos(-2\,a+2\,c)+1)\cos(bx+a)-\sin(bx+a)\sin}{\sqrt{\cos(-2\,a+2\,c)+1}}}{2\,\cos(bx+a)^2\cos(-2\,a+2\,c)-2\,\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\cos(-2\,a+2\,c)-}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2
*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c)
+ 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
 a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) + 4*cos(b*x + a))/
b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((-tan(a/2)^2*tan(c/2)^2+tan(a/2)^2-4*tan(a/2)*tan(c/2)+tan(c/2
)^2-1)/(-2*tan(a/2)^2*tan(c/2)^2-2*tan(a/2)^2-2*tan(c/2)^2-2)*ln(abs(tan(b*
x/2)+tan(c/2)))+(-tan(a/2)^2*tan(c/2)^3+tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(
c/2)^2+tan(c/2)^3-tan(c/2))/(2*tan(a/2)^2*tan(c/2)^3+2*tan(a/2)^2*tan(c/2)+
2*tan(c/2)^3+2*tan(c/2))*ln(abs(tan(b*x/2)*tan(c/2)-1))+(-2*tan(b*x/2)*tan(
a/2)-tan(a/2)^2+1)/(tan(a/2)^2+1)/(tan(b*x/2)^2+1))
```

maple [C] time = 0.75, size = 93, normalized size = 3.21

$$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos\left(a-c\right)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos\left(a-c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c),x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a-c)))/
b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.35, size = 105, normalized size = 3.62

$$-\frac{\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos(c)+\cos(c)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin(c)+\sin(c)^2\right)-\cos\left(-a+c\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^
2 - 2*sin(b*x)*sin(c) + sin(c)^2) - cos(-a + c)*log(cos(b*x)^2 - 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*cos(b*x
 + a))/b
```

mupad [B] time = 5.39, size = 231, normalized size = 7.97

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x),x)
```

```
[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) - (log(- exp(a*1i)*ex
p(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)
*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/
(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c
*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2
i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a+bx\right)\cot\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c), x)
```

3.251 $\int \cos(a+bx)\cot^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4577, 4578, 2637, 3770, 2606, 8}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Sin
[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \int \cot(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(a-c)\,\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} - \sin(a-c)\int \csc(c+bx)\,dx - \int c$$
$$= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [C] time = 0.10, size = 112, normalized size = 2.43

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sin(a)\cos(bx)}{b}-\frac{\cos(a)\sin}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) - (Cos[b*x]*Sin[a])/b + ((2*I)*ArcTan[((Cos[
c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(
b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

fricas [B] time = 0.49, size = 316, normalized size = 6.87

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^2-4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\,\mathrm{si}\right.}{}}{4\left(b\cos\left(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*s
in(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c
) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a +
 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a
+ 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*
c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a +
 2*c) + 1) - 8*cos(-2*a + 2*c) - 8)/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*co
s(-2*a + 2*c) + b)*sin(b*x + a))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((tan(c/2)^2*tan(a/2)-tan(c/2)*tan(a/2)^2+tan(c/2)-tan(a/2))/(t
an(c/2)^2*tan(a/2)^2+tan(c/2)^2+tan(a/2)^2+1)*ln(abs(tan(b*x/2)+tan(c/2)))+
(tan(c/2)^3*tan(a/2)-tan(c/2)^2*tan(a/2)^2+tan(c/2)^2-tan(c/2)*tan(a/2))/(-
tan(c/2)^3*tan(a/2)^2-tan(c/2)^3-tan(c/2)*tan(a/2)^2-tan(c/2))*ln(abs(tan(b
*x/2)*tan(c/2)-1))+(tan(b*x/2)^3*tan(c/2)^4*tan(a/2)^2-tan(b*x/2)^3*tan(c/2
```

```
)^4+4*tan(b*x/2)^3*tan(c/2)^3*tan(a/2)-6*tan(b*x/2)^3*tan(c/2)^2*tan(a/2)^2
+6*tan(b*x/2)^3*tan(c/2)^2-4*tan(b*x/2)^3*tan(c/2)*tan(a/2)+tan(b*x/2)^3*ta
n(a/2)^2-tan(b*x/2)^3-6*tan(b*x/2)^2*tan(c/2)^3*tan(a/2)^2+6*tan(b*x/2)^2*t
an(c/2)^3+6*tan(b*x/2)^2*tan(c/2)*tan(a/2)^2-6*tan(b*x/2)^2*tan(c/2)+tan(b*
x/2)*tan(c/2)^4*tan(a/2)^2-tan(b*x/2)*tan(c/2)^4+12*tan(b*x/2)*tan(c/2)^3*t
an(a/2)+2*tan(b*x/2)*tan(c/2)^2*tan(a/2)^2-2*tan(b*x/2)*tan(c/2)^2-12*tan(b
*x/2)*tan(c/2)*tan(a/2)+tan(b*x/2)*tan(a/2)^2-tan(b*x/2)-2*tan(c/2)^3*tan(a
/2)^2+2*tan(c/2)^3-16*tan(c/2)^2*tan(a/2)+2*tan(c/2)*tan(a/2)^2-2*tan(c/2))
/(-4*tan(c/2)*tan(a/2)^2-4*tan(c/2))/(tan(b*x/2)^4*tan(c/2)+tan(b*x/2)^3*ta
n(c/2)^2-tan(b*x/2)^3+tan(b*x/2)*tan(c/2)^2-tan(b*x/2)-tan(c/2)))
```

maple [C] time = 0.84, size = 145, normalized size = 3.15

$$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}+\frac{i\left(e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^2,x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+I/b/(-exp(2*I*(b*x+a+c))+exp
(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+exp(I*(a-c
)))/b*sin(a-c)-ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

maxima [B] time = 0.36, size = 613, normalized size = 13.33

$$\frac{(\sin(3bx+a+2c)-\sin(bx+a))\cos(4bx+2a+2c)+3(\sin(2bx+2a)+\sin(2bx+2c))\cos(3bx+a+2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 3*(sin(
2*b*x + 2*a) + sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*
x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(3*b*x + a
 + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log(cos(b*x)
^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)
^2) + (cos(3*b*x + a + 2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x
+ a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(
-a + c) - 2*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*
sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(c) + sin(c)^2) - (cos(3*b*x + a + 2*c) - cos(b*x + a))*sin(4*
b*x + 2*a + 2*c) - (3*cos(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x
+ a + 2*c) - 3*cos(b*x + a)*sin(2*b*x + 2*a) - 3*cos(b*x + a)*sin(2*b*x + 2
*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a) + 3*cos(2*b*x + 2*c)*sin(b*x + a) - s
in(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

mupad [B] time = 5.44, size = 289, normalized size = 6.28

$$-\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i
 + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x
*2i)*1i)) - (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2)
)*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log(exp(a*1i)*
exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*e
xp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(
2*b*(-exp(a*2i - c*2i))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c)**2, x)
```

3.252 $\int \cos(a+bx)\cot^3(c+bx)\,dx$

Optimal. Leaf size=73

$$\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\cos(a+bx)}{b}$$

```
[Out] 3/2*arctanh(cos(b*x+c))*cos(a-c)/b-cos(b*x+a)/b-1/2*cos(a-c)*cot(b*x+c)*csc
(b*x+c)/b+csc(b*x+c)*sin(a-c)/b
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4577, 4578, 2638, 3770, 2606, 8, 2611}

$$\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - Cos[a + b*x]/b - (Cos[a - c]*C
ot[c + b*x]*Csc[c + b*x])/(2*b) + (Csc[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \cos(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx - \int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \sin(a-$$
$$= \frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \cos(a-$$
$$= \frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\cot(c+bx)\csc(c}{2b}$$

Mathematica [A] time = 0.34, size = 71, normalized size = 0.97

$$\frac{\csc^2(bx+c)(2\cos(a-bx-2c)+\cos(a+3bx+2c)-5\cos(a+bx))+12\cos(a-c)\tanh^{-1}\left(\cos(c)-\sin(c)\,\text{ta}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + (2*Cos[a - 2*c - b*x
] - 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Csc[c + b*x]^2)/(4*b)
```

fricas [B] time = 0.51, size = 385, normalized size = 5.27

$$-\frac{16\cos(bx+a)^3\cos(-2a+2c) - 4\left(4\cos(bx+a)^2+1\right)\sin(bx+a)\sin(-2a+2c) - 4(\cos(-2a+2c)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
+ a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) + 5)*cos(b*x + a) + 3*sqrt(2)*(2*
(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(cos(-2
*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + 2*cos(-
2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) -
sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) +
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1))/(2*b*cos(b*x +
a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*co
s(-2*a + 2*c) - b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/b*((2*tan(b*x/2)*tan(a/2)+tan(a/2)^2-1)/(tan(a/2)^2+1)/(tan(b*x/2
)^2+1)+(-2*tan(b*x/2)^3*tan(c/2)^7*tan(a/2)^2+2*tan(b*x/2)^3*tan(c/2)^7-6*t
an(b*x/2)^3*tan(c/2)^5*tan(a/2)^2+6*tan(b*x/2)^3*tan(c/2)^5-16*tan(b*x/2)^3
*tan(c/2)^4*tan(a/2)+6*tan(b*x/2)^3*tan(c/2)^3*tan(a/2)^2-6*tan(b*x/2)^3*ta
n(c/2)^3+2*tan(b*x/2)^3*tan(c/2)*tan(a/2)^2-2*tan(b*x/2)^3*tan(c/2)-tan(b*x
/2)^2*tan(c/2)^8*tan(a/2)^2+tan(b*x/2)^2*tan(c/2)^8+4*tan(b*x/2)^2*tan(c/2)
^7*tan(a/2)-2*tan(b*x/2)^2*tan(c/2)^6*tan(a/2)^2+2*tan(b*x/2)^2*tan(c/2)^6-
20*tan(b*x/2)^2*tan(c/2)^5*tan(a/2)+22*tan(b*x/2)^2*tan(c/2)^4*tan(a/2)^2-2
2*tan(b*x/2)^2*tan(c/2)^4+20*tan(b*x/2)^2*tan(c/2)^3*tan(a/2)-2*tan(b*x/2)^
2*tan(c/2)^2*tan(a/2)^2+2*tan(b*x/2)^2*tan(c/2)^2-4*tan(b*x/2)^2*tan(c/2)*t
an(a/2)-tan(b*x/2)^2*tan(a/2)^2+tan(b*x/2)^2+2*tan(b*x/2)*tan(c/2)^7*tan(a/
2)^2-2*tan(b*x/2)*tan(c/2)^7-16*tan(b*x/2)*tan(c/2)^6*tan(a/2)+14*tan(b*x/2
)*tan(c/2)^5*tan(a/2)^2-14*tan(b*x/2)*tan(c/2)^5+16*tan(b*x/2)*tan(c/2)^4*t
an(a/2)-14*tan(b*x/2)*tan(c/2)^3*tan(a/2)^2+14*tan(b*x/2)*tan(c/2)^3-16*tan
(b*x/2)*tan(c/2)^2*tan(a/2)-2*tan(b*x/2)*tan(c/2)*tan(a/2)^2+2*tan(b*x/2)*t
an(c/2)-2*tan(c/2)^6*tan(a/2)^2+2*tan(c/2)^6+8*tan(c/2)^5*tan(a/2)-12*tan(c
/2)^4*tan(a/2)^2+12*tan(c/2)^4-8*tan(c/2)^3*tan(a/2)-2*tan(c/2)^2*tan(a/2)^
2+2*tan(c/2)^2)/(-16*tan(c/2)^2*tan(a/2)^2-16*tan(c/2)^2)/(tan(b*x/2)^2*tan
(c/2)+tan(b*x/2)*tan(c/2)^2-tan(b*x/2)-tan(c/2))^2+(3*tan(c/2)^2*tan(a/2)^2
-3*tan(c/2)^2+12*tan(c/2)*tan(a/2)-3*tan(a/2)^2+3)/(-4*tan(c/2)^2*tan(a/2)^
2-4*tan(c/2)^2-4*tan(a/2)^2-4)*ln(abs(tan(b*x/2)+tan(c/2)))+(3*tan(c/2)^3*t
an(a/2)^2-3*tan(c/2)^3+12*tan(c/2)^2*tan(a/2)-3*tan(c/2)*tan(a/2)^2+3*tan(c
/2))/(4*tan(c/2)^3*tan(a/2)^2+4*tan(c/2)^3+4*tan(c/2)*tan(a/2)^2+4*tan(c/2)
)*ln(abs(tan(b*x/2)*tan(c/2)-1)))
```

maple [C] time = 1.01, size = 179, normalized size = 2.45

$$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{-3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3\,e^{i(bx+3a+2c)}}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos\left(a-c\right)}{2b}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^3,x)
```

```
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-1/2/b/(-exp(2*I*(b*x+a+c))+exp(
2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a)
)-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)-3/
2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

maxima [B] time = 0.40, size = 1254, normalized size = 17.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) - 2*c
os(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) - 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(
3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x +
a + 2*c) - cos(b*x + a))*cos(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) - 5*cos(2
*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) - 1
0*cos(2*b*x + 2*c)*cos(b*x + a) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4
*cos(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*c
os(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^
2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*
c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*co
s(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a +
```

```
c)*sin(3*b*x + a + 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*l
og(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(
c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*cos(3*b*x + a +
2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(
b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2 + 4*cos(-a + c)
*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) +
 cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*cos(-a + c) - cos(b
*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a + c)*sin(3*b*x + a
+ 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) +
2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*b*x
+ 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) - 2*sin(2*b
*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x + a
+ 2*c) - sin(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a + 2*c) -
 sin(b*x + a))*sin(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c
))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) - 10*sin(2*b*x +
2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b
*x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2
+ b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
 + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*c
os(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
 + a))*sin(5*b*x + a + 4*c))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x)^3,x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**3,x)
[Out] Integral(cos(a + b*x)*cot(b*x + c)**3, x)
```

3.253 $\int \cos(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=134

$$-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

```
[Out] 1/2/b/exp(I*(b*x+a))-1/2*exp(I*(b*x+a))/b-hypergeom([1, -1/2*b/d],[1-1/2*b/
d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+exp(I*(b*x+a))*hypergeom([1, 1/2*b/d
],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

Rubi [A] time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4560, 2194, 2251}

$$-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] 1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -b/(
2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4560

```
Int[Cos[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(I/(
E^(I*(a + b*x))*2)) - (I*E^(I*(a + b*x)))/2 + I/(E^(I*(a + b*x))*(1 + E^(2*
I*(c + d*x)))) + (I*E^(I*(a + b*x)))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{
a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\tan(c+dx)\,dx &= \int \left(-\frac{1}{2}ie^{-i(a+bx)}-\frac{1}{2}ie^{i(a+bx)}+\frac{ie^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{ie^{i(a+bx)}}{1+e^{2i(c+dx)}}\right)dx\\
&= -\left(\frac{1}{2}i\int e^{-i(a+bx)}\,dx\right)-\frac{1}{2}i\int e^{i(a+bx)}\,dx+i\int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx+i\int \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\,d\\
&= \frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1\right.}{b}
\end{aligned}$$

Mathematica [A] time = 1.66, size = 114, normalized size = 0.85

$$\frac{e^{-i(a+bx)}\left(2e^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)-e^{2i(a+bx)}-2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (1 - E^((2*I)*(a + b*x)) - 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -E
^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1
 + b/(2*d), -E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(bx+a\right)\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*tan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

maple [F] time = 1.59, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*tan(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\tan\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + d*x),x)
```

```
[Out] int(cos(a + b*x)*tan(c + d*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a + bx)\tan(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x)
[Out] Integral(cos(a + b*x)*tan(c + d*x), x)
```

3.254 $\int \cos(a+bx)\cot(c+dx)\,dx$

Optimal. Leaf size=130

$$\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

```
[Out] -1/2/b/exp(I*(b*x+a))+1/2*exp(I*(b*x+a))/b+hypergeom([1, -1/2*b/d],[1-1/2*b
/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-exp(I*(b*x+a))*hypergeom([1, 1/2*b/d
],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4558, 2194, 2251}

$$\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + d*x],x]
```

```
[Out] -1/(2*b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1, -b/
(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4558

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I/(E^
(I*(a + b*x))*2) + (I*E^(I*(a + b*x)))/2 - I/(E^(I*(a + b*x))*(1 - E^(2*I*(
c + d*x)))) - (I*E^(I*(a + b*x)))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot(c+dx)\,dx &= \int \left(\frac{1}{2}ie^{-i(a+bx)}+\frac{1}{2}ie^{i(a+bx)}-\frac{ie^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{ie^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx\\ &= \frac{1}{2}i\int e^{-i(a+bx)}\,dx+\frac{1}{2}i\int e^{i(a+bx)}\,dx-i\int \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx-i\int \frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\,dx\\ &= -\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}+\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d}\right.}{}\end{aligned}$$

Mathematica [A] time = 1.79, size = 108, normalized size = 0.83

$$\frac{e^{-i(a+bx)}\left(-2e^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)+e^{2i(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + d*x],x]
[Out] (-1 + E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), E
^((2*I)*(c + d*x))] - 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1
 + b/(2*d), E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cos\left(bx+a\right)\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(d*x+c),x)
[Out] int(cos(b*x+a)*cot(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\cot\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + d*x),x)
```

```
[Out] int(cos(a + b*x)*cot(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x)
[Out] Integral(cos(a + b*x)*cot(c + d*x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```